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Robust Stabilization for Uncertain Time-Delay Systems under Time-Varying Sampling

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Abstract

This paper discusses robust sampled-data control for uncertain systems with time-varying delay. We allow a time-varying sampling.

When we use digital devices to control systems, we usually apply the zero-order control input. In this case, the closed-loop system with such a state feedback control input becomes a system with time-varying delays in state. We first give a sufficient condition for the stability of the closed-loop system with sampled-data control, in terms of linear matrix inequalities(LMIs). The key techniques to obtain such a stability condition are to employ generalized Lyapunov function and Leibniz-Newton formula. These lead to a less conservative stability condition. Based on such a stability condition, we also propose a design method of sampled-data state feedback controller for time-delay systems. Furthermore, we extend our results to a class of uncertain time-delay systems.

Mathematics Subject Classification: 93E20

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1 Introduction

The control of sampled-data systems is an important practical problem. The dynamics of the systems are naturally continuous, and control inputs are usually applied at discrete time instants. This form of the sampled-data systems arises in various applications and system formulations, such as manufacturing systems and industrial systems. Thus, the theoretical design of controllers for the sampled-data systems is essentially required in many applications. The numerous works for the sampled-data systems have ever been made and the significant results have appeared in the literature(for example, Chen and Francis [2], Sun et al. [11]). The stochastic counterpart for the sampled-data systems has also appeared in Jazwinski [9] and Yoneyama et al. [14]. Jazwinski [9] considered the stochastic filtering for the sampled-data systems and Yoneyama et al. [14] have given a design method for sampled-data control systems via jump system approach.

Astrom and Wittenmark [1] and Fridman et al. [5] introduced a delay system approach to sampled-data stabilization of linear systems. The continuous-time linear system with sampled-data control input results in the closed-loop system with time-varying state-delays. Sufficient stability conditions for such a linear state-delayed system were obtained and a design method of a stabilizing sampled-data state feedback controller was proposed. Further development has been made in Fridman and Shaked [6] and Suplin et al. [12] where H_∞ control was concerned, respectively. On the other hand, stability analysis and control design for linear time-delay systems are active [3], [4]. [10]. Recently, the same research has been carried out for time-varying delay case of time-delay systems and some techniques that reduce the conservatism in the stability conditions have employed([7], [8], [13]).

In this paper, we consider the sampled-data stabilization for systems with time-varying delay. Few works on the sampled-data stabilization problem for time-varying delayed systems have appeared in the literature, and an input delay approach to this problem is new. When we consider the delayed control to a time-delay system, the closed-loop system becomes a system with multiple time-varying delays. Free weighting matrix method and Leibniz-Newton formula are used to obtain a sufficient stability condition of the closed-loop system. It is known that those techniques reduce the conservatism in the stability condition. A design method of sampled-data state feedback stabilization of time-delay systems is proposed by delay-dependent stability conditions that are given in terms of LMIs. A numerical example is given to illustrate a design method of sampled-data state feedback stabilization controllers for time-delay systems.

2 Time-Delay Systems

Consider the following uncertain time-delay system:

$$\dot{x}(t) = \ (A+\Delta A)x(t) + (B+\Delta B)u(t) + (A_d+\Delta A_d)x(t-\tau(t)) \tag{1}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control input. The matrices A, A_d and B are constant matrices of appropriate dimensions. $\tau(t)$ is an unknown time-varying delay that satisfies $0 \leq \tau(t) \leq \tau_M$ where τ_M is known constant. The time-varying uncertainties are of the form

$$[\,\Delta A \quad \Delta A_d \quad \Delta B\,] = HF(t)\,[\,E_1 \quad E_2 \quad E_3\,]$$

where $F(t) \in \Re^{l \times j}$ is an unknown time-varying matrix satisfying $F^T(t)F(t) \leq I$ and $H,\ E_1,\ E_2$ and E_3 are known constant matrices of appropriate dimensions. We consider the sampled-data control input. It may be represented as delayed control as follows;

$$u(t) = u_d(t_k) = u_d(t-(t-t_k)) = u_d(t-h(t)),\ t_k \leq t \leq t_{k+1}$$

where u_d is a discrete-time control signal and the time varying delay $h(t) = t - t_k$ is piecewise linear with the derivative $\dot{h}(t) = 1$ for $t \neq t_k$. t_k is the sampling instant satisfying $0 < t_1 < t_2 < \cdots < t_k < \cdots$. Define the maximum sampling interval h_M such that we have $h(t) \leq t_{k+1} - t_k = h_M$ for all t_k.

Our problem is to find a sampled-data state feedback controller

$$u(t) = Kx(t_k) \tag{2}$$

where K is to be determined, which robustly stabilizes the system (1). We represent a piecewise control law as a continuous-time control with a time-varying piecewise continuous(continuous from the right) delay. Thus we look for a state feedback controller of the form

$$u(t) = Kx(t-h(t)) \tag{3}$$

Then, the closed-loop system (1) with (3) is given by

$$\begin{aligned} \dot{x}(t) = \ & (A + HF(t)E_1)x(t) + (A_d + HF(t)E_2)x(t-\tau(t)) \\ & +(B + HF(t)E_3)Kx(t-h(t)) \end{aligned} \tag{4}$$

3 Sampled-Data Stabilization

Here we consider the sampled-data stabilization of a time-delay system. We first give a sufficient condition for a nominal closed-loop system to be stable. Then we propose a design method of a sampled-data state feedback controller for a nominal system. Finally, we extend the result to a class of uncertain time-delay systems.

3.1 Stability Analysis

We make a stability analysis of nominal closed-loop system (4).

Theorem 3.1 *Given a control gain matrix* K*, the nominal time-delay system (4) with* $H = 0,\ E_i = 0,\ i = 1, 2, 3$ *is asymptotically stable if there exist* $P > 0,\ Q \geq 0,\ R \geq 0,\ Y_1 > 0,\ Y_2 > 0,\ Z_1 > 0,\ Z_2 > 0,$

$$N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix}, M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{bmatrix} L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix},$$

$$W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \end{bmatrix} \text{ and } T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

such that

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0 \tag{5}$$

where

$$\begin{aligned}
\Phi_{11} = &\ \Phi_1 + \Phi_2 + \Phi_2^T + \Phi_3 + \Phi_3^T \\
\Phi_1 = &\ \begin{bmatrix} Q+R & 0 & 0 & 0 & 0 & P \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & -R & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & \tau_M(Y_1+Y_2)+h_M(Z_1+Z_2) \end{bmatrix} \\
\Phi_2 = &\ [\, N+M+L+W \quad -N+S \quad -M-S \quad -L+V \quad -V-W \quad 0 \,] \\
\Phi_3 = &\ [\, -TA \quad -TBK \quad 0 \quad -TA_d \quad 0 \quad T \,] \\
\Phi_{12} = &\ [\, h_M N \quad h_M S \quad h_M M \quad \tau_M L \quad \tau_M V \quad \tau_M W \,] \\
\Phi_{22} = &\ diag[\, -h_M Z_1 \quad -h_M Z_1 \quad -h_M Z_2 \quad -\tau_M Y_1 \quad -\tau_M Y_1 \quad -\tau_M Y_2 \,]
\end{aligned}$$

Proof: First, it follows from the Leibniz-Newton formula that the following equations are true for any matrices $N,\ S,\ M,\ L, V$ and W:

$$2\zeta^T(t)N\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds\right] = 0 \tag{6}$$

$$2\zeta^T(t)S\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s)ds\right] = 0 \tag{7}$$

$$2\zeta^T(t)M\left[x(t) - x(t-h_M) - \int_{t-h_M}^{t} \dot{x}(s)ds\right] = 0 \tag{8}$$

$$2\zeta^T(t)L\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds\right] = 0 \tag{9}$$

$$2\zeta^T(t)V\left[x(t-\tau(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds\right] = 0 \tag{10}$$

$$2\zeta^T(t)W\left[x(t) - x(t-\tau_M) - \int_{t-\tau_M}^{t} \dot{x}(s)ds\right] = 0 \tag{11}$$

where

$$\zeta(t) = [\, x^T(t) \;\; x^T(t-h(t)) \;\; x^T(t-h_M) \;\; x^T(t-\tau(t)) \;\; x^T(t-\tau_M) \;\; \dot{x}^T(t)\,]^T$$

The following is also true for any matrix T:

$$2\zeta^T(t)T[\dot{x}(t) - Ax(t) - A_d x(t-\tau(t)) - BKx(t-h(t))] = 0 \tag{12}$$

Now, we consider the following Lyapunov functional:

$$V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t)$$

where $x_t = x(t+\theta),\ -max(h_M,\ \tau_M) \leq \theta \leq 0$,

$$\begin{aligned} V_1(x) = &\ x^T(t)Px(t) \\ V_2(x_t) = &\ \int_{t-\tau_M}^{t} x^T(s)Qx(s)ds + \int_{t-h_M}^{t} x^T(s)Rx(s)ds \\ V_3(x_t) = &\ \int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)(Y_1+Y_2)\dot{x}(s)dsd\theta \\ &+ \int_{-h_M}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)dsd\theta \end{aligned}$$

and $P > 0,\ Q \geq 0,\ R \geq 0,\ Y_1 > 0,\ Y_2 > 0,\ Z_1 > 0,\ Z_2 > 0$ are to be determined. Then, we take the derivative of $V(x_t)$ with respect to t along the solution of the nominal system (4) and add the left-hand-side of (6)-(12):

$$\begin{aligned} \frac{d}{dt}V(x_t) = &\ 2\dot{x}^T(t)Px(t) + x^T(t)(Q+R)x(t) - x^T(t-\tau_M)Qx(t-\tau_M) \\ &-x^T(t-h_M)Rx(t-h_M) + \tau_M\dot{x}(t)(Y_1+Y_2)\dot{x}(t) \\ &+h_M\dot{x}(t)(Z_1+Z_2)\dot{x}(t) \\ &-\int_{t-\tau_M}^{t} \dot{x}^T(s)(Y_1+Y_2)\dot{x}(s)ds - \int_{t-h_M}^{t} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds \\ = &\ 2\dot{x}^T(t)Px(t) + x^T(t)(Q+R)x(t) - x^T(t-\tau_M)Qx(t-\tau_M) \\ &-x^T(t-h_M)Rx(t-h_M) + \tau_M\dot{x}(t)(Y_1+Y_2)\dot{x}(t) \\ &+h_M\dot{x}(t)(Z_1+Z_2)\dot{x}(t) - \int_{t-\tau(t)}^{t} \dot{x}^T(s)Y_1\dot{x}(s)ds \end{aligned}$$

$$
\begin{aligned}
&-\int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)Y_1\dot{x}(s)ds - \int_{t-h(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_M}^{t-h(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
&-\int_{t-\tau_M}^{t} \dot{x}^T(s)Y_2\dot{x}(s)ds - \int_{t-h_M}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
&+2\zeta^T(t)N\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)S\left[x(t-h(t)) - x(t-h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)M\left[x(t) - x(t-h_M) - \int_{t-h_M}^{t} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)L\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)V\left[x(t-\tau(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)W\left[x(t) - x(t-\tau_M) - \int_{t-\tau_M}^{t} \dot{x}(s)ds\right] \\
&+2\zeta^T(t)T[\dot{x}(t) - Ax(t) - A_dx(t-\tau(t)) - BKx(t-h(t))]
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & \zeta^T(t)[\Phi_{11} + \tau_M(LY_1^{-1}L^T + VY_1^{-1}V^T + WY_2^{-1}W^T) \\
&+h_M(NZ_1^{-1}N^T + SZ_1^{-1}S^T + MZ_2^{-1}M^T)]\zeta(t) \\
&-\int_{t-h(t)}^{t}[\zeta^T(t)N + \dot{x}^T(s)Z_1]Z_1^{-1}[N^T\zeta(t) + Z_1\dot{x}(s)]ds \\
&-\int_{t-h_M}^{t-h(t)}[\zeta^T(t)S + \dot{x}^T(s)Z_1]Z_1^{-1}[S^T\zeta(t) + Z_1\dot{x}(s)]ds \\
&-\int_{t-h_M}^{t}[\zeta^T(t)M + \dot{x}^T(s)Z_2]Z_2^{-1}[M^T\zeta(t) + Z_2\dot{x}(s)]ds \qquad (13) \\
&-\int_{t-\tau(t)}^{t}[\zeta^T(t)L + \dot{x}^T(s)Y_1]Y_1^{-1}[L^T\zeta(t) + Y_1\dot{x}(s)]ds \\
&-\int_{t-\tau_M}^{t-\tau(t)}[\zeta^T(t)V + \dot{x}^T(s)Y_1]Y_1^{-1}[V^T\zeta(t) + Y_1\dot{x}(s)]ds \\
&-\int_{t-\tau_M}^{t}[\zeta^T(t)W + \dot{x}^T(s)Y_2]Y_2^{-1}[W^T\zeta(t) + Y_2\dot{x}(s)]ds
\end{aligned}
$$

Since $Y_1 > 0,\ Y_2 > 0,\ Z_1 > 0,\ Z_2 > 0$, the last six integral terms in (13) are all less than 0. Hence, if $\Phi_{11} + \tau_M(LY_1^{-1}L^T + VY_1^{-1}V^T + WY_2^{-1}W^T) + h_M(NZ_1^{-1}N^T + SZ_1^{-1}S^T + MZ_2^{-1}M^T) < 0$, we have $\dot{V}(x) < -\varepsilon||x(t)||^2$ for a sufficiently small $\varepsilon > 0$ which implies that the nominal system (4) is asymptotically stable. The above condition can be written as (5) by Schur complement formula.

Remark 3.2 *Theorem 3.1 employs Leibniz-Newton formula in (6)-(11), and free weighting matrix method in (12). These methods are known to reduce the conservatism in stability conditions.*

3.2 State Feedback Control

In this section, we seek a design method of the sampled-data control for a time-delay system. Unfortunately, Theorem 3.1 does not give a feasible LMI condition for obtaining a state feedback control gain matrix K. Hence, we look for another stability condition. To this end, we make a congruence transformation and obtain a feasible LMI stability condition. Based on it, we give a design method of sampled-data state feedback controllers.

Theorem 3.3 *Given scalars $\rho_i,\ i = 1, \cdots, 6$, the sampled-data controller (2) stabilizes the nominal system (1) with $H = 0,\ E_i = 0,\ i = 1, 2, 3$ if there exist $\bar{P} > 0,\ \bar{Q} \geq 0,\ \bar{R} \geq 0,\ \bar{Y}_1 > 0,\ \bar{Y}_2 > 0,\ \bar{Z}_1 > 0,\ \bar{Z}_2 > 0,\ \bar{G},\ U,$*

$$\bar{N} = \begin{bmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \\ \bar{N}_4 \\ \bar{N}_5 \\ \bar{N}_6 \end{bmatrix}, \bar{S} = \begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \\ \bar{S}_3 \\ \bar{S}_4 \\ \bar{S}_5 \\ \bar{S}_6 \end{bmatrix}, \bar{M} = \begin{bmatrix} \bar{M}_1 \\ \bar{M}_2 \\ \bar{M}_3 \\ \bar{M}_4 \\ \bar{M}_5 \\ \bar{M}_6 \end{bmatrix}, \bar{L} = \begin{bmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \\ \bar{L}_4 \\ \bar{L}_5 \\ \bar{L}_6 \end{bmatrix}, \bar{V} = \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \\ \bar{V}_6 \end{bmatrix} \text{ and } \bar{W} = \begin{bmatrix} \bar{W}_1 \\ \bar{W}_2 \\ \bar{W}_3 \\ \bar{W}_4 \\ \bar{W}_5 \\ \bar{W}_6 \end{bmatrix}$$

such that

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix} < 0 \qquad (14)$$

where

$$\begin{aligned}
\Theta_{11} &= \Theta_1 + \Theta_2 + \Theta_2^T + \Theta_3 + \Theta_3^T, \\
\Theta_1 &= \begin{bmatrix} \bar{Q} + \bar{R} & 0 & 0 & 0 & 0 & \bar{P} \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & -\bar{R} & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & -\bar{Q} & 0 \\ * & * & * & * & * & \tau_M(\bar{Y}_1 + \bar{Y}_2) + h_M(\bar{Z}_1 + \bar{Z}_2) \end{bmatrix} \\
\Theta_2 &= \begin{bmatrix} \bar{N} + \bar{M} + \bar{L} + \bar{W} & -\bar{N} + \bar{S} & -\bar{M} - \bar{S} & -\bar{L} + \bar{V} & -\bar{W} - \bar{V} & 0 \end{bmatrix} \\
\Theta_3 &= \begin{bmatrix} -\rho_1 A\bar{G}^T & -\rho_1 BU & 0 & -\rho_1 A_d\bar{G}^T & 0 & \rho_1\bar{G}^T \\ -\rho_2 A\bar{G}^T & -\rho_2 BU & 0 & -\rho_2 A_d\bar{G}^T & 0 & \rho_2\bar{G}^T \\ -\rho_3 A\bar{G}^T & -\rho_3 BU & 0 & -\rho_3 A_d\bar{G}^T & 0 & \rho_3\bar{G}^T \\ -\rho_4 A\bar{G}^T & -\rho_4 BU & 0 & -\rho_4 A_d\bar{G}^T & 0 & \rho_4\bar{G}^T \\ -\rho_5 A\bar{G}^T & -\rho_5 BU & 0 & -\rho_5 A_d\bar{G}^T & 0 & \rho_5\bar{G}^T \\ -\rho_6 A\bar{G}^T & -\rho_6 BU & 0 & -\rho_6 A_d\bar{G}^T & 0 & \rho_6\bar{G}^T \end{bmatrix} \\
\Theta_{12} &= \begin{bmatrix} h_M\bar{N} & h_M\bar{S} & h_M\bar{M} & \tau_M\bar{L} & \tau_M\bar{V} & \tau_M\bar{W} \end{bmatrix} \\
\Theta_{22} &= diag\begin{bmatrix} -h_M\bar{Z}_1 & -h_M\bar{Z}_1 & -h_M\bar{Z}_2 & -\tau_M\bar{Y}_1 & -\tau_M\bar{Y}_1 & -\tau_M\bar{Y}_2 \end{bmatrix}
\end{aligned}$$

In this case, a state feedback gain in (2) is given by

$$K = U\bar{G}^{-T} \qquad (15)$$

Proof: We let $T_i = \rho_i G,\ i = 1, \cdots, 6$ where ρ_i are given scalars, and substitute T_i into the condition (5). If (5) holds, it follows that $(6,6)$-block of Φ_{11} must be negative definite. This leads that $T_6 + T_6^T = \rho_6(G + G^T) < 0$, which implies that G is nonsingular if $\rho_6 \neq 0$.

Now we make a congruence transformation to (5) with

$$\Sigma = \mathrm{diag}\,[\bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}\quad \bar{G}]$$

where $\bar{G} = G^{-1}$ to calculate $\Sigma\Phi\Sigma^T$. Letting

$$\bar{P} = \bar{G}P\bar{G}^T,\ \bar{Q} = \bar{G}Q\bar{G}^T,\ \bar{R} = \bar{G}R\bar{G}^T,\ \bar{Y}_i = \bar{G}Y_i\bar{G}^T,\ \bar{Z}_i = \bar{G}Z_i\bar{G}^T,\ i = 1, 2$$

$$\bar{N} = \Sigma N\bar{G}^T, \bar{S} = \Sigma S\bar{G}^T, \bar{M} = \Sigma M\bar{G}^T, \bar{L} = \Sigma L\bar{G}^T, \bar{V} = \Sigma V\bar{G}^T, \bar{W} = \Sigma W\bar{G}^T$$

we obtain $\Sigma\Phi\Sigma^T = \Theta$ in (14) where we define $U = K\bar{G}^T$. If the condition (14) holds, a state feedback control gain matrix K is obviously given by (15).

3.3 Robust Stabilization

Now we extend the result in the previous sections to robust stabilization.

Theorem 3.4 *Given scalars $\rho_i,\ i = 1, \cdots, 6$, the sampled-data controller (2) robustly stabilizes the system (1) if there exist $\bar{P} > 0,\ \bar{Q} \geq 0,\ \bar{R} \geq 0,\ \bar{Y}_1 > 0,\ \bar{Y}_2 > 0,\ \bar{Z}_1 > 0,\ \bar{Z}_2 > 0,\ \bar{G},\ U,\ \bar{N},\ \bar{S},\ \bar{M},\ \bar{L},\ \bar{V},\ \bar{W}$ and a scalar λ such that*

$$\Lambda = \begin{bmatrix} \Theta + \lambda\hat{H}\hat{H}^T & \hat{E}^T \\ \hat{E} & -\lambda I \end{bmatrix} < 0$$

where Θ is given in Theorem 3.3 and

$$\begin{aligned} \hat{H} &= -[\rho_1 H^T\quad \rho_2 H^T\quad \rho_3 H^T\quad \rho_4 H^T\quad \rho_5 H^T\quad \rho_6 H^T\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]^T, \\ \hat{E} &= [E_1\bar{G}^T\quad E_3 U\quad 0\quad E_2 G^T\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0]. \end{aligned}$$

In this case, a state feedback gain in (2) is given by (15).

Proof: Replacing $A,\ A_d,\ B$ in Θ with $A + HF(t)E_1,\ A_d + HF(t)E_2,\ B + HF(t)E_3$, we obtain

$$\Theta + \lambda\hat{H}\hat{H}^T + \frac{1}{\lambda}\hat{E}^T\hat{E} < 0$$

for some $\lambda > 0$, from which the result is deduced by Schur complement formula.

Remark 3.5 *In order to obtain the optimal $\rho_i,\ i = 1, \cdots, 6$, we may use a numerical software like Matlab with optimization toolbox* `fminsearch`*[4].*

4 Numerical Examples

The following numerical examples illustrate our results. First, we compares our result with other results in the literature. Then we give a design method of stabilizing sampled-data controllers for nominal time-delay system and uncertain time-delay system.

Consider the system (1) with the following matrices

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ H = 0,\ E_1 = E_2 = 0,\ E_3 = 0. \tag{16}$$

We first compare our result with Fridman et al.[5] and Suplin et al.[12] where a sampled-data stabilization for a non-delay system with $\tau(t) = 0$ is considered. Suplin et al.[12] and Fridman et al.[5] guarantee the sampled-data stabilization for $h_M \leq 1.3928$ and $h_M \leq 3.0201$, respectively, while Theorem 3.2 does for $h_M \leq 9.99$ when $\rho_1 = 1, \rho_2 = -0.65, \rho_3 = 0.06, \rho_4 = -0.03, \rho_5 = -0.15, \rho_6 = 1200.10$ and $\tau(t) = 0$. This shows that our result is less conservative than other results.

Next, we consider the sampled-data stabilization of the above time-delay system for $\tau_M = 0.9999$. Theorem 3.2 shows the stabilizing sampled-data state feedback controller exists for $h_M \leq 1.0$. When $\rho_1 = 1, \rho_2 = -0.66, \rho_3 = 0, \rho_4 = -0.03, \rho_5 = -0.15, \rho_6 = 154.8$, we achieve the sampled-data stabilization with $h_M = 1.0$ and obtain the stabilizing controller (2) with $K = [0.0000\ \ -0.1001]$ by Theorem 3.2.

Finally, we consider the robust sampled-data stabilization for the system (1) with (16) and H, E_1, E_2, E_3 replaced by

$$H = I,\ E_1 = E_2 = 0.15I,\ E_3 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

Given $\tau_M \leq 0.5$, Theorem 3.3 shows the robust stabilizing sampled-data state feedback controller exists for $h_M \leq 0.5$. When $\rho_1 = 1, \rho_2 = 0, \rho_3 = 0, \rho_4 = -0.04, \rho_5 = -0.33, \rho_6 = 1.12$, we achieve the robust sampled-data stabilization with $h_M = 0.5$ and obtain the stabilizing controller (2) with $K = [-0.0080\ \ -0.8485]$ by Theorem 3.3.

5 Conclusions

We considered the sampled-data control problem for time-delay systems. Our method was based on a control input delay approach, which was new challenge to such a control problem. We first showed the closed-loop system with the sampled-data state feedback control became the state delayed system. First, we gave a sufficient condition for the closed-loop system to be stable. Then we proposed a design method of the robust sampled-data state feedback controller for uncertain time-delay systems. Finally, we gave illustrative examples and showed our result is less conservative than other existing ones.

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