Applied Mathematical Sciences, Vol. 1, 2007, no. 4, 165 – 181

# Sequential Design for Comparing Two Poisson Distributions with Delayed Observations

**Afaf M. Mady**

Department of Statistics
Faculty of Economics and Political Science
Cairo University, Egypt

**Abstract**

Given two Poisson populations with unknown means; it is required to select N units in three phases. Colton (1963) assumed that the cost of sampling is equal to the cost of an incorrect choice for half of the sample size. In this paper Colton's decision procedure is used to determine a sequential method of sampling when the choice is between two Poisson distributions, the standard performance being determined by the mean. This model is modified by considering delayed observations. This modification leads to major changes in the results. Minimax and maximin approaches are used for determining the optimal position of the boundaries of a sequential experiment.

**Keywords**: Decision theory, delayed observations, minimax approach, maximin approach.

## 1. Introduction

There are many practical problems associated with the selection or the ranking of populations based on a given attribute. The following represent some typical examples. Which process is best for the manufacture of an electronic component or an appliance? Which drug or drugs of a given set are most promising in the cure of a given disease? What type of fertilizer gives the largest expected yield from a given variety of grain?

Mady (2000) and Mendoza and Iglewicz (1977) discussed three phase procedures for choosing the better of two Exponential (Normal) distributions with unknown means. The sequential version of this model is reconsidered in this article when the choice is between two Poisson distributions.

It is assumed that N units of an article are to be manufactured using one of two processes. The finite N is reasonable in that the decision between two processes is not everlasting. We shall use the selected process at least until some new process has been developed. An experiment will be performed on 2n of the units, n on each process. In some manufacturing experiments only a small fraction of the manufactured article requiring manufacturing during the duration of the experiment, are actually entered in the experiment. It is therefore assumed that, while the experiment is in progress, 2n units are entered in the experiment and Kn units are not. That is, it is assumed that N units are subdivided into three phases based on the type of allocation rule used. The three phases consist of the delay phase containing Kn units, the experimental phase containing 2n units and the post experimental phase containing N- Kn- 2n units (will be manufactured using the process selected as the better at the conclusion of the experiment) . It is assumed that a proportion $q_i$ of units are to be manufactured using process i during the delay phase, where $q_1 + q_2=1$. For simplicity assume that $q_1 \geq q_2$ and define process 1 as the standard process.

It is assumed that we obtain defects or no defects units using one of two processes. The number of defects on a manufactured article using each process are assumed to be Poissonally distributed with unknown means $\lambda_i$ (i=1, 2). Assume that smaller number of defects is associated with better process. Then, letting

$\lambda = \lambda_2 - \lambda_1$, we should like the experiment to select process 1 if $\lambda$ is positive and process 2 if $\lambda$ is negative.

In formulating the consequences of right and wrong decisions, we consider Colton's approaches which differ in location of a base line. One is the "Cost" or "Loss" approach. For this we assume that for each manufactured unit using the inferior process we incur a loss or cost directly proportional to $\lambda$ (without loss of generality we take $\lambda$ as positive). We will obtain the equation of expected loss for all N units in the next section.

The other approach is the "Net Gain" approach. For this we assume that each time one unit that is manufactured using the superior process we gain in direct proportion to $\lambda$, while if manufacture using the inferior process we lose in direct proportion to $\lambda$ (i.e., we have a negative gain). We will obtain the equation of expected net gain for all N units in sec. (3).

It will be shown that the modification of the delay phase leads to results considerably different from those obtained when there is no delay phase.

There have been a number of papers on the delayed observations including Sung (1999), Williamson (1998), Douk (1994) and Hoel, Sobel and Weiss (1975).

## 2. Minimax Method with Expected Loss

It is assumed that we require manufacturing N units of an article using one of two processes. The random variables $x_{ij}$, j=1,…n denote the number of defects on a manufactured article using the process i (i= 1,2). They are assumed to be Poisson distributed with unknown means $\lambda_i$. That is,

$$f(x_{ij}, \lambda_i) = \lambda_i^{x_{ij}} e^{-\lambda_i} / x_{ij}!, \qquad \lambda_i \geq 0, \; x_{ij} = 0, 1, ..$$

It is assumed that independent pairs of units, are manufactured at a time and compute the observed difference $d_j = x_{1j} - x_{2j}$. Here $x_{ij}$ represents the $j^{th}$ unit that is manufactured using the $i^{th}$ process.

There is no loss of generality in assuming that small defects are desirable. It is also assumed that a loss of zero is incurred for each manufactured unit using the superior process and a loss of $|\lambda| = |\lambda_2 - \lambda_1|$ for a unit that is manufactured using the inferior process. Sampling is continued as long as

$$-\tau < \sum_{j=1}^{n} d_j < \tau \quad , \qquad (1)$$

where $\tau$ is a constant less than zero. Process 1 is chosen as superior if $\sum d_j \geq \tau$, while process 2 is chosen as superior if $\sum_{j=1}^{n} d_j \leq -\tau$. This is equivalent to a sequential probability ratio test of $H_0 : \lambda = \lambda_0 > 0$ versus $H_1 : \lambda = -\lambda_0$ (the size of the Type I $(\alpha)$ and Type II $(\beta)$ errors are the same and they are fixed in advance of the experiment). Using the usual sequential probability ratio test approximation, one obtains Pr (inf.) = $(1+x)^{-1}$, where Pr (inf.) is the probability of selecting the inferior process and $x = \exp[\tau \log(\lambda_1 / \lambda_2)]$ .

Furthermore,

$$E(n/\lambda) = \frac{\tau(1-x)}{|\lambda|(1+x)} \qquad \lambda \neq 0$$

$$= \tau^2 / 2\lambda_1 \qquad \lambda = 0 \ .$$

Defining P = n / N, one obtains the expected loss per unit as,

$$E\,[L\,(\lambda\,,\tau\,)] = |\lambda|\{K\,E\,(P/\lambda\,)[q_1\,\varphi\,(\lambda) + (1-q_1)(1-\varphi\,(\lambda)]$$

$$+ E(P/\lambda\,) + [1-(K+2)\,E(P/\lambda\,)\,]\,Pr\,(inf.)\}$$

$$= |\lambda|\left(\frac{\tau(1\text{-}x)}{N|\lambda|(x+1)}\right)[\,1+K\{q_1\,\varphi(\lambda)+(1-q_1)(1-\varphi(\lambda\,))\}]$$

$$+\ |\lambda|\left(1+\frac{\tau(K+2)(x-1)}{N|\lambda|(x+1)}\right)(x+1)^{-1}\ . \tag{2}$$

Here $\phi\,(\lambda)=1$ for $\lambda > 0$ and $\phi\,(\lambda) = 0$ for $\lambda \le 0$.

Because $q_1 \ge q_2$, it can be easily seen that $E[L(|\lambda^*|,\tau)] \ge E[L(-|\lambda^*|,\tau)]$ for every $K \ge 0$. One needs therefore, only to consider the case $\lambda \ge 0$ in order to obtain the minimax solution.

For $\lambda > 0$, (2) reduces to

$$E[L(\lambda\,,\tau)] = \frac{\lambda}{x+1} - \frac{\tau(x-1)}{N(x+1)^2}[x-1+Kq_1\,x+K(q_1-1)]\,. \tag{3}$$

Solving $\partial\,E\,[L\,(\lambda\,,\tau)\,]\;/\partial\,\lambda_i = 0$, one obtains,

$$x\,+1+\frac{\lambda\,\tau x}{\lambda_1} = -\frac{\tau^2\,x}{N\lambda_1(x+1)}[\{x-1+Kq_1x+K(q_1-1)\}(3-x)$$

$$+(x^2-1)(1+Kq_1)]\,, \tag{4}$$

$$x+1+\frac{\lambda\,\tau x}{\lambda_2} = -\frac{\tau^2\,x}{N\,\lambda_2(x+1)}[\{x-1+Kq_1x+K(q_1-1)\}(3-x)$$

$$+(x^2-1)(1+Kq_1)\,]. \tag{5}$$

Similarly, $\partial\,E[L\,(\lambda,\tau)]\,/\partial\,\tau = 0$ yields

$$N = \frac{\tau(x-1)(1+Kq_1)}{\lambda} + \frac{\tau(3-x)[x-1+Kq_1x+K(q_1-1)]}{\lambda(x+1)} + \frac{(x-1)[x-1+Kq_1x+K(q_1-1)]}{\lambda x \log(\lambda_1/\lambda_2)}. \tag{6}$$

Combining equations (4) and (5), one obtains, $\lambda_1 = \lambda_2 = \lambda^*$ .It is easy to see that $\lambda$ = **0** is the unique solution of (4) and (5).

When $\lambda$ = **0**, the right hand side of equation (6) is the indeterminate form zero/zero. There is no solution for equation (6) unless $q_1 = 0.5$ . In this case applying L'Hopital's Rule the right hand side of (6) approaches $[3(K+2)\tau^2]/2\lambda^*$ as $\lambda$ approaches zero and $q_1 = 0.5$ .

Hence,

$$\tau^* = -\sqrt{2N\lambda^*/[3(K+2)]} \ , \tag{7}$$

and this solution is unique for each K.

Note that $\Pr(\text{inf.}) = 0.5$ and this value is an acceptable because $\lambda_1 = \lambda_2 (i.e., \lambda = 0)$ and it does not depend on K, $q_1$ and N.

### 2.1. Numerical Results

A numerical example will now be given in order to illustrate the use of minimax method and to summarize the results.

Table 1 gives values for $\lambda^*$ = 2,5 , 25 and a number of values of K. Thus, if $q_1$ = 0.5, K=10, $\lambda^*$ = 2 and N=10000, then $\tau^*$ = -33.33, E(n/ $\lambda^*$ ) = 278.

Table 1 shows that the expected sample sizes change considerably as K increases. In the above example, K = 0 yields E(n /$\lambda^*$ =2)= 1667 as compared to E(n /$\lambda^*$ =2)= 278 for K=10. The two $\tau^*$ 's have different values in the above comparison. Also $\lambda^*$ has no effect on E(n /$\lambda^*$ ) for all values of K.

Consequently, if there is an actual delay, then the expected sample size seems to be very low.

**Table 1**

Some needed constants and performance parameters for the sequential minimax case $[\lambda_1=\lambda_2=\lambda^*, \tau^*=-\sqrt{2N\lambda^*/[3(K+2)]}, E(n/\lambda^*)=\tau^{*2}/2\lambda^*]$

| | $\lambda^*=2$ | | $\lambda^*=5$ | | $\lambda^*=25$ | |
|---|---|---|---|---|---|---|
| **K** | $\tau^*$ | $E(n/\lambda^*)$ | $\tau^*$ | $E(n/\lambda^*)$ | $\tau^*$ | $E(n/\lambda^*)$ |
| 0 | -81.65 | 1667 | -129.10 | 1667 | -288.68 | 1667 |
| 5 | -43.64 | 476 | -69.01 | 476 | -154.30 | 476 |
| 10 | -33.33 | 278 | -52.70 | 278 | -117.85 | 278 |
| 25 | -22.22 | 123 | -35.14 | 123 | -78.57 | 123 |
| 50 | -16.01 | 64 | -25.32 | 64 | -56.61 | 64 |
| 75 | -13.16 | 43 | -20.81 | 43 | -46.52 | 43 |
| 100 | -11.43 | 33 | -18.08 | 33 | -40.42 | 33 |

**3. Maximin Method with Expected Net Gain**

For this approach we assume that each time a unit that is manufactured using the superior process we gain in direct proportion to $\lambda$, while if manufacture using the inferior process we lose in direct proportion to $\lambda$ (i.e., we have a negative gain). We then get the expected net gain for all N units,

$$E(\text{Net Gain})= \lambda\ [N-(k+2)\ E(n)][\Pr(\text{sup.})-\Pr(\text{inf.})]$$

$$=\lambda\ [N-(k+2)\ E(n)][1-2\Pr(\text{inf.})]. \qquad (8)$$

The problem is to determine $\tau$, the position of the boundary, so that expected net gain is maximized.

At $\lambda$ =0 (i.e., $\lambda_1=\lambda_2=\lambda^*$) the derivatives of E (Net Gain) with respect to $\lambda_1$ and $\lambda_2$ respectively vanishe. It can be shown without difficulty that this is the only value of $\lambda$ for which this is true (this result is obvious since the least value of E (Net Gain) is, of course, zero when $\lambda = 0$).

Differentiating E(Net Gain ) with respect to $\tau$ gives the equation

$$2\lambda N x \log(\lambda_1/\lambda_2)\ (x+1)^2+(K+2)(x^2-1)[x^2-1+4\tau x\log(\lambda_1/\lambda_2)].$$

Setting this derivative equal to zero gives

$$\frac{-2N}{(K+2)}=\frac{(x-1)[x^2-1+4\tau x\log(\lambda_1/\lambda_2)]}{\lambda x(x+1)\log(\lambda_1/\lambda_2)} \quad . \tag{9}$$

When $\lambda = 0$ the right hand side is the indeterminate form 0/0. Applying L'Hopital's Rule twice, as $\lambda$ approaches zero, the right hand side approaches $-3\tau^2/\lambda_1$ . Thus,

$$\tau^* = -\sqrt{2N\lambda^*/[3\,(K+2)]}, \tag{10}$$

is the maximin solution .

Note that both minimax and maximin results are equivalent.

## 4. The Case When One Population Mean is Already Known

The above approaches may be more difficult to the case when one population mean is already known and it is desired to decide whether or not to change to the alternative population.

Consider two Poisson populations $\Pi_1$ and $\Pi_2$ with parameters $\lambda_1$ and $\beta$ (known) respectively. It is assumed that independent units are to be manufactured using the process with unknown mean and compute $d_j = \sum_{j=1}^{n} x_{1j}$ .Here $x_{1j}$ represents the $j^{th}$ unit that is manufactured using process 1. Sampling is continued as long as

$$-\tau_1 - n\,\tau_2 < d_j < \tau_1 - n\,\tau_2\ , \tag{11}$$

where $\tau_1$ is a constant greater than zero and $\tau_2$ is a constant less than zero. Process 1 is chosen as superior if $d_j \geq \tau_1 - n\,\tau_2$, while process 2 is chosen as superior if $d_j \le -\tau_1 - n\,\tau_2$. This is equivalent to a sequential probability ratio test of $H_0 : \lambda = \lambda_0$ versus $H_1 : \lambda = -\lambda_0$, $\lambda = \beta - \lambda_1$ (the size of the type I ($\alpha$) and type II ($\beta$ ) errors are the same and they are fixed in advance of the experiment). Using the usual sequential probability ratio test approximation, one obtains $\Pr(\text{inf.}) = (1+x)^{-1}$, where $\Pr(\text{inf.})$ is the probability of selecting the inferior pro-cess, x = exp(ah) and h can be obtained from the equation $-\lambda+\beta = \dfrac{ah\tau_2 / \tau_1}{\exp(-ah/\tau_1) - 1}$, $a = -\tau_1 \ln(\gamma_2/\gamma_1)$ (recall that $\gamma_1 = \beta - \lambda_0$, $\gamma_2 = \beta + \lambda_0$). Furthermore,

$$E(n/\lambda) = \frac{\tau_1 (1-x)}{(x+1)(\tau_2 - \lambda + \beta)} \qquad \lambda \ne 0$$

$$= \quad -\tau_1^2 / \tau_2 \qquad \lambda = 0 \tag{12}$$

**4.1. Minimax Method with Expected Loss:**

Defining P= n /N, one obtains the expected loss per unit as

$$E[L(\lambda,\tau_1,\tau_2)]=|\lambda|\ [0.5\,E(P/\lambda)+KE(P/\lambda)\ \{q_1\,\varphi(\lambda)+(1-q_1)(1-\varphi(\lambda))\}]$$

$$+|\lambda|\,[1-(K+1)E(P/\lambda)]\Pr(\text{inf.})$$

$$=|\lambda|\,\frac{\tau_1\,(1-x)}{2N(x+1)(\tau_2-\lambda+\beta)}[1+2K\{q_1\,\varphi(\lambda)+(1-q_1)(1-\varphi(\lambda))\}]$$

$$+|\lambda|\left(1+\frac{\tau_1\,(x-1)(K+1)}{(x+1)N(\tau_2-\lambda+\beta)}\right)(x+1)^{-1}. \qquad (13)$$

Here $\varphi(\lambda)=1$ for $\lambda>0$ and $\varphi(\lambda)=0$ for $\lambda\leq 0$.

Because $q_1>q_2$ it can be easily seen that $E[L(\lambda^*,\tau_1,\tau_2)]\geq E[L(-\lambda^*,\tau_1,\tau_2)]$ for every $K\geq 0$.

One needs, therefore, only to consider the case $\lambda\geq 0$ in order to obtain the minimax solution.

For $\lambda>0$, (13) reduces to

$$E[L(\lambda\ ,\tau_1,\tau_2)]=\frac{\lambda}{x+1}-\frac{\lambda\tau_1(x-1)\ [x-1+2Kq_1x+2K(q_1-1)]}{2N(x+1)^2(\tau_2-\lambda+\beta)}. \qquad (14)$$

Solving $\partial E[L(\lambda\ ,\tau_1,\tau_2)]/\partial\tau_i=0$, one obtains

$$N=\frac{\tau_1[x-1-hxa][x(1+2Kq_1)-1+2K(q_1-1)]}{2axh(\tau_2-\lambda+\beta)}$$

$$-\frac{\tau_1(x-1)[-x(1+2Kq_1)+3-2K(q_1-2)]}{2(x+1)(\tau_2-\lambda+\beta)},$$

$$N=\frac{\tau_1[x(1+2Kq_1)-1-2K(1-q_1)][x-1-xa(\tau_2-\lambda+\beta)\frac{\partial h}{\partial\tau_2}]}{2ax\frac{\partial h}{\partial\tau_2}(\tau_2-\lambda+\beta)^2}$$

(15)

$$+\frac{\tau_1(1-x)[-x(1+2Kq_1)+3+2K(2-q_1)]}{2(x+1)(\tau_2-\lambda+\beta)}.$$

(16)

Combining equations (15) and (16), one obtains h=0 (i.e., $\lambda-\beta=\tau_2$).

Now, we will differentiate between two cases, these cases include $\tau_2=-\beta$ (i.e ., $\lambda=0$) and $\tau_2\neq-\beta$ (i.e., $\lambda=(\tau_2+\beta)$ )

a) <u>The case</u> $\lambda^*=0$ (i.e., $\tau_2=-\beta$ )

When $\lambda=0$ , the right hand side of equation (15) is the indeterminate form zero /zero .There is no solution for equation (15) unless $q_1=0.5$. In this case applying L'Hopital's Rule the right hand side of (15) approaches $-\tau_1^2(K+1)/\tau_2$ as $\lambda$ approaches zero and $q_1=0.5$. Hence

$$\tau_1^* = \sqrt{N\beta/(K+1)}$$

and $\tau_2 = -\beta$ .

(17)

Table 2 gives values $\beta$ =2,5 , 50 and a number of values of K. Thus, if $q_1 = 0.5$ , K=10, $\beta$ =2 and N=10000, then $\tau_1^* = 42.64$, E(n/ $\beta$ )= 909.

Table 2 shows that the expected sample sizes change considerably as K increases. In the above example, K =0 yields E(n/ $\beta$ =2) =10000 as compared to E(n/ $\beta$ =2) = 909 for K =10. The two $\tau^*$ 's have different values in the above comparison . Also $\beta$ has no effect on E(n / $\beta$ ) for all values of K.

Consequently, if there is an actual delay, then the expected sample size seems to be very low.

**Table 2 (see next page)**

Some needed constants and performance parameters for the sequential minimax case

$$[\lambda^* = 0, \tau_2^* = -\beta, \tau_1^* = \sqrt{N\beta/(K+1)}], \ E(n/\lambda^*) = -\tau_1^{*2}/\tau_2^*, q_1 = 0.5]$$

| $\beta=2$ | | | $\beta=5$ | | $\beta=50$ | |
|---|---|---|---|---|---|---|
| **K** | $\tau_1^*$ | $E(n/\beta)$ | $\tau_1^*$ | $E(n/\beta)$ | $\tau_1^*$ | $E(n/\beta)$ |
| 0 | 141.42 | 10000 | 223.61 | 10000 | 707.11 | 10000 |
| 5 | 57.74 | 1667 | 91.29 | 1667 | 288.68 | 1667 |
| 10 | 42.64 | 909 | 67.42 | 909 | 213.20 | 909 |
| 25 | 27.74 | 385 | 43.85 | 385 | 138.68 | 385 |
| 50 | 19.80 | 196 | 31.31 | 196 | 99.01 | 196 |
| 75 | 16.22 | 132 | 25.65 | 132 | 81.11 | 132 |
| 100 | 14.07 | 99 | 22.25 | 99 | 70.36 | 99 |

b) <u>The case</u> $\lambda_1^*=-\tau_2$ (i.e., $\tau_2 \neq -\beta$)

When $\lambda_1^*=-\tau_2$, the right hand side of equation (15) is the indeterminate form zero /zero. Applying L'Hopital's Rule the right hand side of (15) approaches $-\tau_1^2[1+2K(1-q_1)]/\tau_2$ as $\lambda_1^*$ approaches $-\tau_2$. Hence

$$\tau_1^*=\sqrt{N\lambda_1^*/[1+2K(1\text{-}q_1)]}\ , \qquad (q_1\neq 0.5) \qquad (18)$$

and $\tau_2^*=-\lambda_1^*$.

Table 3 gives values $q_1$ =0.25, 0.75 and a number of values of K. Thus, if $q_1$ =0.25, K=10, $\lambda_1^*$=2 and N=10000, then $\tau_1^*$=35.36, $E(n/\lambda_1^*)$ = 625.

Table 3 shows that the expected sample sizes change considerably as K increases. In the above example, K =0 yields $E(n/\lambda_1^*=2)$ =10000 as compared to $E(n/\lambda_1^*=2)$ = 625 for K =10. The two $\tau_1^*$ have different values in the above comparison. Also $\lambda_1^*$ has no effect on $E(n/\lambda_1^*)$ for all values of K but $q_1$ has a stronger effect for all values of K.

Consequently, if there is an actual delay, then the expected sample size seems to be very low.

**Table 3**

Some needed constants and performance parameters for the sequential minimax case

$$[\lambda_1 = \lambda^* \ , \tau_2^* = -\lambda_1^*, \tau_1^* = \sqrt{N\lambda_1^* \ /[1+2K(1-q_1)]}, E(n/\lambda^*) = -\tau_1^{*2}/\tau_2^* \ ]$$

| $\lambda_1^* = 2$ / **K** | $\tau_1^*$ **when** $q_1 = 0.25$ | $E(n/\lambda_1^*)$ **when** $q_1 = 0.25$ | $\tau_1^*$ **when** $q_1 = 0.75$ | $E(n/\lambda_1^*)$ **when** $q_1 = 0.75$ |
|---|---|---|---|---|
| 0 | 141.42 | 10000 | 141.42 | 10000 |
| 5 | 48.51 | 1176 | 75.59 | 2857 |
| 10 | 35.36 | 625 | 57.74 | 1667 |
| 25 | 22.79 | 260 | 38.49 | 741 |
| 50 | 16.22 | 132 | 27.74 | 385 |
| 75 | 13.27 | 88 | 22.79 | 260 |
| 100 | 11.51 | 66 | 19.80 | 196 |

| $\lambda_1^* = 5$ / **K** | $\tau_1^*$ **when** $q_1 = 0.25$ | $E(n/\lambda_1^*)$ **when** $q_1 = 0.25$ | $\tau_1^*$ **when** $q_1 = 0.75$ | $E(n/\lambda_1^*)$ **when** $q_1 = 0.75$ |
|---|---|---|---|---|
| 0 | 223.61 | 10000 | 223.61 | 10000 |
| 5 | 76.70 | 1176 | 119.52 | 2857 |
| 10 | 55.90 | 625 | 91.29 | 1667 |
| 25 | 36.04 | 260 | 60.86 | 741 |
| 50 | 25.65 | 132 | 43.85 | 385 |
| 75 | 20.99 | 88 | 36.04 | 260 |
| 100 | 18.20 | 66 | 31.31 | 196 |

**4.2. Maximin Method with Expected Net Gain:**

It is not difficult to show that the expected net gain for all N units is,

E(Net Gain) = $\lambda$ [N-(K+1) E(n)] [Pr(sup.)-Pr(inf.)]

$$= \frac{\lambda N\,(x-1)}{x+1} - \frac{\lambda \tau_1\,(K+1)\,(x-1)^2}{(\tau_2 - \lambda + \beta)\,(x+1)^2} \quad . \tag{19}$$

The problem is to determine $\tau_1$ and $\tau_2$ , the position of the boundaries, so that expected net gain is maximized.

At $\lambda = 0$ the derivative $\frac{\partial E(NetGain)}{\partial \lambda}$ vanishes. It can be shown that this is the only value of $\lambda$ for which this is true.

Differentiating E(Net Gain) with respect to $\tau_1$ ($\tau_2 = -\beta$ at $\lambda = 0$) gives the equation

-2N x ah (x+1) ( $\tau_2 - \lambda + \beta$ ) – $\tau_1$ (K+1) (x-1) [ $x^2$ - 1- 4 ahx].

Setting this derivative equal to zero gives

$$-\frac{2aN}{\tau_1(K+1)} = \frac{(x-1)\ [x^2 - 1 - 4ahx]}{hx\ (x+1)\ (\tau_2 - \lambda + \beta)} \quad . \tag{20}$$

When $\lambda = 0$, $\tau_2 = -\beta$ the right hand side is the indeterminate form 0/0. Applying L'Hopital's Rule twice as $\lambda$ approaches zero, the right hand side approaches $2a\,\tau_1/\tau_2$ . Thus,

$$\tau_1^* = \sqrt{N\beta / (K+1)}$$

$$\text{and} \quad \tau_2^* = -\beta, \tag{21}$$

is the maximin solution.

Note that both minimax and maximin ($\lambda^*$ = 0) results are equivalent.

**5. Conclusion**

When a choice has to be made in favour of one of two populations the cost of sampling (experimenting) in order to obtain information on which to base the decision must be balanced against the cost of making the wrong choice.

Also, we have shown that the delay phase leads to results considerably different from those obtained when there is no delay phase. The nature of the delay could, in general, be quite important. For instance, an assumption in the Colton model is that the response to the treatments is instantaneous, or that there is no time lag between the treatment of the patients during the trial stage and the availability of all the treatment results. In practice, however, the response to the treatments is often delayed, causing a "waiting period" between the two stages, and an accumulation of new patients who have to be treated before the beginning of the treatment stage. The allocation of treatments to theses patients is an important issue, especially when their number is large relative to N.

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Received: July 14, 2006