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Robust H_∞ Disturbance Attenuation for Uncertain Discrete-Time Systems with Time-Varying Delay

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Abstract

This paper discusses robust H_∞ disturbance attenuation of uncertain discrete-time systems that have time-varying delay in state. There have been only a few results on H_∞ disturbance attenuation for discrete-time delay systems. Especially, few results on robust H_∞ disturbance attenuation for a class of uncertain discrete-time delay systems have appeared in the literature. Therefore, the study of robust H_∞ disturbance attenuation for uncertain discrete-time delay systems is a very important problem. We first obtain an H_∞ disturbance attenuation condition for nominal discrete-time systems with time varying delay via linear matrix inequalities (LMIs). To this end, we define a new Lyapunov function and use Leibniz-Newton formula and free weighting matrix method, which reduce the conservativeness and unnecessary LMI slack variables in our robust H_∞ disturbance attenuation conditions. Then we extend to a robust H_∞ disturbance attenuation condition for uncertain discrete-time delay systems. Finally, we give some numerical examples to show that our conditions are less conservative than other results in the literature.

Mathematics Subject Classification: 93E20

Keywords: Delay systems, discrete-time systems, uncertain systems, linear matrix inequality

1 Introduction

When we consider control problems of physical systems, we often see time-delays in the process of control algorithms. Time-delays often appear in many practical systems and mathematical formulations such as electrical system, mechanical system, biological system, and transportation system. Hence, a system with time-delay is a natural representation for them, and its analysis and synthesis are of theoretical and practical importance. In the past decades, research on continuous-time delay systems has been active. Difficulty that arises in continuous time-delay systems is that it is infinite dimensional and a corresponding controller can be a memory feedback. This class of controllers may minimize a certain performance index, but it is difficult to implement it to practical systems due to a memory feedback. To overcome such a difficulty, a memoryless controller is used for time-delay systems. In the last decade, sufficient stability conditions have been given via linear matrix inequalities (LMIs), and stabilization methods by memoryless controllers have been investigated by many researchers. Since Li and de Souza considered robust stability and stabilization problems in [8], less conservative robust stability conditions for continuous time-delay systems have been obtained ([7], [10]). Recently, H_∞ disturbance attenuation conditions have also been given ([9], [14], [15]).

On the other hand, research on discrete-time delay systems has not attracted as much attention as that of continuous-time delay systems. In addition, most results have focused on discrete-time systems with time-invariant delays ([3], [11], [13], [17]). Only some results on discrete-time systems with time-varying delays have appeared in the literature. Gao and Chen [4], Hara and Yoneyama [5], [6] gave robust stability conditions. Fridman and Shaked [1] solved a guaranteed cost control problem. Fridman and Shaked [2], Yoneyama [16], Zhang and Han [18] considered the H_∞ disturbance attenuation. They have given sufficient conditions via LMIs for corresponding control problems. Nonetheless, their conditions still show the conservatism. Hara and Yoneyama [5] and Yoneyama [16] gave least conservative conditions but their conditions require many LMI slack variables, which in turn require a large amount of computations. Furthermore, to authors' best knowledge, no result on robust H_∞ disturbance attenuation problem for uncertain discrete-time systems with time-varying delays has given in the literature.

In this paper, we consider H_∞ disturbance attenuation for nominal discrete-time systems with time-varying delay and robust H_∞ disturbance attenuation for uncertain system counterpart. First, we obtain a new H_∞ disturbance attenuation condition for a nominal time-delay system. To this end, we define a new Lyapunov function and use Leibniz-Newton formula and free weighting matrix method. These methods are known to reduce the conservatism in our H_∞ disturbance attenuation condition. Our method requires fewer LMI variables than the existing results, and hence leads to a smaller amount of computations. Then, we extend our H_∞ disturbance attenuation condition to robust H_∞ disturbance attenuation condition for uncertain discrete-time systems with time-varying delay. Finally, we give some numerical examples to illustrate our results and to compare with existing results.

2 Time-Delay Systems

Consider the following discrete-time system with a time-varying delay and uncertainties in the state.

$$\begin{aligned} x(k+1) &= (A+\Delta A)x(k) + (A_d + \Delta A_d)x(k-d_k) + (B+\Delta B)w(k) \\ z(k) &= Cx(k) + Dw(k), \\ x(k) &= 0, k \in [-d_M, 0] \end{aligned} \tag{1}$$

where $x(k) \in \Re^n$ is the state, $w(k) \in \Re^m$ is the disturbance, $z(k) \in \Re^q$ is the controlled output. A, A_d, B, C and D are system matrices with appropriate dimensions. d_k is a time-varying delay and satisfies $0 \le d_m \le d_k \le d_M$ and $d_{k+1} \le d_k$ where d_m, d_M are known constants. Uncertain matrices are of the form

$$\Delta A = HF(k)E, \ \Delta A_d = HF(k)E_d, \ \Delta B = HF(k)E_1 \tag{2}$$

where $F(k) \in \Re^{l \times j}$ is an unknown time-varying matrix satisfying $F^T(k)F(k) \le I$ and H, E, E_d and E_1 are constant matrices of appropriate dimensions.

Definition 2.1 *The system (1) is said to be robustly stable if it is asymptotically stable for all admissible uncertainties (2).*

Our problem is to find conditions such that the system (1) is robustly stable with $w = 0$ and satisfies

$$J = \sum_{k=0}^{\infty} (z^T(k)z(k) - \gamma^2 w^T(k)w(k)) < 0 \ \ \forall \ w \neq 0$$

for a prescribed $\gamma > 0$. If such conditions are satisfied, we say the system (1) achieves the robust H_∞ disturbance attenuation γ.

When we discuss a nominal system, we consider the following system.

$$\begin{aligned} x(k+1) &= Ax(k) + A_d x(k-d_k) + Bw(k), \\ z(k) &= Cx(k) + Dw(k), \\ x(i) &= 0, i \in [-d_M, 0]. \end{aligned} \tag{3}$$

The following lemma is useful to prove our results.

Lemma 2.2 *([12]) Given matrices $Q = Q^T$, H, E and $R = R^T > 0$ with appropriate dimensions.*

$$Q + HF(k)E + E^T F^T(k)H^T < 0$$

for all $F(k)$ satisfying $F^T(k)F(k) \leq R$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \frac{1}{\varepsilon} HH^T + \varepsilon E^T RE < 0.$$

3 Analysis of H_∞ Disturbance Attenuation

This section analyzes H_∞ disturbance attenuation for discrete-time delay systems. Section 3.1 gives an H_∞ disturbance attenuation condition for nominal systems and Section 3.2 extend to robust H_∞ disturbance attenuation.

3.1 H_∞ Disturbance Attenuation

Theorem 3.1 *Given integers d_m and d_M. Then the time-delay system (3) achieves H_∞ disturbance attenuation γ if there exist matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $S > 0$, $M > 0$, L_1, L_2, L_3, N_1, N_2, T_1 and T_2 satisfying*

$$\Phi = \begin{bmatrix} \Phi_1 + \Xi_L + \Xi_N + \Xi_T & \sqrt{d_M} Z_1 \\ * & -S \end{bmatrix} < 0 \tag{4}$$

where

$$\begin{aligned}
\Phi_1 &= \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 \\ * & \Phi_{22} & 0 & 0 & 0 & C^T D \\ * & * & -Q_1 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & -P_2 & 0 \\ * & * & * & * & P_2 - Q_2 & 0 \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}, \\
\Phi_{22} &= -P_1 + Q_1 + (d_M - d_m)M + C^T C, \\
\Phi_{44} &= P_2 + Q_2 + d_M S, \\
\Phi_{66} &= -\gamma^2 I + D^T D, \\
Z_1 &= \begin{bmatrix} 0 \\ N_1 \\ 0 \\ -P_2 + N_2 \\ P_2 \end{bmatrix},
\end{aligned}$$

$$\Xi_L = \begin{bmatrix} L_1+L_1^T & L_2^T-L_1 & 0 & L_3^T-L_1 & 0 & 0 \\ * & -L_2-L_2^T & 0 & -L_3^T-L_2 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & -L_3-L_3^T & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix},$$

$$\Xi_N = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ * & N_1+N_1^T & -N_1 & N_2^T & 0 & 0 \\ * & * & 0 & -N_2^T & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix},$$

$$\Xi_T = \begin{bmatrix} T_1+T_1^T & T_2^T-T_1A & -T_1A_d & 0 & 0 & -T_1B \\ * & -T_2A-A^TT_2^T & -T_2A_d & 0 & 0 & -T_2B \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}.$$

Proof: Consider a Lyapunov function

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k)$$

where $e(k) = x(k+1) - x(k)$ and

$$\begin{aligned} V_1(k) &= x^T(k)P_1x(k) + \sum_{i=k-d_k}^{k-1} e^T(i)P_2 \sum_{i=k-d_k}^{k-1} e(i), \\ V_2(k) &= \sum_{i=k-d_k}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-d_k}^{k-1} e^T(i)Q_2e(i), \\ V_3(k) &= \sum_{i=-d_k}^{-1} \sum_{j=k+i}^{k-1} e^T(j)Se(j), \\ V_4(k) &= \sum_{j=-d_M+1}^{-d_m} \sum_{i=k+j}^{k-1} x^T(i)Mx(i), \end{aligned}$$

and P_1, P_2, Q_1, Q_2, S and M are positive definite matrices to be determined. Then we calculate the difference $\Delta V = V(k+1) - V(k)$ and add following zero quantities.

$$2[x^T(k+1)L_1 + x^T(k)L_2 + e^T(k)L_3][x(k+1) - x(k) - e(k)] = 0,$$

$$2[x^T(k)N_1 + e^T(k)N_2][x(k) - x(k-d_k) - \sum_{i=k-d_k}^{k-1} e(i)] = 0,$$

$$2[x^T(k+1)T_1 + x^T(k)T_2]$$
$$[x(k+1) - Ax(k) - A_d x(k-d_k) - Bw(k)] = 0.$$

Since $\Delta V_i(k),\ i = 1, \cdots, 4$ are calculated as follows;

$$\begin{aligned}
\Delta V_1(k) &= x^T(k+1)P_1x(k+1) + \sum_{i=k+1-d_{k+1}}^{k} e^T(i)P_2 \sum_{i=k+1-d_{k+1}}^{k} e(i) \\
&\quad -x^T(k)P_1x(k) - \sum_{i=k-d_k}^{k-1} e^T(i)P_2 \sum_{i=k-d_k}^{k-1} e(i) \\
&\le x^T(k+1)P_1x(k+1) - x^T(k)P_1x(k) + e^T(k)P_2e(k) \\
&\quad -2e^T(k)P_2e(k-d_k) + 2e^T(k)P_2 \sum_{i=k-d_k}^{k-1} e(i) \\
&\quad +e^T(k-d_k)P_2e(k-d_k) - 2e^T(k-d_k)P_2 \sum_{i=k-d_k}^{k-1} e(i), \\
\Delta V_2(k) &= \sum_{i=k+1-d_{k+1}}^{k} x^T(i)Q_1x(i) + \sum_{i=k+1-d_{k+1}}^{k} e^T(i)Q_2e(i) \\
&\quad - \sum_{i=k-d_k}^{k-1} x^T(i)Q_1x(i) - \sum_{i=k-d_k}^{k-1} e^T(i)Q_2e(i) \\
&\le x^T(k)Q_1x(k) + e^T(k)Q_2e(k) - x^T(k-d_k)Q_1x(k-d_k) \\
&\quad -e^T(k-d_k)Q_2e(k-d_k), \\
\Delta V_3(k) &= d_{k+1}e^T(k)Se(k) - \sum_{i=k-d_{k+1}}^{k-1} e^T(i)Se(i) \cdots - \sum_{i=k-d_k}^{k-1} e^T(i)Se(i) \\
&\le d_Me^T(k)Se(k) - \sum_{i=k-d_k}^{k-1} e^T(i)Se(i), \\
\Delta V_4(k) &= (d_M - d_m)x^T(k)Mx(k) - \sum_{i=k-d_M+1}^{k-d_m} x^T(i)Mx(i) \\
&\le (d_M - d_m)x^T(k)Mx(k),
\end{aligned}$$

we have

$$\begin{aligned}
&\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \\
= \ &\Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \\
\le \ &\xi^T(k)[\Phi_1 + \Xi_L + \Xi_N + \Xi_T]\xi(k) + \sum_{i=k-d_k}^{k-1} \xi^T(k)Z_1S^{-1}Z_1^T\xi(k) \\
&- \sum_{i=k-d_k}^{k-1} (\xi^T(k)Z_1 + e^T(i)S)S^{-1}(Z_1^T\xi(k) + Se(i)) \\
\le \ &\xi^T(k)[\Phi_1 + \Xi_L + \Xi_N + \Xi_T + d_MZ_1S^{-1}Z_1^T]\xi(k)
\end{aligned}$$

where $\xi^T(k) = [x^T(k+1)\ \ x^T(k)\ \ x^T(k-d_k)\ \ e^T(k)\ \ e^T(k-d_k)\ \ w^T(k)]$. If (4) is satisfied, then by Schur complement formula, we have $\Phi_1 + \Xi_L + \Xi_N + \Xi_T + d_M Z_1 S^{-1} Z_1^T < 0$. It follows that $\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0$. This leads to $\Delta V(k) < 0$ when $w(k) = 0$ and hence the stability with $w(k) = 0$ is established. Summing up from $k = 0$ to $k = \infty$, we get $V(\infty) - V(0) + J < 0$. Since $V(\infty) \geq 0$ and $V(0) = 0$, we have $J < 0$, which proves the H_∞ disturbance attenuation γ.

Remark 3.2 *We employ* $\sum_{i=k-d_k}^{k-1}(\star)$ *in our Lyapunov function instead of* $\sum_{i=k-d_M}^{k-1}(\star)$. *This gives a less conservative* H_∞ *disturbance attenuation condition.*

Remark 3.3 *[16] and Theorem 3.1 have the same number of LMI slack variables. However, a size of LMI slack variables in [16] is larger than that of Theorem 3.1. It implies that our method requires a shorter computation time than [16].*

3.2 Robust H_∞ Disturbance Attenuation

By extending Theorem 3.1, we obtain a condition for robust H_∞ disturbance attenuation γ of uncertain system (1).

Theorem 3.4 *Given integers d_m and d_M. Then the time-delay system (1) achieves H_∞ disturbance attenuation γ if there exist matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $S > 0$, $M > 0$, L_1, L_2, L_3, N_1, N_2, T_1 and T_2 and a scalar $\lambda > 0$ satisfying*

$$\Pi = \begin{bmatrix} \Phi + \lambda \bar{E}^T \bar{E} & \bar{H}^T \\ * & -\lambda I \end{bmatrix}, \tag{5}$$

where

$$\bar{H} = [-H^T T_1^T \quad -H^T T_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

and

$$\bar{E} = [0 \quad E \quad E_d \quad 0 \quad 0 \quad E_1 \quad 0].$$

Proof: Replacing A, A_d and B in (4) with $A+HF(k)E$, $A_d+HF(k)E_d$ and $B+HF(k)E_1$, respectively, we obtain the robust H_∞ disturbance attenuation (4) corresponding to the system (1):

$$\Phi + \bar{H}^T F(k)\bar{E} + \bar{E}^T F^T(k)\bar{H} < 0 \tag{6}$$

By Lemma 2.2, a necessary and sufficient condition that guarantees (6) is that there exists a scalar $\lambda > 0$ such that

$$\Phi + \lambda \bar{E}^T \bar{E} + \frac{1}{\lambda}\bar{H}^T \bar{H} < 0 \tag{7}$$

Applying Schur complement formula, we can show that (7) corresponding to (5).

4 Examples

In this section, the following example is provided to illustrate the advantage of the proposed results.

Example 4.1 *Consider the following discrete-time delay system:*

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0.8+\alpha & 0 \\ 0 & 0.97 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix} x(k-d_k) \\ &\qquad + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} w(k), \\ z(k) &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} x(k) \end{aligned}$$

where α satisfies $|\alpha| \le \bar{\alpha}$ for $\bar{\alpha}$ is an upper bound of $\alpha(k)$. First, we consider the H_∞ disturbance attenuation for a nominal time-delay system with $\alpha(k) = 0$. Theorem 3.1 gives minimum lower bound of γ for different time-delay d_k in Table 1. When a constant delay $d_k = 10$ is considered, Theorem 3.1 gives a smaller γ than [11]. For a time-varying delay $0 \le d_k \le 6$, Table 1 shows Theorem 3.1 gives better results than [2] and [18].

Table 1. The minimum lower bound of γ

Time-Delay	*Approach*	*Minimum γ*
Time-Invariant	*[11]*	*4.7470*
$d_k = 10$	*Theorem 3.1*	*1.3102*
Time-Varying	*[2]*	*1.0847*
	[18]	*0.9265*
$0 \le d_k \le 6$	*Theorem 3.1*	*0.5448*

Next, we consider the robust H_∞ disturbance attenuation for the uncertain time-delay system with $\alpha(k) \neq 0$. In this case, system matrices can be represented in the form of (1) with matrices given by

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \; E = [\,1 \quad 0\,], E_d = E_1 = [\,0 \quad 0\,],$$

$$B = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \; C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \; H = \begin{bmatrix} \bar{\alpha} \\ 0 \end{bmatrix}, \; F(k) = \frac{\alpha(k)}{\bar{\alpha}}.$$

For a time-varying delay $0 \leq d_k \leq 6$, Theorem 3.4 gives minimum lower bound of γ for different $\bar{\alpha}$ in Table 2.

Table 2. The minimum lower bound of γ

Time-Delay	$\bar{\alpha}$	*Minimum γ*
$0 \leq d_k \leq 6$	*0.05*	*0.5760*
	0.10	*0.6687*
	0.15	*1.0318*
	0.20	*4.1300*

5 Conclusions

In this paper, we proposed new H_∞ disturbance attenuation condition and robust H_∞ disturbance attenuation condition for discrete-time systems with time-varying delay. Our conditions were obtained by introducing new Lyapunov function and using Leibniz-Newton formula and free weighting matrix method. They have less LMI slack variables than those of the existing methods. Numerical examples showed that our conditions are less conservative than other existing results.

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