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Subclass of Univalent Functions with Negative Coefficients and Starlike with Respect to Symmetric and Conjugate Points

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Abstract

Let S be the class of functions analytic and univalent in the open unit disc given by $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ and $a_k \in \mathcal{C}$. In this paper we have studied three subclasses $S_s^* M(\alpha, \beta, \delta), S_c^* M(\alpha, \beta, \delta)$ and $S_{sc}^* M(\alpha, \beta, \delta)$ consisting of analytic functions with negative coefficients and starlike with respect to symmetric points, starlike with respect to conjugate points and starlike with respect to symmetric conjugate points, respectively. Here we discuss coefficient inequality, growth, distortion, extreme points, convex combination and convolution properties of the three classes.

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1. Introduction

Let S be the class of functions analytic and univalent in the open unit disc given by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

and $a_k \in \mathcal{C}.M$ be the subclass of S consisting of functions f of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k \tag{2}$$

where $a_k \geq 0, a_k \in I\!R$. In [5], Sakaguchi introduced the class of analytic functions which are univalent and starlike with respect to symmetric points. This class is denoted by S_s^* and satisfies $Re\left\{\frac{zf'}{f(z)-f(-z)}\right\} > 0$ for $z \in \mathbb{D}$. This definition has given rise to many generalized and extended classes of functions. The subclasses $S_s^*M(\alpha,\beta,\delta), S_c^*M(\alpha,\beta,\delta)$ and $S_{sc}^*M(\alpha,\beta,\delta)$ consisting of analytic functions with negative coefficients were introduced by Halim and et. al. in [1], and are respectively starlike with respect to symmetric points, starlike with respect to conjugate points, and starlike with respect to symmetric conjugate points. Here α, β satisfy the conditions $0 \leq \alpha < 1, 0 < \beta < 1, 0 \leq \delta < 1$ and $0 < \frac{2(1-\beta)}{1+\alpha\beta} < 1$. This paper extends the results in [2] to other properties namely, distortion, convex combination and convolution. Let S^* be the subclass of S consisting of functions starlike in D. Notice that $f \in S^*$ iff $Re\left(z\frac{f'}{f}\right) > 0$ for $z \in D$.

Consider S_s^*, the subclass of S consisting of functions given by (1) satisfying $Re\left\{\frac{zf'(z)}{f(z)-f(-z)}\right\} > 0, z \in D$. These functions are called starlike with respect to symmetric points and were introduced by Sakaguchi in [5]. The same class is also considered by Robertson [8], Stankiewicz [4], Wu [14], Owa et. al. [12], and Aini Janteng, M. Darus [2]. El-Ashwah and Thomas in [10], have introduced two other subclasses namely S_c^* and S_{sc}^*.

In [13], Sudharsan et. al. and Aini Janteng, M. Darus in [2] have discussed the subclass $S_s^*(\alpha,\beta,\delta)$ of functions f analytic and univalent in $I\!I$ given by (1) and satisfying the condition

$$\left|\frac{zf'(z)}{f(z)-f(-z)} - (1+\delta)\right| < \beta\left|\frac{\alpha zf'(z)}{f(z)-f(-z)} + (1-\delta)\right|$$

for some $0 \leq \alpha \leq 1, 0 < \beta \leq 1$ and $z \in I\!D$.

However, in this paper we consider the subclass M defined by (2).

Definition 1.1 : *A function $f \in S_s^*M(\alpha,\beta,\delta)$ is said to be starlike with respect to symmetric points if it satisfies*

$$\left|\frac{zf'(z)}{f(z)-f(-z)} - (1+\delta)\right| < \beta\left|\frac{\alpha zf'(z)}{f(z)-f(-z)} + (1-\delta)\right| \text{ for } z \in D.$$

Definition 1.2 : *A function $f \in S_c^*M(\alpha,\beta,\delta)$ is said to be starlike with respect* *to conjugate points if it satisfies*

$$\left|\frac{zf'(z)}{f(z)+\overline{f(\overline{z})}}-(1+\delta)\right|<\beta\left|\frac{\alpha zf'(z)}{f(z)+\overline{f(\overline{z})}}+(1-\delta)\right| \quad \text{for} \quad z\in D.$$

Definition 1.3. *A function $f\in S^*_{sc}M(\alpha,\beta,\delta)$ is said to be starlike with respect to symmetric conjugate points if it satisfies*

$$\left|\frac{zf'(z)}{f(z)-\overline{f(-\overline{z})}}-(1+\delta)\right|<\beta\left|\frac{\alpha zf'(z)}{f(z)-\overline{f(-\overline{z})}}+(1-\delta)\right| \quad \text{for} \quad z\in I\!D.$$

Notice that the above conditions imposed on α,β and δ in the introduction are necessary to ensure that these classes form a subclass of S.

First we state the preliminary results similar to those obtained by Halim et. al. in [1], required for proving our main results.

2. Preliminaries

Theorem 2.1: *$f\in S^*_sM(\alpha,\beta,\delta)$ if and only if*

$$\sum_{k=2}^{\infty}\frac{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta))-(1+2\delta)}a_k\le 1. \tag{3}$$

Corollary 2.1 : *If $f\in S^*_sM(\alpha,\beta,\delta)$ then*

$$a_k\le\frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)},\quad k\ge 2.$$

Theorem 2.2 : *$f\in S^*_cM(\alpha,\beta,\delta)$ if and only if*

$$\sum_{k=2}^{\infty}\frac{k(1+\beta\alpha)+2[\beta-1-\delta(1+\beta)]}{\beta(\alpha+2(1-\delta))-1}a_k\le 1. \tag{4}$$

Corollary 2.2 : *If $f\in S^*_cM(\alpha,\beta,\delta)$ then*

$$a_k\le\frac{\beta(\alpha+2(1-\delta))-1}{k(1+\beta\alpha)+2[\beta-1-\delta(1+\beta)]},\quad k\ge 2.$$

Theorem 2.3: *$f\in S^*_{sc}M(\alpha,\beta,\delta)$ if and only if*

$$\sum_{k=2}^{\infty}\frac{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta))-(1+2\delta)}a_k\le 1. \tag{5}$$

Corollary 2.3 : *If* $f \in S^*_{sc}M(\alpha,\beta,\delta)$ *then*

$$a_k \leq \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)}, \quad k \geq 2.$$

3. Growth and Distortion Theorems

Theorem 3.1 : *Let the function* f *be defined by (2) and belong to the class* $S^*_sM(\alpha,\beta,\delta)$. *Then for* $\{z : 0 < |z| = r < 1\}$,

$$r - \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{2(1+\beta\alpha)}r^2 \leq |f(z)| \leq r + \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{2(1+\beta\alpha)}r^2.$$

Proof. Let

$$\begin{aligned} f(z) &= z - \sum_{k=2}^{\infty} a_k z^k \\ |f(z)| &\leq |z| + \sum_{k=2}^{\infty} a_k |z|^k \leq r + r^2 \sum_{k=2}^{\infty} a_k \leq r + r^2 \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{2(1+\beta\alpha)} \quad (6) \end{aligned}$$

since (3) implies

$$2(1+\beta\alpha)\sum_{k=2}^{\infty} a_k \leq \beta(\alpha+2(1-\delta))-(1+2\delta).$$

Similarly

$$\begin{aligned} |f(z)| &\geq |z| - \sum_{k=2}^{\infty} a_k |z|^k \\ &\geq |z| - |z|^2 \sum_{k=2}^{\infty} a_k = r - r^2 \sum_{k=2}^{\infty} a_k \geq r - r^2 \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{2(1+\beta\alpha)}. \quad (7) \end{aligned}$$

Hence the result.

The result is sharp for

$$f(z) = z - \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{2(1+\beta\alpha)}z^2 \quad \text{at} \quad z = \pm r.$$

Next we state similar results for functions belonging to $S^*_cM(\alpha,\beta,\delta)$ and $S^*_{sc}M(\alpha,\beta,\delta)$. Method of proof is same as in Theorem 3.1.

Theorem 3.2 : *Let the function* f *be defined by (2) and belong to the class* $S^*_cM(\alpha,\beta,\delta)$. *Then for* $\{z : 0 < |z| = r < 1\}$,

$$r - \frac{\beta(\alpha+2(1-\delta))-1}{2[\beta(\alpha+1)-\delta(1+\beta)]}r^2 \leq |f(z)| \leq r + \frac{\beta(\alpha+2(1-\delta))-1}{2[\beta(\alpha+1)-\delta(1+\beta)]}r^2.$$

the result is sharp for

$$f(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - 1}{2[\beta(\alpha+1) - \delta(1+\beta)]} z^2 \quad \text{at} \quad z = \pm r.$$

Theorem 3.3 : *Let the function f be defined by (2) and belong to the class $S^*_{sc}M(\alpha, \beta, \delta)$. Then for $\{z : 0 < |z| = r < 1\}$,*

$$r - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{2(1+\beta\alpha)} r^2 \leq |f(z)| \leq r + \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{2(1+\beta\alpha)} r^2.$$

The result is sharp for

$$f(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{2(1+\beta\alpha)} z^2 \quad \text{at} \quad z = \pm r.$$

Next we state the distortion theorems.

Theorem 3.4 : *Let $f \in S^*_s M(\alpha, \beta, \delta)$. Then for $\{z : 0 < |z| = r < 1\}$*

$$1 - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{(1+\beta\alpha)} r \leq |f'(z)| \leq 1 + \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{(1+\beta\alpha)} r.$$

The result is sharp for

$$f(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{2(1+\beta\alpha)} z^2.$$

Theorem 3.5 : *Let f be the function defined by (2) and belonging to the class $S^*_c M(\alpha, \beta, \delta)$. Then for $\{z : 0 < |z| = r < 1\}$*

$$1 - \frac{\beta(\alpha + 2(1-\delta)) - 1}{\beta(\alpha+1) - \delta(1+\beta)} r \leq |f'(z)| \leq 1 + \frac{\beta(\alpha + 2(1-\delta)) - 1}{\beta(\alpha+1) - \delta(1+\beta)} r.$$

The result is sharp for

$$f(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - 1}{2[\beta(\alpha+1) - \delta(1+\beta)]} z^2 \quad \text{at} \quad z = \pm r.$$

Theorem 3.6 : *Let f be the function defined by (2) and belonging to the class $S^*_{sc} M(\alpha, \beta, \delta)$. Then for $\{z : 0 < |z| = r < 1\}$*

$$1 - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{(1+\beta\alpha)} r \leq |f'(z)| \leq 1 + \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{(1+\beta\alpha)} r.$$

The result is sharp for

$$f(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{2(1+\beta\alpha)} z^2 \quad \text{at} \quad z = \pm r.$$

4. Closure Theorems

All the three subclasses discussed here are closed under convex linear combinations. We prove for the class $S_s^*M(\alpha,\beta,\delta)$. It can easily be proved similarly for $S_c^*M(\alpha,\beta,\delta)$ and $S_{sc}^*M(\alpha,\beta,\delta)$.

Theorem 4.1 : *Consider* $f_j(z) = z - \sum_{k=2}^{\infty} a_{k,j} z^k \in S_s^*M(\alpha,\beta,\delta)$ *for* $j = 1, 2, \cdots, \ell$ *then* $g(z) = \sum_{j=1}^{\ell} c_j f_j(z) \in S_s^*M(\alpha,\beta,\delta)$ *where* $\sum_{j=1}^{\ell} c_j = 1$.

Proof. Let

$$\begin{aligned} g(z) &= \sum_{j=1}^{\ell} c_j \left(z - \sum_{k=2}^{\infty} a_{k,j} z^k \right) \\ &= z - \sum_{k=2}^{\infty} z^k \sum_{j=1}^{\ell} c_j a_{k,j} \\ &= z - \sum_{k=2}^{\infty} e_k z^k \quad \text{where} \quad e_k = \sum_{j=1}^{\ell} c_j a_{k,j}. \end{aligned}$$

Now $g(z) \in S_s^*M(\alpha,\beta,\delta)$ since

$$\begin{aligned} &\sum_{k=2}^{\infty} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)}{\beta(\alpha + 2(1-\delta)) - (1+2\delta)} e_k \\ &\leq \sum_{k=2}^{\infty} \sum_{j=1}^{\ell} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)}{\beta(\alpha + 2(1-\delta) - (1+2\delta)} c_j a_{k,j} \\ &\leq \sum_{j=1}^{\ell} c_j = 1 \quad \text{since} \quad \sum_{k=2}^{\infty} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)](-1)^k - 1)}{\beta(\alpha + 2(1-\delta)) - (1+2\delta) a_{k,j}} \leq 1. \end{aligned}$$

5. Extreme Points

Theorem 5.1 : *Let* $f_1(z) = z$, $f_k(z) = z - \frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)} z^k$ *for* $k \geq 2$. *Then* $f \in S_s^*M(\alpha,\beta,\delta)$ *if and only if it can be expressed in the form* $f(z) = \sum_{k=1}^{\infty} \lambda_k f_k(z)$ *where* $\lambda_k \geq 0$ *and* $\sum_{k=1}^{\infty} \lambda_k = 1$.

Proof. Let

$$\begin{aligned} f(z) &= \sum_{k=1}^{\infty} \lambda_k f_k(z) \\ &= z - \sum_{k=2}^{\infty} \lambda_k \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)} z^k. \qquad (8) \end{aligned}$$

Now $f(z) \in S_s^* M(\alpha, \beta, \delta)$ since

$$\sum_{k=2}^{\infty} \left\{ \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)}{\beta(\alpha + 2(1-\delta)) - (1+2\delta)} \right\}$$
$$\frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)} \lambda_k = \sum_{k=2}^{\infty} \lambda_k = 1 - \lambda_1 \leq 1.$$

Conversely, suppose that $f \in S_s^* M(\alpha, \beta, \delta)$. Then by Corollary 2.1

$$a_k \leq \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)}, \quad k \geq 2$$

set

$$\lambda_k = \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)}{\beta(\alpha + 2(1-\delta)) - (1+2\delta)} a_k, \quad k \geq 2 \tag{9}$$

and $\lambda_1 = 1 - \sum_{k=2}^{\infty} \lambda_k$ then $f(z) = \sum_{k=1}^{\infty} \lambda_k f_k(z)$. Similarly extreme points for functions belonging to $S_s^* M(\alpha, \beta, \delta)$ and $S_{sc}^* M(\alpha, \beta, \delta)$ are found.

Methods of proving Theorem 5.2 and Theorem 5.3 are similar to that of Theorem 5.1.

Theorem 5.2 : *Let* $f_1(z) = z$,

$$f_k(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - 1}{k(1+\beta\alpha) + 2[\beta - 1 - \delta(1+\beta)]} z^k, \quad k \geq 2.$$

Then $f \in S_c^* M(\alpha, \beta, \delta)$ *if and only if it can be expressed in the form* $f(z) = \sum_{k=1}^{\infty} \lambda_k f_k(z)$ *where* $\lambda_k \geq 0$ *and* $\sum_{k=1}^{\infty} \lambda_k = 1$.

Theorem 5.3 : *Let* $f_1(z) = z$,

$$f_k(z) = z - \frac{\beta(\alpha + 2(1-\delta)) - (1+2\delta)}{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)} z^k, \quad k \geq 2.$$

Then $f \in S_{sc}^* M(\alpha, \beta, \delta)$ *if and only if it can be expressed in the form* $f(z) = \sum_{k=1}^{\infty} \lambda_k f_k(z)$ *where* $\lambda_k \geq 0$ *and* $\sum_{k=1}^{\infty} \lambda_k = 1$.

6. Convolution Theorems

The three subclasses $S_s^* M(\alpha, \beta, \delta)$, $S_c^* M(\alpha, \beta, \delta)$ and $S_{sc}^* M(\alpha, \beta, \delta)$ are closed under convolution. We prove for the class $S_s^* M(\alpha, \beta, \delta)$.

Theorem 6.1 : *Let* $f, g \in S_s^*M(\alpha,\beta,\delta)$ *where* $f(z) = z - \sum_{k=2}^{\infty} a_k z^k$ *and* $g(z) = z - \sum_{k=2}^{\infty} b_k z^k$ *then* $f * g \in S_s^*M(\alpha,\beta,\gamma)$ *for*

$$\gamma \geq \frac{k + (1+\delta)((-1)^k - 1)[N]^2 + (1+2\delta)[D]^2}{(\alpha+2(1-\delta))[D]^2 - (\alpha k + (\delta-1)((-1)^k-1)[N]^2}$$

where $N = \beta(\alpha + 2(1-\delta)) - (1+2\delta)$ *and*
$D = k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k - 1)$.
Proof : We have $f \in S_s^*M(\alpha,\beta,\delta)$ if and only if

$$\sum_{k=2}^{\infty} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta)) - (1+2\delta)} a_k \leq 1. \qquad (10)$$

Similarly $g \in S_s^*M(\alpha,\beta,\delta)$ if and only if

$$\sum_{k=2}^{\infty} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta)) - (1+2\delta)} b_k \leq 1. \qquad (11)$$

To find a smallest number γ such that

$$\sum_{k=2}^{\infty} \frac{k(1+\gamma\alpha) = [1-\gamma+\delta(1+\gamma)]((-1)^k-1)}{\gamma(\alpha+2(1-\delta)) - (1+2\delta)} a_k b_k \leq 1 \qquad (12)$$

By Cauchy Schwarz inequality (10) and (11) imply

$$\sum_{k=2}^{\infty} \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta)) - (1+2\delta)} \sqrt{a_k b_k} \leq 1 \qquad (13)$$

(12) will hold for

$$\frac{k(1+\gamma\alpha) + [1-\gamma+\delta(1+\gamma)]((-1)^k-1)}{\gamma(\alpha+2(1-\delta)) - (1+2\delta)} a_k b_k \leq \frac{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)}{\beta(\alpha+2(1-\delta)) - (1+2\delta)} \sqrt{a_k b_k}.$$

That is if

$$\sqrt{a_k b_k} \leq \frac{[\gamma(\alpha+2(1-\delta)) - (1+2\delta)][k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)]}{[\beta(\alpha+2(1-\delta)) - (1+2\delta)][k(1+\gamma\alpha) + [1-\gamma+\delta(1+\gamma)]((-1)^k-1)]} \qquad (14)$$

(13) implies

$$\sqrt{a_k b_k} \leq \frac{\beta(\alpha+2(1-\delta)) - (1+2\delta)}{k(1+\beta\alpha) + [1-\beta+\delta(1+\beta)]((-1)^k-1)}. \qquad (15)$$

Thus it is enough to show that

$$\frac{\beta(\alpha+2(1-\delta))-(1+2\delta)}{k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)k-1)}$$
$$\leq \frac{[\gamma(\alpha+2(1-\delta))-(1+2\delta)][k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1)]}{[\beta(\alpha+2(1-\delta))-(1+2\delta)][k(1+\gamma\alpha)+[1-\gamma+\delta(1+\gamma)]((-1)^k-1)]}$$

which simplifies to

$$\gamma \geq \frac{[k+(1+\delta)(-1)^k-1)][N]^2+(1+2\delta)[D]^2}{(\alpha+2(1-\delta))[D]^2-(\alpha k+(\delta-1)((-1)^k-1)[N]^2}$$

where

$$N=\beta(\alpha+2(1-\delta))-(1+2\delta) \quad \text{and} \tag{16}$$

$$D=k(1+\beta\alpha)+[1-\beta+\delta(1+\beta)]((-1)^k-1) \tag{17}$$

Theorem 6.2 : *Let* $f, g \in S_c^*M(\alpha,\beta,\delta)$ *then* $f * g \in S_c^*M(\alpha,\gamma,\delta)$ *where*

$$\gamma \geq \frac{(k-2(1+\delta))[N]^2+[D]^2}{(\alpha+2(\delta-1))[D]^2-(k\alpha+2(1-\delta))[N]^2}$$

where $N=\beta(\alpha+2(1-\delta))-1$ *and* $D=k(1+\beta\alpha)+2[\beta(1-\delta)-(1+\delta)]$.

Convolution theorem for subclass $S_{sc}^*M(\alpha,\beta,\delta)$ is similar to Theorem 6.1.

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