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On Restricted Edge-Connectivity of Vertex-Transitive Graphs*

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Abstract: It is known that for connected vertex-transitive graphs of degree $k(\geqslant 2)$, the restricted edge-connectivity $k \leqslant \lambda' \leqslant 2k-2$ and the bounds can be attained. Two necessary and sufficient conditions for a vertex-transitive graph G of degree k to admit $\lambda'(G) = k$ are presented. Afterwards, for any connected graph G_0, $\lambda'(K_2 \times G_0)$ is determined to be $\lambda'(K_2 \times G_0) = \min\{2\delta(G_0), 2\lambda'(G_0), v(G_0)\}$, and then for any given integer s with $0 \leqslant s \leqslant k-3$, there is a connected vertex-transitive graph G of degree k and $\lambda'(G) = k + s$ if and only if either k is odd or s is even.

Key words: connectivity; restricted edge-connectivity; transitive graphs; circulant graphs

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0 Introduction

We follow [1] for graph-theoretical terminology and notation not defined here. A graph $G = (V, E)$ always means a simple graph (without loops and multiple edges), with vertex-set $V = V(G)$ and edge-set $E = E(G)$. In this paper, we consider the restricted edge-connectivity, which is a new graph-theoretical parameter introduced by Esfahanian and Hakimi [3]. For the sake of convenience, the graph considered in this note is a connected graph, not a triangle or a star.

Let $S \subseteq E(G)$. If $G - S$ is disconnected and contains no isolated vertices, then S is called a restricted edge-cut of G. The restricted edge-connectivity of G, denoted by $\lambda'(G)$, is defined as the minimum cardinality over all restricted edge-cuts of G. The restricted edge-connectivity provides a more accurate measure of fault-tolerance of networks than the classical edge-connectivity (see [2]). Thus, it has received much attention recently (see, for example, [2 – 13]).

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For $e = xy \in E(G)$, let $\xi_G(e) = d_G(x) + d_G(y) - 2$. The minimum edge-degree of G is defined to be $\xi(G) = \min\{\xi_G(e) | e \in E(G)\}$. It was shown in [3] that

$$\lambda(G) \leqslant \lambda'(G) \leqslant \xi(G), \qquad (1)$$

where $\lambda(G)$ is the edge-connectivity of G. A graph G is said to be optimal if $\lambda'(G) = \xi(G)$, and non-optimal otherwise.

From inequality (1), it is clear that $k \leqslant \lambda'(G) \leqslant 2k - 2 = \xi(G)$ for a connected vertex-transitive graph G of degree $k(\geqslant 2)$ since $\lambda(G) = k$. There are optimal and vertex-transitive graphs, such as the complete graph K_{k+1} and the hypercube Q_k. Recently, it has been shown in [10,12] that for any non-optimal and vertex-transitive graph G of degree k there is an integer $m(\geqslant 2)$ such that $\lambda'(G) = \frac{n}{m}$. In this note, it is pointed out that for any given integers $k(\geqslant 3)$ and s with $0 \leqslant s < k - 2$, there is a connected vertex-transitive graph G with degree k and $\lambda'(G) = k + s$ if and only if either k is odd or s is even.

The rest of the note is organized as follows. Section 1 contains necessary definitions and known results. Section 2 gives two necessary and sufficient conditions for a vertex-transitive graph G of degree k to admit $\lambda'(G) = k$. Section 3 proves $\lambda'(K_2 \times G_0) = \min\{2\delta(G_0), 2\lambda'(G_0), v(G_0)\}$ for any connected graph G_0, and constructs a class of non-optimal and vertex-transitive graphs with degree k and $\lambda' = k + s$ for any odd k or even s with $k \geqslant 3$ and $0 \leqslant s < k - 2$.

1 Notation and Lemmas

Let $G = (V, E)$ be a graph. For two disjoint non-empty subsets X and Y of $V(G)$, let $(X, Y)_G = \{e = xy \in E(G): x \in X \text{ and } y \in Y\}$. If $Y = \overline{X} = V(G) \backslash X$, then we write $\partial_G(X)$ for $(X, \overline{X})_G$ and $d_G(X)$ for $|\partial_G(X)|$.

A restricted edge-cut S of G is called a λ'-cut if $|S| = \lambda'(G)$. It is easy to see that $G - S$ has just two connected components for any λ'-cut S. A non-empty and proper subset X of $V(G)$ is called a λ'-fragment of G if $\partial_G(X)$ is a λ'-cut of (G). The minimum λ'-fragment over all λ'-fragments of G is called a λ'-atom of G. The cardinality of a λ'-atom of G is denoted by $a(G)$.

Lemma 1[13] Let G be a non-optimal graph. Then any two distinct λ'-atoms of G are disjoint, and $a(G) \geqslant k \geqslant 3$ if G is k-regular.

Lemma 2[10,12] Let G be a non-optimal and vertex-transitive graph of degree $k(\geqslant 3)$, and X a λ'-atom of G. Then,

(i) $G[X]$ is a vertex-transitive subgraph of degree $k - 1$;

(ii) There is a partition $\{X_1, X_2, \cdots, X_m\}$ of $V(G)$ such that $G[X_i] \cong G[X]$ for each $i = 1, 2, \cdots, m$, $m \geqslant 2$.

Lemma 3(Theorem 2.3.5 in [12]) The Cartesian product of vertex-transitive graphs is a vertex-transitive graph.

2 Two necessary and sufficient conditions

In this section, we will give two necessary and sufficient conditions for a non-optimal vertex-transitive graph G of degree k to admit $\lambda'(G) = k$.

Theorem 1 Let G be a non-optimal and vertex-transitive graph of degree k. Then $\lambda'(G) = k$ if and only if the induced subgraph $G[X]$ is a complete graph of order k for any λ'-atom X of G.

Proof Let X be a λ'-atom of G, and $s = |X|$. Then $G[X]$ is a vertex-transitive subgraph of G of degree $k-1$ by Lemma 2. It follows that

$$sk = \sum_{x \in X} d_G(x) = \sum_{x \in X} d_{G[X]}(x) + \lambda'(G) = s(k-1) + \lambda'(G). \tag{2}$$

Suppose that $\lambda'(G) = k$. From (2), we have $sk = s(k-1) + k$, which implies that $s = k$, and $G[X]$ is a complete graph of order k.

Conversely, suppose that $G[X]$ is a complete graph of order k. Then from (2), we have $k^2 = k(k-1) + \lambda'(G)$, which means that $\lambda'(G) = k$.

Lemma 4 Let G be a non-optimal, k-regular and connected graph. If G contains a complete graph K_k, then $X = V(K_k)$ is a λ'-atom of G, and hence $\lambda'(G) = k$.

Proof Since G is a non-optimal k-regular graph, by Lemma 1, $k \geqslant 3$. Let X be the vertex-set of a complete subgraph K_k of G. Then $|X| = k \geqslant 3$. We will first prove that $G - \partial_G(X)$ contains no isolated vertices. Suppose to the contrary that $G - \partial_G(X)$ contains an isolated vertex x. Then $x \in V(G) \backslash X$ and $N_G(x) \subseteq X$. Noting that $d_G(x) = k = |X|$, we have $N_G(x) = X$. Since G is k-regular and connected, G is a complete graph of order $k+1$, which is optimal. This contradicts the assumption that G is non-optimal. Therefore, $G - \partial_G(X)$ contains no isolated vertices. Thus, $\partial_G(X)$ is a restricted edge-cut of G. It follows from (1) that

$$k = \lambda(G) \leqslant \lambda'(G) \leqslant |\partial_G(X)| = d_G(X) = k,$$

which means $\lambda'(G) = k$, namely, $\partial_G(X)$ is a λ'-cut of G. By Lemma 1, $k \leqslant |X| = k$, which means X is a λ'-atom of G.

By Theorem 1 and Lemma 4, we have the following result immediately.

Theorem 2 Let G be a non-optimal and connected vertex-transitive graph of degree $k (\geqslant 3)$. Then $\lambda'(G) = k$ if and only if G contains a complete graph of order k.

Theorem 3 Let G be a non-optimal and connected vertex-transitive graph. Then G has a prefect matching, and hence G has even order.

Proof By Lemma 2, there is a λ'-atom partition $\{X_1, X_2, \cdots, X_m\}$ of $V(G)$ such that $G[X_i]$ is a vertex-transitive subgraph of G of degree $k-1$, where $m \geqslant 2$. Let

$$M = E(G) \backslash (E(G[X_1]) \cup \cdots \cup E(G[X_m])).$$

It is clear that M is a matching of G since any two distinct edges in M have no end-vertices in common. On the other hand, since $G[X_i]$ is a $(k-1)$-regular subgraph of G, for any $x \in V(G)$, there must exist one edge $e \in M$ such that x is an end-vertex of e. This means M is a prefect matching of G.

3 Main results

We present our main results in this section. We consider the Cartesian product $K_2 \times G_0$ of K_2 and G_0, where K_2 is the complete graph of order 2 and G_0 is a connected graph of order $v(G_0) \geqslant 2$. Let $V(K_2) = \{0,1\}$ and $V(G_0) = \{x_1, x_2, \cdots, x_n\}$. By the definition of the Catesian product, $K_2 \times G_0$ is obtained from two copies of G_0 by connecting (via a new edge) vertex x_i in one copy to the vertex x_i in the other copy for each $i = 1, 2, \cdots, n$. Let $G = K_2 \times G_0$, then G can be expressed as the union of two disjoint subgraphs of G that are isomorphic to G_0. Let G_1 and G_2 be such two subgraphs of G and

$$V_1 = V(G_1) = \{0x_i : 1 \leqslant i \leqslant n\}, \quad V_2 = V(G_2) = \{1x_i : 1 \leqslant i \leqslant n\}.$$

It is clear that $\xi(K_2 \times G_0) = 2\delta(G_0)$, and hence $\lambda'(K_2 \times G_0) \leqslant 2\delta(G_0)$. We denote $\lambda'(G_0) = \infty$ if $\lambda'(G_0)$ does not exist (such graphs are only K_2, K_3 and the star $K_{1,n}$). The following two facts are also clear. If $v(G_0) \geqslant 2$, then $\partial_G(V_1)$ is a restricted edge-cut of G, thus, $\lambda'(G) \leqslant |\partial_G(V_1)| = |V_1| = v(G_0)$. If X_0 is a λ'-atom of G_0, then $\partial_G(0X_0 \cup 1X_0)$ is a restricted edge-cut of G, thus, $\lambda'(G) \leqslant |\partial_G(0X_0 \cup 1X_0)| = 2|\partial_{G_0}(X_0)| = 2\lambda'(G_0)$. It follows that

$$\lambda'(K_2 \times G_0) \leqslant \min\{v(G_0),\ 2\delta(G_0),\ 2\lambda'(G_0)\}. \tag{3}$$

We will prove below that the equality in (3) holds.

Theorem 4 Let G_0 be a connected graph of order $v(G_0)(\geqslant 2)$. Then

$$\lambda'(K_2 \times G_0) = \min\{v(G_0), 2\delta(G_0), 2\lambda'(G_0)\}.$$

Proof Let $G = K_2 \times G_0$. It is easy to check that the theorem holds if $G_0 = K_2$, K_3 or $K_{1,n}$. Then we may suppose $\lambda'(G_0)$ is well defined. Also, it is clear from the definition of $G = K_2 \times G_0$ that every edge of G is included in a cycle of G, which deduces $\lambda'(G) \geqslant 2$. Thus, if $\delta(G_0) = 1$, then $\lambda'(G) = 2 = 2\delta(G_0)$, and the theorem is true. Suppose $\delta(G_0) \geqslant 2$ below.

Let X be a λ'-atom of G. Then $d_G(X) = \lambda'(G)$. We consider three cases according to the behavior of X respectively.

Case 1 If $X = V_1$ (or V_2), then clearly $\lambda'(G) = v(G_0)$.

Case 2 $X \subset V_1$ (or V_2). In this case, we assert that $\partial_G(X)$ is a restricted edge-cut of G_0. Suppose to the contrary that there is an isolated vertex x in $G_0 - \partial_G(X)$. Then $N_{G_0}(x) \subseteq X$. Note that $G - \partial_G(X \cup \{x\})$ has no isolated vertices. Therefore, $\partial_G(X \cup \{x\})$ is a restricted edge-cut of G. Since $d_{G_0}(x) = |N_{G_0}(x)| \geqslant \delta(G_0) \geqslant 2$,

$$\lambda'(G) \leqslant d_G(X \cup \{x\}) = d_G(X) - d_{G_0}(x) + 1 < d_G(X) = \lambda'(G).$$

It's a contradiction. Thus, $\partial_G(X)$ is a restricted edge-cut of G_0, which means $d_{G_0}(X) \geqslant \lambda'(G_0)$. It follows that

$$2\lambda'(G_0) \geqslant \lambda'(G) = d_G(X) = |X| + d_{G_0}(X) \geqslant |X| + \lambda'(G_0),$$

which means $|X| \leqslant \lambda'(G_0)$. If there exists some $0x \in X$ such that $N_{G_1}(0x) \subseteq X$, then

$$2\delta(G_0) \leqslant 2d_{G_0}(x) \leqslant 2(|X| - 1) \leqslant |X| + \lambda'(G_0) - 2 <$$
$$d_G(X) = \lambda'(G) \leqslant 2\delta(G_0),$$

which is impossible. Thus, we may suppose that for any $0x \in X$, there is at least one edge in

$\partial_{G_0}(X)$, among the edges incident with x in G_0. Thus, for any two adjacent vertices $0x$ and $0y$ in $G[X]$, the number of edges in $\partial_{G_0}(X)$ incident with the two vertices is at most $(d_G(X)-|X|)-(|X|-2)$. It follows that

$$2\delta(G_0)\leqslant d_{G_0}(x)+d_{G_0}(y)\leqslant 2(|X|-1)+(d_G(X)-|X|)-(|X|-2)=$$
$$d_G(X)=\lambda'(G)\leqslant 2\delta(G_0),$$

which means that $\lambda'(G)=2\delta(G_0)$.

Case 3 $X_1=X\cap V_1\neq\emptyset$ and $X_2=X\cap V_2\neq\emptyset$. Let

$$X'_1=N_G(X_2)\cap V_1\qquad X'_2=N_G(X_1)\cap V_2.$$

We first show that

$$X'_1=X_1\qquad X'_2=X_2. \tag{4}$$

Suppose to the contrary that the equalities in (4) both are not true. Then at least one of the sets $X'_1\backslash X_1$ and $X'_2\backslash X_2$ is non-empty. We can, without loss of generality, suppose that $X'_1\backslash X_1\neq\emptyset$. Let $Y_1=X\cup(X'_1\backslash X_1)$, and $U_1=V_1\backslash(X_1\cup X'_1)$. It is clear that $G[Y_1]$ and $G[\overline{Y}_1]$ both contain no isolated vertices. Therefore, $\partial_G(Y_1)$ is a restricted edge-cut of G, and thus, $d_G(Y_1)\geqslant\lambda'(G)=d_G(X)$. We have then

$$d_G(X)\leqslant d_G(Y_1)=d_G(X\cup(X'_1\backslash X_1))=$$
$$d_G(X)-|X'_1\backslash X_1|-|(X'_1\backslash X_1,X_1)_G|+|(X'_1\backslash X_1,U_1)_G|,$$

which means that $|X'_1\backslash X_1|+|(X'_1\backslash X_1,X_1)_G|-|(X'_1\backslash X_1,U_1)_G|\leqslant 0$, namely,

$$|X'_1\backslash X_1|\leqslant|(X'_1\backslash X_1,U_1)_G|-|(X'_1\backslash X_1,X_1)_G|. \tag{5}$$

We consider the sets $Y_2=(X_2\cap X'_2)\cup X_1$ and $U_2=V(G_2)\backslash X_2$. It is clear that $|Y_2|\leqslant|X|$, and the subgraphs $G[Y_2]$ and $G[\overline{Y}_2]$ both contain no isolated vertices. Therefore, $\partial_G(Y_2)$ is a restricted edge-cut of G, and

$$\lambda'(G)\leqslant d_G(Y_2)=d_G((X_2\cap X'_2)\cup X_1)=$$
$$d_G(X)-|X_2\backslash X'_2|-|(X_2\backslash X'_2,U_2)_G|+|(X_2\backslash X'_2,X_2\cap X'_2)_G|.$$

By the construction of G, it is clear that $G[X_2\backslash X'_2]\cong G[X'_1\backslash X_1]$, and so

$$\begin{aligned}&|X_2\backslash X'_2|=|X'_1\backslash X_1|,\\&|(X_2\backslash X'_2,\ U_2)_G|\geqslant|(X'_1\backslash X_1,\ U_1)_G|,\\&|(X_2\backslash X'_2,\ X_2\cap X'_2)_G|\leqslant|(X'_1\backslash X_1,\ X_1)_G|.\end{aligned} \tag{7}$$

By inequalities (6), (7) and (5), we have that

$$\lambda'(G)\leqslant d_G(Y_2)\leqslant d_G(X)-|X'_1\backslash X_1|-|(X'_1\backslash X_1,U_1)_G|+|(X'_1\backslash X_1,X_1)_G|\leqslant$$
$$d_G(X)-2|X'_1\backslash X_1|<d_G(X)=\lambda'(G).$$

This contradiction implies that the equalities in (4) hold. Thus, $|X_1|=|X_2|$ and $d_G(X)=2d_{G_1}(X_1)$. If $|X_1|=1$, say $X=\{0x\}$, then $d_{G_1}(x)\geqslant\delta(G_0)$, and thus, $2\delta(G_0)\leqslant 2d_{G_0}(x)=d_G(X)\leqslant 2\delta(G_0)$, which means that $\lambda'(G)=2\delta(G_0)$.

Suppose $|X_1|\geqslant 2$. It is clear that $G[X_1]$ is connected as $G[X]$ is connected. In other words, $G_0[X_1]$ contains no isolated vertices. By the same consideration in case 2, it is easy to see $G_0[\overline{X}_1]$ contains no isolated vertices, where $\overline{X}_1=V(G_0)\backslash X_1$. Therefore, $\partial_{G_0}(X_1)$ is a restricted edge-cut of G_0, and so $d_{G_0}(X_1) \geqslant \lambda'(G_0)$. It follows that

$$2\lambda'(G_0) \leqslant 2d_{G_0}(X_1) = \lambda'(G) \leqslant 2\lambda'(G_0).$$

This means that $\lambda'(G) = 2\lambda'(G_0)$.

Summing up the three cases, we have that

$$\lambda'(K_2 \times G_0) \geqslant \min\{v(G_0),\ 2\delta(G_0),\ 2\lambda'(G_0)\}. \tag{8}$$

The theorem follows.

As consequences of Theorem 4, we can obtain the following results.

Corollary 1 Let G_0 be a connected vertex-transitive graph of degree k. Then $\lambda'(K_2 \times G_0) = \min\{2k,\ v(G_0)\}$.

Proof It is known $k = \lambda \leqslant \lambda'$ for any connected vertex-transitive graph of degree k. It follows that $\min\{2\delta(G_0),\ 2\lambda'(G_0),\ v(G_0)\} = \min\{2k,\ v(G_0)\}$.
By Theorem 4, $\lambda'(K_2 \times G_0) = \min\{2k,\ v(G_0)\}$.

Corollary 2[2] For hypercube Q_k ($k \geqslant 2$), $\lambda'(Q_k) = 2k - 2$.

Proof Since Q_{k-1} is a connected and vertex-transitive graph of degree $k-1$ for $k \geqslant 2$,

$$\min\{2(k-1),\ v(Q_{k-1})\} = \min\{2k-2,\ 2^{k-1}\} = 2k-2.$$

By $Q_k = K_2 \times Q_{k-1}$ and Corollary 1, $\lambda'(Q_k) = 2k-2$.

Theorem 5 For any given integers k and s with $k \geqslant 3$, $0 \leqslant s \leqslant k-3$, there is a connected vertex-transitive graph G with degree k and $\lambda'(G) = k+s$ if and only if either k is odd or s is even.

Proof Let k be even and s odd. Suppose to the contrary that there is a vertex-transitive graph G of degree k and $\lambda'(G) = k+s \leqslant 2k-3$. Then G is non-optimal. Let X be a λ'-atom of G. Consider the subgraph $G[X]$. By Lemma 2(i), $|X| = \lambda'(G) = k+s$, $k \geqslant 3$ and $G[X]$ is $(k-1)$-regular. It follows that

$$2|E(G[X])| = \sum_{x \in X} d_{G[X]}(x) = (k-1)|X| = (k-1)(k+s). \tag{9}$$

The left-hand side of (9) is even, but the right-hand side is odd, which is a contradiction. The necessity follows.

To prove the sufficiency, we consider the circulant graph $G(n; a_1, a_2, \cdots, a_k)$, where $0 < a_1 < \cdots < a_k \leqslant \frac{n}{2}$, having vertices $0, 1, 2, \cdots, n-1$ and edge ij if and only if $|j-i| \equiv a_t \pmod n$ for some t, $1 \leqslant t \leqslant k$. The circulant graph is vertex-transitive, and is $2k$-regular if $a_k \neq \frac{n}{2}$, and $(2k-1)$-regular otherwise.

Let $G = K_2 \times G_0$, where G_0 is a circulant graph. Then G is vertex-transitive by Lemma 3.

We show the sufficiency by selecting a circulant graph G_0 with degree $k-1$ and $\lambda'(G) = k+s$ according to the parity of k and s.

For $m \geqslant 1$, we select

$$G_0 = \begin{cases} G(k+s; 1, 2, \cdots, m), & \text{if } k = 2m+1, \\ G(k+s; 1, 2, \cdots, m-1, m+\frac{1}{2}s), & \text{if } k = 2m \text{ and } s \text{ is even.} \end{cases}$$

It is easy to check that $\delta(G_0) = k - 1$. Since for any s with $0 \leqslant s < k-2$,

$$2\delta(G_0) = 2k - 2 > k + s = \upsilon(G_0),$$

by Corollary 1, $\lambda'(G) = \upsilon(G_0) = k + s$, as required.

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点可迁图的限制边连通度

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摘要: 对于度 $k(\geqslant 2)$ 的点可迁连通图的限制边连通度 λ',已知 $k \leqslant \lambda' \leqslant 2k-2$,且 λ' 的界可以达到. 在此基础上,对度为 k 的点可迁图 G 进一步给出了满足 $\lambda'(G) = k$ 的两个充要条件. 接着,对任意的连通图 G_0 证明了 $\lambda'(K_2 \times G_0) = \min\{2\delta(G_0), 2\lambda'(G_0), \upsilon(G_0)\}$. 最后证明了对任意满足 $0 \leqslant s \leqslant k-3$ 的整数 s,存在度为 k 的点可迁连通图 G 满足 $\lambda'(G) = k + s$ 当且仅当 k 为奇数或者 s 为偶数.

关键词: 连通度;限制边连通度;可迁图;循环图