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Numerical Solution of Steady Two-Dimensional MHD Forward Stagnation-Point Flow

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Abstract

Steady two dimensional stagnation point flow of an incompressible viscous electrically conducting fluid over a flat plate is investigated. It is shown that the velocity at a point decreases/increases with increase in the magnetic field when free stream velocity is less/greater than the velocity of the plate.

Nomenclature:

MHD - Magneto hydrodynamics
u, v, w - Velocity components in X, Y, Z directions respectively
U, W- The mainstream velocity components
ρ - Field density.
ψ - Stream function

η - The single independent variable
σ - Electrical conductivity
g- Gravity, b- Unknown constant
B_0 - Imposed magnetic induction parallel to y – axis.
S - Magnetic field strength
S_0 - Modified Hartmann number
C_f - Skin friction coefficient
λ - Crocco independent variable
ϕ - New dependent variable, R_n - Reynold's number
α_1, α_2 - Flow parameter

1. Introduction:

Magneto hydrodynamics is the study of the motion of an electrically conducting fluid in the presence of a magnetic field. Due to the motion of an electrically conducting fluid in a magnetic field the electrical currents are induced in the fluid which produces their own magnetic field, called induced magnetic field, and these modify the original magnetic field. In addition to this the induced currents interacts with the magnetic field to produce electromagnetic forces perturbing the original motion. Thus the two important basic effects of Magneto hydrodynamics are (1) the motion of the fluid affects the magnetic field and (2) the magnetic field affects the motion of the fluid.

Boundary layer flow of an electrically conducting fluid over moving surfaces emerges in a large variety of industrial and technological applications. It has been investigated by many researchers; Wu [1] has studied the effects of suction or injection in a steady two-dimensional MHD boundary layer flow of on a flat plate. Takhar et.al. [2] studied a MHD asymmetric flow over a semi infinite moving surface and numerically obtained the solutions. An analysis of heat and mass transfer characteristics in an electrically conducting fluid over a linearly stretching sheet with variable wall temperature was investigated by Vajravelu and Rollins [3]. Mahapatra and Gupta [4] treated the steady two-dimensional stagnation-point flow of an incompressible viscous electrically conducting fluid towards a stretching surface, the flow being permeated by a uniform transverse magnetic field. Jean-David Hoernel [5] has been investigated the similarity solutions for the steady laminar incompressible boundary layer governing MHD flow near forward stagnation-point of two-dimensional and axisymmetric bodies.

The purpose of the present paper is to study the steady laminar, incompressible, viscous electrically conducting non-Newtonian power-law fluid at a stagnation point. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field is neglected [6]. The governing momentum equation is solved numerically.

2. Flow Analysis:

The MHD equations for steady two-dimensional laminar incompressible stagnation point flow for viscous, electrically conducting non-Newtonian power-law fluid are, in the usual notation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \textbf{------- (1)}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \frac{\partial}{\partial y} \tau_{yx} - \frac{\sigma B_0^{\,2}}{\rho} (U - u) \qquad \text{------- (2)}$$

where the induced magnetic field is neglected .It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. In (2), U(x) stands for the stagnation- point velocity in the inviscid free stream.

The appropriate boundary conditions are

$$\begin{aligned} &u = 0, \quad v = 0 \qquad at \qquad y = 0 \\ &u = U(x) \qquad at \qquad y = \infty \end{aligned} \qquad \text{------- (3)}$$

Introducing stream function ψ (x, y), where;

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \psi}{\partial x} \end{aligned} \qquad \text{------- (4)}$$

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial y\, \partial x} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = U \frac{\partial U}{\partial x} + \frac{n}{R_n} \frac{\partial}{\partial y} \left\{ \left[\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} \frac{\partial^2 \psi}{\partial y^2} \right\} + S \left[U - \frac{\partial \psi}{\partial y} \right]$$

-------(5)

where $S = \frac{\sigma B_0^{\,2}}{\rho}$ is a magnetic field strength.

$$\begin{aligned} &\frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = 0 \qquad at \qquad y = 0 \\ &\frac{\partial \psi}{\partial y} = U(x) \qquad at \qquad y = \infty \end{aligned} \qquad \text{--------- (6)}$$

Now on applying the free-parameter method (one of the methods of similarity transformation) introducing function $f(\eta)$,where $f'(\eta) = \frac{u}{U} = \frac{1}{U} \frac{\partial \psi}{\partial y}$, $\eta = (R_n)^{\frac{1}{n+1}} \frac{y}{\xi(x,t)}$ here $\xi(x, t)$ is an arbitrary function we get The laminar two dimensional incompressible boundary layer equation for stagnation point non-Newtonian MHD flow can written as;

$$f''' + f\left(f''\right)^{2-n} + \left(\frac{1+n}{2n}\right)\left(1-f'^{2}\right)\left(f''\right)^{1-n} + S_0\left(\frac{1+n}{2n}\right)\left(1-f'\right)\left(f''\right)^{1-n} = 0 \quad \text{------- (7)}$$

with boundary condition

$$\begin{aligned} &f = 0,\ \ f' = 0 \qquad \eta = 0 \\ &f' = 1 \qquad\qquad \eta \to \infty \end{aligned} \quad \text{------- (8)}$$

and $U = \alpha_1 x\ \ , S_1 = S_0\,\alpha_1\left(x+\alpha_2\right)^{m-1}$

where it is assumed that the magnetic induction is constant i.e. B = constant and therefore S_0 = constant. To solve equation (7) approximately let's introduce the Crocco independent variables λ instead of the independent variable η

$$\lambda = -f' = \frac{df}{d\eta} \quad \text{------- (9)}$$

Now, equation (7) becomes,

$$\phi\,\phi'' - \frac{\phi'^{2}}{n+1} - 2\lambda\phi - \left(1-\lambda^{2}\right)\phi' - S_0\left[\left(1-\lambda\right)\phi' + \left(n+1\right)\phi\right] = 0 \quad \text{------- (10)}$$

where $\phi = \dfrac{2\eta\left(f''\right)^{\eta+1}}{\eta+1}$ -------(11)

and $\phi' = \dfrac{d\phi}{d\lambda}$ -------(12)

ϕ is the new dependent variable. The corresponding boundary conditions are obtained by use of (7), (8) and (11) and the fact that $f'' = 0\ \ at\ \ \eta \to \infty$.

$$\begin{aligned} &\phi' = -\left(n+1\right)\left(1+S_0\right)\ at\ \lambda = 0 \\ &\phi = 0 \qquad\qquad at\ \ \lambda = 1 \end{aligned} \quad \text{------- (13)}$$

Combining (10) and (13) we obtain the so-called supplementary boundary conditions for equation (10) in the form

$$\begin{aligned} &\phi'' = \left(n+1\right)S_0 \quad at \quad \lambda = 0 \\ &\phi' = 0 \qquad\qquad at \quad \lambda = 1 \end{aligned} \quad \text{------- (14)}$$

Using the Galerkin's let us assume an approximate solution of equation (10) in the form,

$$\phi = \left(n+1\right)\left[b\left(1-4\lambda^{3}+3\lambda^{4}\right) + S_0\,\frac{\left(1-\lambda\right)^{2}}{2} + \frac{1}{3}\left(2-3\lambda+\lambda^{3}\right)\right] \quad \text{------- (15)}$$

Which satisfies the boundary conditions (13) – (14) where b is an unknown constant substituting equation (15) into equation (10) and integrating this result with respect to λ, from λ = 1 to λ = 1, we obtain an equation that can be solved for the constant b,

$$b^2 + \frac{7}{24}\left[S_0 + \frac{(n+6)}{(2n+4)}\right]b - \frac{7n(8+5S_0)}{[576(n+2)]} = 0 \qquad \text{------- (16)}$$

and whose solution is

$$b = \frac{7}{48}\left\{\left[S_0 + \frac{(n+6)}{(2n+4)2} + \frac{4n}{7}\frac{(8+5S_0)}{(n+2)}\right]\frac{1}{2} - \left[S_0 + \frac{(n+6)}{n(n+2)}\right]\right\} \qquad \text{------- (17)}$$

The skin-friction coefficient is

$$\frac{C_f}{\mathrm{Re}^{\frac{1}{n+1}}} = C\ \ (n, S_0) \qquad \text{------- (18)}$$

where, $C(n,\ S_0) = \left[(n+1)\ \left(b + \frac{2}{3}\ +\ \frac{S_0}{2}\right)/n\right]^{\frac{n}{n+1}}$ -------(19)

Putting the values of ϕ and λ in (15) we get,

$$(f'')^{n+1} = \frac{(n+1)^2}{2n}\left[b\left(1-4f'^3+3f'^4\right)+\frac{S_0}{2}\left(1-f'^2\right)+\frac{\left(2-3f'+f'^3\right)}{3}\right]$$

$$\therefore f''(0) = \left\{\frac{(n+1)^2}{2n}\left(b+\frac{S_0}{2}+\frac{2}{3}\right)\right\}^{\frac{1}{n+1}} \qquad \text{------- (20)}$$

where b is given in (17)

Table for the graph f" (0) → S_0 is given below.

Table 1: Values of the wall shear stress and so for various values of n.

n = 0.1

S_0	0	0.2	0.4	0.6	0.8
f'' (0)	3.6031	4.0837	4.5587	5.0288	5.4946

n = 0.2

S_0	0	0.2	0.4	0.6	0.8
f'' (0)	2.125	2.3809	2.6305	2.8755	3.1164

n = 0.4

S_0	0	0.2	0.4	0.6	0.8
f'' (0)	1.4750	1.6219	1.7637	1.9010	2.0345

n = 0.6

S_0	0	0.2	0.4	0.6	0.8
f'' (0)	1.3027	1.4137	1.5196	1.6213	1.7193

n = 0.8

S_0	0	0.2	0.4	0.6	0.8
f'' (0)	1.2419	1.3338	1.4208	1.5038	1.5833

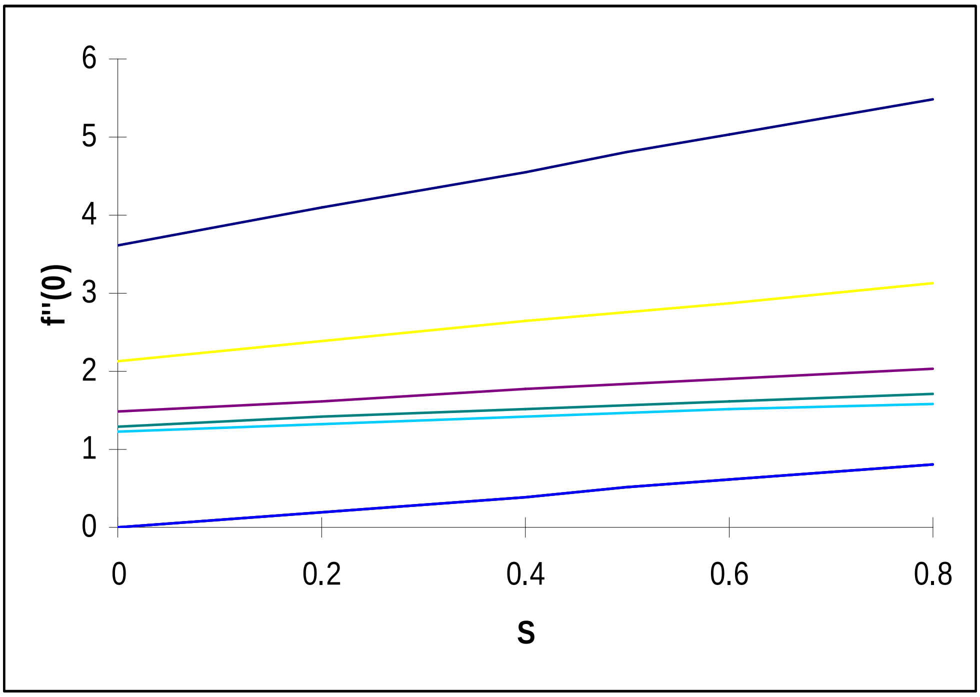

Figure 1: Values of the wall shear stress

Conclusion:

Due to the increase in the modified Hartmann number S_0 , there is an increase in the wall shear stress. Also the wall shear stress increase with increasing magnetic field.

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