

# Calculation of vibratory power transmission of complex floating raft system by FEM

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**Abstract:** A method was proposed for investigating the characteristic of the vibratory power transmission of the complex floating raft system by the finite element method (FEM). This method allows straightforward generation of a structural model of a complex floating raft system in FEM software ANSYS. By means of the harmonic response analysis technique, the response of the linear system under multiple harmonic excitations can be determined. Thus the desired vectors required to calculate the power transmission can be obtained. Based on this method, the vibratory power transmission of a practical complex floating raft system was investigated numerically. This work shows the validity of the proposed method.

**Key words:** floating raft system; power flow; FEM; ANSYS

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## 基于有限元方法的复杂浮筏系统功率流计算

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**摘要:** 提出一种基于有限元理论的复杂浮筏系统功率流的计算方法。该方法可应用 ANSYS 等有限元软件直观得生成复杂浮筏系统的模型, 并采用谐响应分析技术求解线性系统在外激励下的响应, 从而求得计算系统功率流所需的力和速度向量。应用该方法研究了一实际复杂浮筏系统的功率流传递特性, 算例结果表明了方法的可靠性。

**关键词:** 浮筏系统; 功率流; 有限元法; ANSYS

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## 0 Introduction

The isolation of a vibrating machine, particularly one mounted on a flexible foundation in marine and aeronautical applications, has been studied by many researchers<sup>[1,2]</sup>. To achieve more efficient vibration cancellation, floating raft system, a type of special vibration isolation structure which is based on two-stage isolation system, was developed about twenty years ago specifically for ships. This new isolation structure can isolate vibration of hosts and auxiliary machines and effectively reduce the structural noise of ships. It can also protect the equipment and instruments in a ship from being damaged and makes them operate properly when the ship is under external loads and sudden shocks. The study of floating raft systems has drawn much attention in recent years. But the conventional floating raft system is limited to passive isolation, which does not deal with the alterable excitation situation effectively.

Active control techniques are able to dynamically adapt the characteristic parameters of the systems or structures so as to meet the strict requirements of vibration isolation. In order to improve the isolation performance of a passive floating raft system, active/semi-active control can be applied to the system. Thus it is necessary to investigate the characteristics of the vibration transmission in the floating raft system and choose the proper cost function to be minimized in the vibration control. Power has been used as a parameter to quantify the structural vibration transmission in many relevant researches. The characterization of vibration transmission using power represents a good way to estimate the energy available for causing radiated noise. On the other hand, the minimization of power transmission to the receiver is one proposed strategy for active vibration control. This is an alternative to the conventional strategy for canceling the velocities or forces at the concerned

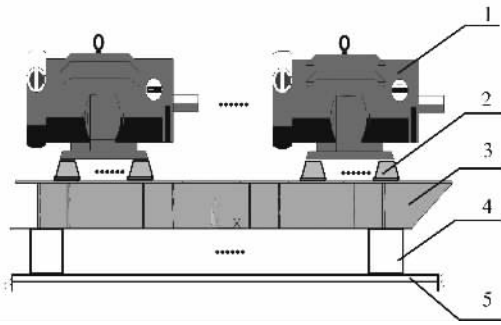
junctions or minimizing the vibration response of the receiver measured at several points. Some researches have been reported for the calculation of power transmissions in a floating raft system<sup>[3~6]</sup>, but the rafts in these papers are all simple structures just like beams or plates, which are far from the complex structures in practical engineering applications.

The FEM has been in general used for dynamical analyses of structures, especially complex practical structures composed of different parts. Along with the development of large-scale FEM programs, the advantage of this method becomes more remarkable. The purpose of the present study is to propose a method for investigating the characteristic of the vibratory power transmission of the complex floating raft system by FEM. After introduction of the proposed method, the analytical model of a practical complex raft is built by finite element software ANSYS, then modal analysis on the finite element model and experimental modal analysis on the full-scale structure are performed to verify the reliability of the FEM model. Based on this, the finite element model of the floating raft system is built and the reduction of vibratory power flow, which transmits from the vibrating machine into a flexible simply supported cylinder through the system, is investigated numerically.

## 1 Description of proposed method

### 1.1 Formulation of the problem

Generally, the complete floating raft system is divided into five parts as shown in Fig. 1; the machines, the upper isolators, the raft, lower isolators and supporting structure. These parts are connected at a finite number of junctions. Each of the  $n$  rigid machines is mounted on the raft through  $l$  upper isolators, and  $m$  lower isolators are mounted between the raft and the supporting structure on the ship. The general raft is a kind of box structure strengthened with girders, therefore, it is not suitable here to model the raft



1. machines 2. upper isolators 3. raft  
4. lower isolators 5. supporting structure

**Fig. 1 The schematic of a general complex floating raft system**

simply as beam or plate.

In the design of the floating raft system, it is important to estimate the total power transmitted to the supporting structure of the ship, defined as  $P_{\text{out}}$ , and the power transmitted into the system from the machines, defined as  $P_{\text{in}}$ . Because  $P_{\text{out}}$  is often used as the cost function minimized to assess vibration control. In addition, the power reduction, defined as  $p_{\text{out}} - p_{\text{in}}$ , which indicates the dissipation of the vibratory energy in the system, can be used as another index to estimate the isolation performance of the passive system and vibration control.

For a linear system, when force and velocity are harmonic the power flow is given by

$$P = \frac{1}{T_0} \int_0^{T_0} \text{Re}\{F_i\} \cdot \text{Re}\{V_i\} dt, \quad (1)$$

where  $F_i$  and  $V_i$  are the instantaneous values of force and velocity at a point,  $T_0$  is the average time. When  $F_i$  and  $V_i$  have the same frequencies, which means  $F_i = F e^{i\omega t}$  and  $V_i = V e^{i\omega t}$ , Eq. (1) can be written as

$$P = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \text{Re}\{F e^{i\omega t}\} \cdot \text{Re}\{V e^{i\omega t}\} dt = \frac{1}{2} \text{Re}\{F V^*\}, \quad (2)$$

where  $\omega$  is the frequency of vibration.  $F$  and  $V$  are complex and include relative phase angles,  $*$  denotes the complex conjugate. The ratio of the complex harmonic velocity to the complex harmonic force is the mobility. One may substitute therefore either the force or the velocity to give

$$P = \frac{1}{2} |F|^2 \text{Re}\{\beta\} = \frac{1}{2} |V|^2 \text{Re}\{\beta\} / |\beta|^2. \quad (3)$$

For the floating raft system as shown in Fig. 1, Assume the system is linear and the harmonic excitation force acts on the  $i$ th machine is  $f_{s_i}$  ( $i=1, \dots, n$ ), and the corresponding velocity of the rigid machine is  $v_{s_i}$  ( $i=1, \dots, n$ ). Thus the power flow input by the excitation forces into the complete isolating system is given by

$$P_{\text{in}} = \frac{1}{T_0} \int_0^{T_0} \text{Re}\{\mathbf{F}_s\}^T \cdot \text{Re}\{\mathbf{V}_s\} dt, \quad (4)$$

where  $\mathbf{F}_s = [f_{s_1}, f_{s_2}, \dots, f_{s_n}]^T$ ,  $\mathbf{V}_s = [v_{s_1}, v_{s_2}, \dots, v_{s_n}]^T$ . In the same way, if the force acts on the supporting structure through the  $j$ th lower isolator is  $f_{r_j}$  ( $j=1, \dots, m$ ) and the corresponding velocity at the junction is  $v_{r_j}$  ( $j=1, \dots, m$ ), the power flow input into the supporting structure is given by

$$P_{\text{out}} = \frac{1}{T_0} \int_0^{T_0} \text{Re}\{\mathbf{F}_r\}^T \cdot \text{Re}\{\mathbf{V}_r\} dt, \quad (5)$$

where  $\mathbf{F}_r = [f_{r_1}, f_{r_2}, \dots, f_{r_m}]^T$ ,  $\mathbf{V}_r = [v_{r_1}, v_{r_2}, \dots, v_{r_m}]^T$ . Among the four vectors required to calculate  $P_{\text{in}}$  and  $P_{\text{out}}$ ,  $\mathbf{F}_s$  is the initial excitation and is prescribed. Thus three unknown vectors  $\mathbf{V}_s$ ,  $\mathbf{F}_r$ , and  $\mathbf{V}_r$  should be determined.

## 1.2 Formulation of the method

Mobility, as the intrinsic property of a linear system, represents the relationship between the output and input of the system as a function of frequency. For the floating raft system as shown in Fig. 1, the dynamic equations of the system can be represented by the mobility matrix expressions as

$$\begin{bmatrix} \mathbf{H1}_{11} & \mathbf{H1}_{12} & \cdots & \mathbf{H1}_{1n} \\ \mathbf{H1}_{21} & \mathbf{H1}_{22} & \cdots & \mathbf{H1}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{H1}_{n1} & \mathbf{H1}_{n2} & \cdots & \mathbf{H1}_{nm} \end{bmatrix} \begin{Bmatrix} f_{s1} \\ f_{s2} \\ \vdots \\ f_{sn} \end{Bmatrix} = \begin{Bmatrix} v_{s1} \\ v_{s2} \\ \vdots \\ v_{sn} \end{Bmatrix}, \quad (6)$$

$$\begin{bmatrix} \mathbf{H2}_{11} & \mathbf{H2}_{12} & \cdots & \mathbf{H2}_{1n} \\ \mathbf{H2}_{21} & \mathbf{H2}_{22} & \cdots & \mathbf{H2}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{H2}_{m1} & \mathbf{H2}_{m2} & \cdots & \mathbf{H2}_{mm} \end{bmatrix} \begin{Bmatrix} f_{s1} \\ f_{s2} \\ \vdots \\ f_{sn} \end{Bmatrix} = \begin{Bmatrix} v_{r1} \\ v_{r2} \\ \vdots \\ v_{rm} \end{Bmatrix}, \quad (7)$$

$$\begin{bmatrix} \mathbf{H3}_{11} & \mathbf{H3}_{12} & \cdots & \mathbf{H3}_{1m} \\ \mathbf{H3}_{21} & \mathbf{H3}_{22} & \cdots & \mathbf{H3}_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \mathbf{H3}_{m1} & \mathbf{H3}_{m2} & \cdots & \mathbf{H3}_{mm} \end{bmatrix} \begin{Bmatrix} f_{r1} \\ f_{r2} \\ \vdots \\ f_{rm} \end{Bmatrix} = \begin{Bmatrix} v_{r1} \\ v_{r2} \\ \vdots \\ v_{rm} \end{Bmatrix}, \quad (8)$$

where  $\mathbf{H1}$ ,  $\mathbf{H2}$ ,  $\mathbf{H3}$  are mobility matrixes. Eqs. (6), (7), (8) indicate that after the mobility matrixes are obtained by experiment or FEM, the unknown vectors required by Eqs. (4) and (5) to calculate power flow can be determined.

The commercial finite element numerical package ANSYS is applied to obtain the dynamic characteristic of the system here. The finite element model of the complete floating raft system should be built in ANSYS preprocessor before the analysis. For building the finite element model of complex structure such as a floating raft system, the general approach is to build each part of the system separately with proper elements and then combine the separate models together with the boundary conditions at the junctions between different parts. After the model of the system is built, one can determine the response of the linear system under multiple harmonic excitations by using the harmonic response analysis technique in ANSYS. The idea of the technique is to calculate the structure's response at several frequencies and obtain a graph of some response quantity (usually displacements) versus frequency. Three harmonic response analysis methods are available in ANSYS: full, reduced, and mode superposition. Among these methods, the mode superposition method sums factored mode shapes (eigenvectors) from a modal analysis to calculate the structure's response and is faster and less expensive than either the reduced or the full method for many problems. After the harmonic response analysis, one obtains the amplitude and the phase angle of displacement response of the nodes. For a linear harmonic system, the relationship between displacement and velocity response is given by

$$v = \dot{x} = X\omega \sin(\omega t + \varphi + \pi/2), \quad (9)$$

where  $X$  and  $\varphi$  are the amplitude and phase angle of displacement response. Thus the velocity response of each node of the finite element model can be obtained by applying Eq. (9) to the result of the harmonic response analysis in ANSYS.

For obtaining the mobility matrixes list in

Eqs. (6)~(8), apply a unit excitation force  $\mathbf{I}_s$  on the  $i$ th ( $i = 1, \dots, n$ ) machine and perform the harmonic response analysis individually. After the  $i$ th analysis and applying Eq. (9) to the results, one obtains the desired velocity response  $\mathbf{V}_s = [v_{s1i}, v_{s2i}, \dots, v_{sni}]^T$  and  $\mathbf{V}_r = [v_{r1i}, v_{r2i}, \dots, v_{rni}]^T$  ( $i = 1, \dots, n$ ). Substituting  $\mathbf{V}_s$ ,  $\mathbf{V}_r$  and  $\mathbf{I}_s$  into Eqs. (6) and (7) yields

$$\mathbf{H1} \cdot \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ \mathbf{I}_s \\ 0 \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} v_{s1i} \\ v_{s2i} \\ \vdots \\ v_{sni} \end{Bmatrix}, \quad (10)$$

$$\mathbf{H2} \cdot \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ \mathbf{I}_s \\ 0 \\ \vdots \\ 0 \end{Bmatrix} = \begin{Bmatrix} v_{r1i} \\ v_{r2i} \\ \vdots \\ v_{rni} \end{Bmatrix}. \quad (11)$$

Solving Eqs. (10) and (11), one obtains the  $i$ th column of  $\mathbf{H1}$  and  $\mathbf{H2}$ . After finishing all the  $n$  harmonic response analysis procedures, the desired mobility matrixes  $\mathbf{H1}$  and  $\mathbf{H2}$  are obtained. Matrix  $\mathbf{H3}$  can be determined by applying the same procedure as above to the supporting structure independently. Thus the desired vectors  $\mathbf{V}_s = \mathbf{H1} \cdot \mathbf{F}_s$ ,  $\mathbf{V}_r = \mathbf{H2} \cdot \mathbf{F}_s$ , and  $\mathbf{F}_r = \mathbf{H3}^{-1} \cdot \mathbf{V}_r$  are all determined.

## 2 Modelling of a practical floating raft system

In this and the succedent sections, the commercial finite element software ANSYS is applied to study a practical floating raft system to illustrate the validity of the proposed method. The complete system is composed of two electromotors, raft, three groups of isolators and a supporting structure. Each of the two electromotors is mounted on the raft through four upper isolators, and four lower isolators are

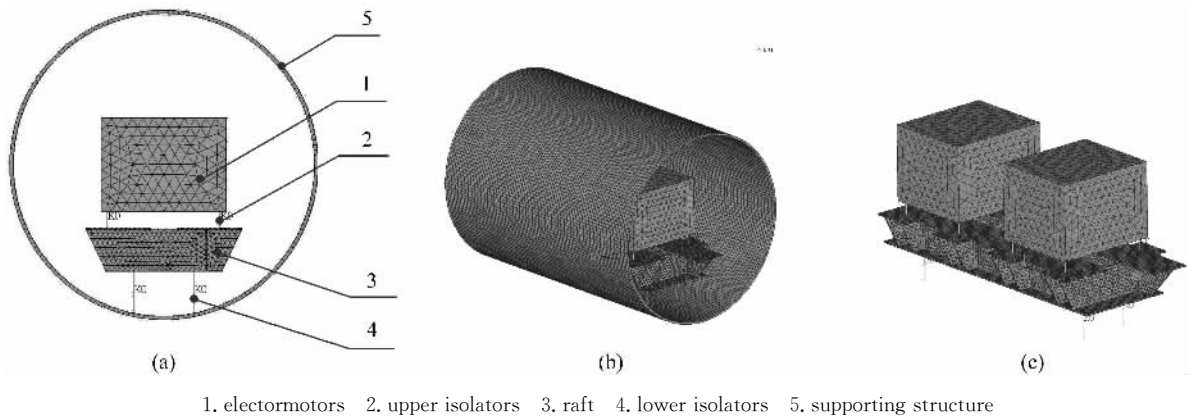


Fig. 2 The FEM model of a practical complex floating raft system (a), 3D view of the FEM model (b) and local plot of the FEM model (c)

1. electromotors 2. upper isolators 3. raft 4. lower isolators 5. supporting structure

mounted between the raft and the supporting structure on the ship. The supporting structure is a thin wall cylinder shell which is 2 m long with the inner radius being 0.7 m and the thickness of the wall 8 mm. Its two ends are simply supported.

As the floating raft system is complex, some approximation and simplification are adopted in the modeling progress of the system to meet the requirement of FE analysis. The cylinder shell is a kind of thin wall structure, thus it is modeled by using the elastic shell element SHELL63 which can better simulate the bending and shear behaviors of the structure. Because the electromotors are much stiffer than the other parts of the system, each electromotor is simulated as a rigid body by a block composed of 3D solid element SOLID45, whose mass, rotary inertia and inertia moment conform with the motor. The raft is modeled with SOLID187 according to its actual size. The isolators are modeled by longitudinal spring-damper element COMBIN14, which is a uniaxial tension compression element with three degrees of freedom at each node. But one spring-damper element can only simulate one directional stiffness of the isolator. Thus to simulate an isolator with three-directional stiffness, three spring-damper element are required and have to overlap each other within two nodes. There are altogether 4 223 shell elements, 18 959 solid elements and 36 spring-damper elements in the floating raft system FE

model, as shown in Fig. 2.

### 3 Results and discussions

#### 3.1 System modal characteristics

The raft is the central part of the whole floating raft system, whose dynamic characteristic directly influences the vibration transmission of the floating raft system. It is necessary to verify the accuracy of the FE model before further analysis, therefore modal analysis on the FE model of the free raft, as shown in Fig. 3, and experimental modal analysis on the full-scale structure are performed. Tab. 1 summarizes the numerical and experimental results for the first two natural frequencies of the raft with free boundary condition. Fig. 4 plots the mode shape for the first two modes. From Tab. 1 and Fig. 4, it can be seen that the FEM simulation results are in good agreement with the experimental results. Thus the

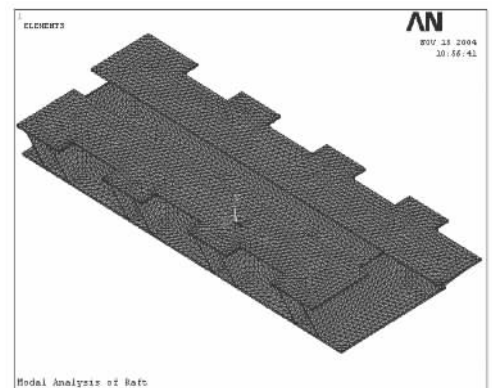


Fig. 3 The FEM model of the free raft

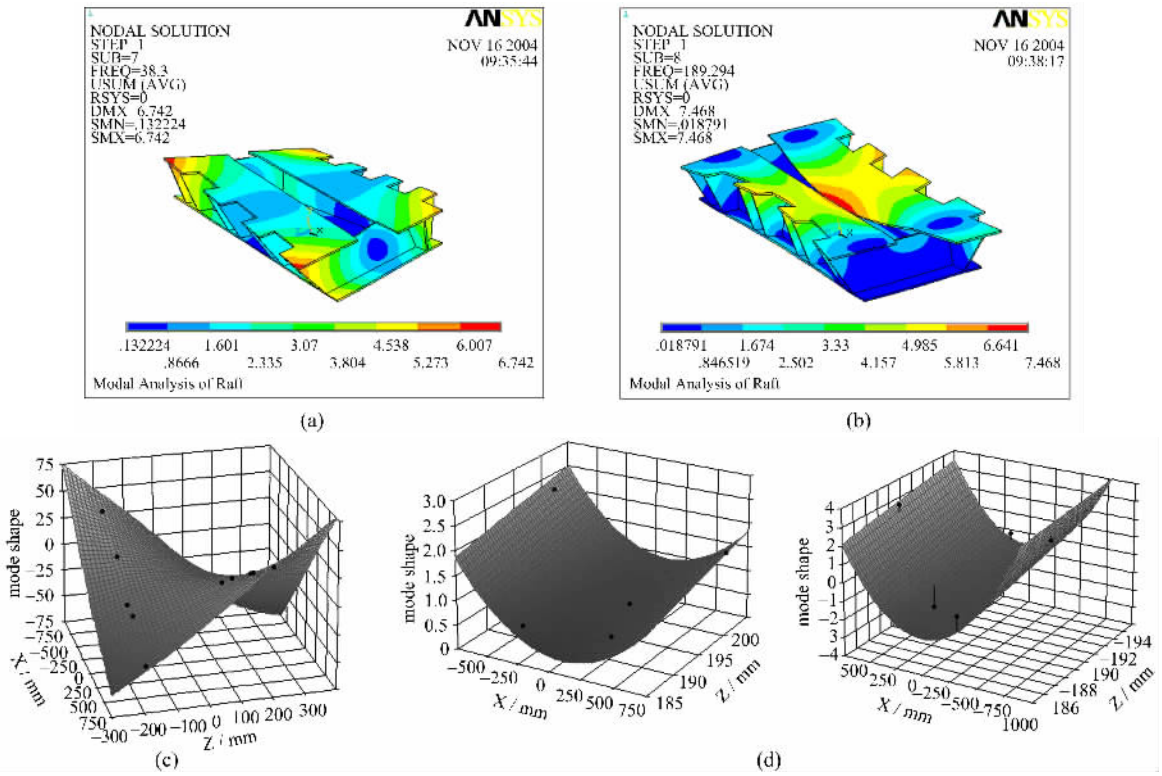


Fig. 4 Mode shapes of first two modes by FEM simulated: (a) 1st mode (torsion) (b) 2nd mode (bending) And by experimental modal analysis: (c) 1st mode (torsion) (d) 2nd mode (bending)

Tab. 1 First two natural frequencies of the free raft

natural frequency	FEM simulated /Hz	experimental results /Hz
$f_1$	37.0	38
$f_2$	188.6	190

simulation results confirm the validity of the current FE model and indicate its suitability for use in the upcoming investigations of the characteristics of the whole floating raft system.

The modal characteristics of the complete floating raft system are then studied. Tab. 2 summarizes the numerical results for modal characteristics of the system and Fig. 5 plots some mode shapes for the corresponding modes listed in Tab. 2. Because the electromotors are mounted on the raft symmetrically, there obviously exist some close modes which have almost the same modal frequencies but different mode shapes.

### 3.2 Power flow characteristics

In this calculation, one driving force combination was used to excite the system and the corresponding power flow ( into the rigid bodies

Tab. 2 Modal characteristics of the floating raft system

mode No	modal frequency /Hz	mode shape
1	3.1	roll
2	3.8	pitch of motors(same direction)
3	3.9	pitch of motors(opposite direction)
4	4.1	roll of motors(opposite direction)
5	6.2	translation(same direction)
6	6.5	translation(opposite direction)
7	8.2	flat turn of motors(same direction)
8	8.3	flat turn of motors(opposite direction)
9	24.9	roll of raft
10	33.6	roll and Translation
11	40.7	pitch
12	53.5	torsion of raft
13	139.7	motion of shell
14	152.1	motion of shell
15	159.9	motion of shell

and into the cylinder shell) was calculated in the frequency range 1~500 Hz. Each driving force on the two rigid bodies, which is 1N in the vertical direction, is located at the center of the top surface of each rigid body. The results are presented in Fig. 6 and the reference power is  $10^{-12}$  Watt. The

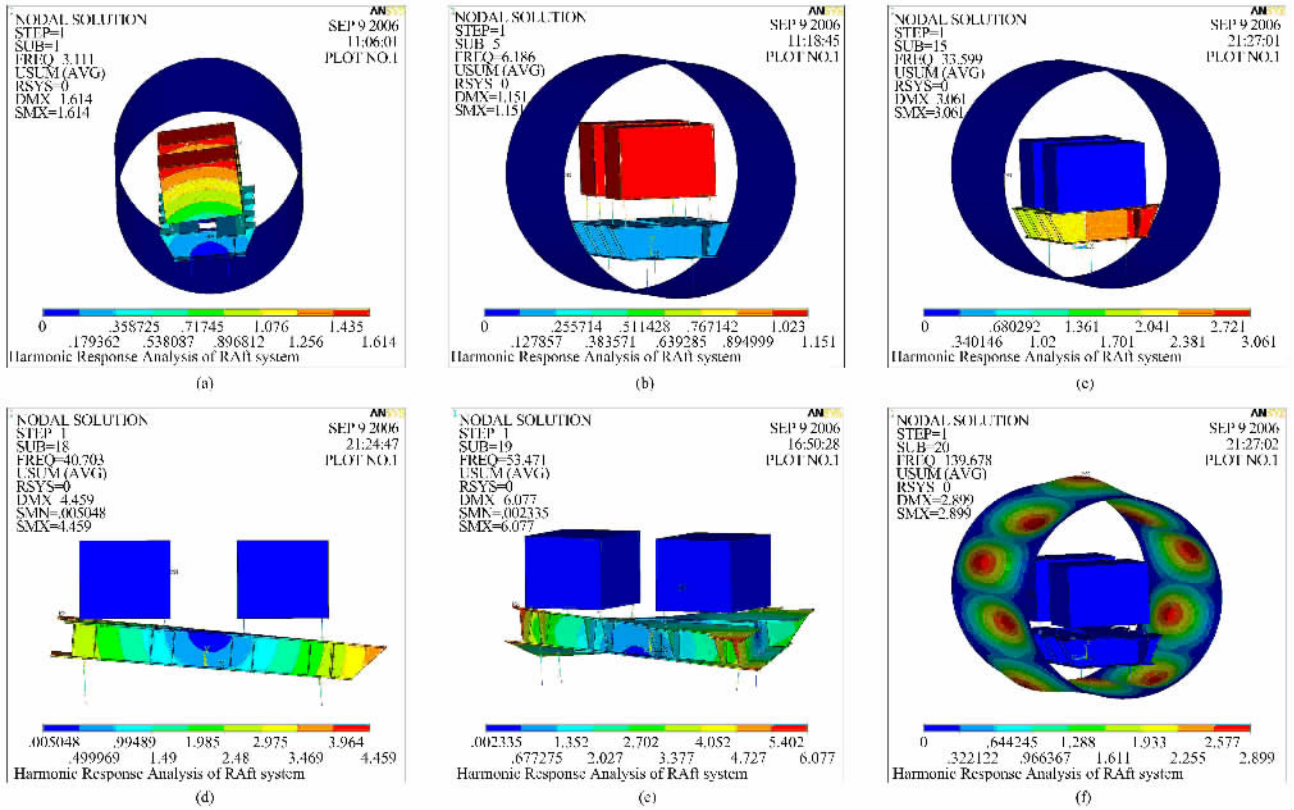


Fig. 5 Mode shapes of several modes: (a) 1st mode (b) 5th mode (c) 10th mode (d) 11th mode (e) 12th mode (f) 13th mode

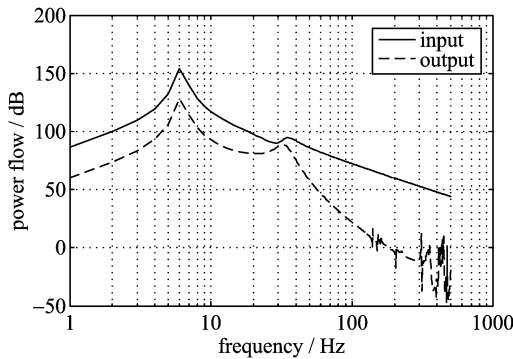


Fig. 6 Power flow transmission of the floating raft system

solid curve represents the input power flow from the rigid bodies ( $P_{in}$ ) and the dashed curve represents the output power flow to the cylinder shell ( $P_{out}$ ). The frequencies corresponding to first two main peaks in the power flow curves are 6 Hz and 34 Hz, which means that these two peaks correspond, respectively, to the 5th and 10th resonance modes of the floating raft system as listed in Tab. 2. And the peaks in the output power flow curve, whose corresponding

frequencies are higher than 100 Hz, correspond to the 13th, 14th, 15th and higher resonance modes of the system. Notice that the driving forces are applied to the system symmetrically. It will be easy to understand that there only exist peaks corresponding to symmetrical resonance modes in the curves. Fig. 7 shows the reduction, defined as  $P_{out} - P_{in}$ , of the input power flow from the rigid bodies and output power flow to the cylinder shell. It represents the vibration isolation performance of the floating raft system more directly.

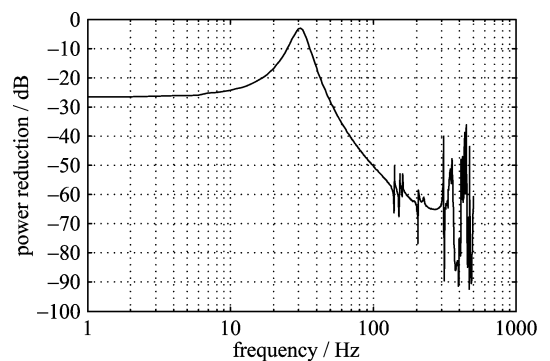


Fig. 7 Power reduction of the floating raft system

## 4 Conclusion

This study has developed a novel computationally efficient method for the calculation of vibratory power flow transmission of a complex floating raft system using an FE numerical package ANSYS. With its advantages of convenience, rapidity, accuracy, low cost, ease of implementation and easy use, the FEA simulator allows not only the straightforward generation of a structural model of a complex floating raft system, but also enables the accurate simulation of its dynamic response which can be further used to study the power flow transmission characteristic. Hence, the proposed method provides a valuable tool for the assessment of vibration isolation performance of a complex structure like the floating raft system.

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