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DOA estimation for nonuniform circular array with mutual coupling based on fourth order cumulants

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Abstract: Based on the special structure of coupling matrix, a direction of arrival (DOA) estimation algorithm in the presence of unknown mutual coupling for the nonuniform circular array (NUCA) is presented. The geometry of the circular array was more general. Due to using the fourth order cumulants (FOC), the number of signals that can be coped with may be larger than the number of sensors in the array, and our algorithm is insensitive to Gaussian noise. Further, iteration is not required in our algorithm, which results in low computational complexity. Simulation results demonstrate the effectiveness of the proposed algorithm.

Keywords: NUCA; mutual coupling; DOA; FOC; calibration

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未知互耦条件下基于四阶累量的非均匀圆阵波达方向估计算法

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摘要:基于互耦矩阵的特殊结构,给出了一种更具一般性的非均匀圆阵模型,提出了一种在未知互耦条件下的非均匀圆阵波达方向估计算法.由于采用了四阶累量,该算法可以估计出比阵元数更多信号的波达方向,并且对高斯噪声不敏感.由于未采用迭代算法,从而降低了算法运算复杂度.仿真实验证实了该算法的有效性.

关键词:非均匀圆阵;互耦;波达方向;四阶累量;校准

0 Introduction

The direction of arrival (DOA) estimation of multiple narrowband signals is a classical problem in array signal processing. Many high-resolution eigendecomposition methods have been proposed[1~4], but the performance of these methods is often critically dependent on the knowledge of the array manifold. However, the array manifold is often affected by unknown mutual coupling. Without array manifold calibration, the performance of DOA estimation

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may degrade substantially because the assumed manifold deviates from the real one with mutual coupling. Many calibration algorithms have thus been proposed in the last decades[5~11].

A maximum-likelihood approach to the calibration is developed[5], with the help of a set of calibration signals in known locations. Although this technique allows for the compensation of all kinds of uncalibration impairments, it may not be easily carried out in practice because of the additional calibration signals. Therefore, blind calibration without the knowledge of calibration signals can be of great advantage.

An iterative procedure among other manifold deviations such as perturbations on gain and phase is presented to compensate for mutual coupling[6]. An alternating minimization procedure based on closed-form solutions is presented[7], which estimates the mutual coupling matrix in the field of complex symmetric Toeplitz matrices. However, there is no guarantee on the convergence of the iteration, and the needed multidimensional search for the associated nonlinear optimal search is computationally very complicated. To avoid iteration, an algorithm based on the GEESE algorithm is described[8], which can estimate the DOA accurately without compensating for mutual coupling. Some decoupling DOA estimation and self-calibration algorithms are proposed by exploiting the symmetric Toepliz structure of the coupling matrix[9,10].

The methods mentioned above are all based on the second-order statistics (SOS). Due to the effect of the mutual coupling, the number of signals that can be estimated by those methods is small. To deal with a bigger number of signals, fourth order cumulants (FOC) are introduced into DOA estimation, which has been studied quite extensively in Refs. [11～14]. By using FOC in DOA estimation, the number of signals that can be coped with may be much larger than the number of sensors, while for SOS based methods the number of signals cannot be greater than the number of sensors. Another major benefit is that FOC-based DOA estimation method is not sensitive to Gaussian noises[11~14]. An algorithm based on FOC is presented to compensate for mutual coupling[11].

However, all of those algorithms can only be used for the uniform linear array (ULA) or uniform circular array (UCA). A recurring problem in array design for both signal reception and parameter estimation is that of how to deploy the elements of a sparse array beneficially[15]. It is shown that nonuniform arrays, such as minimum redundancy (MR) arrays or nonredundant arrays, may lead to significant improvement in performances[16,17]. Few efforts have been made for designing the nonuniform array in the presence of unknown mutual coupling.

In this paper, we present a model for special nonuniform circular array (NUCA) in the presence of sensor mutual coupling. Then, we develop an FOC-based approach to blind calibration for NUCA based on our NUCA model. Similar to the algorithms[9,11], the proposed FOC-based method does not require any iteration and thus is computationally attractive. Furthermore, the geometry of the circular array is more general.

1 Problem formulation

Consider an M-element UCA with radius r and receiving K narrow-band statistically independent signals with carrier wavelength λ. Assume $\boldsymbol{C}$ denotes the mutual coupling matrix (MCM) of the UCA, which is generally considered to be independent of angle[18]. Since the mutual coupling coefficient between two sensors is inversely related to their distances. Let c_{hl} be the complex coupling coefficient between hth and lth sensors. It is easy to see that $c_{hl}=c_{|h-l|}$, and the magnitude of the complex coupling coefficient decreases quite fast as the distance increases, so that just a few non-zero coefficients are enough[6~11]. Assume there are $t+1$ non-zero complex coefficients in MCM with $t+1<M$. Then the $1\times M$ coefficient vector may be written as

$$\hat{\boldsymbol{c}} = [c_0, \cdots, c_t, 0, \cdots, c_t, \cdots, c_1] \tag{1}$$

where $0 < |c_t| < |c_{t-1}| < \cdots < c_0 = 1$. The corresponding $M \times M$ matrix $\boldsymbol{C}$ is given by

$$\boldsymbol{C} = \text{toeplitz}(\hat{\boldsymbol{c}}) = \text{toeplitz}\{[c_0, \cdots, c_t, 0, \cdots, c_t, \cdots, c_1]\} \tag{2}$$

where toeplitz($\hat{\boldsymbol{c}}$) denotes the symmetric toeplitz matrix constructed by vector $\hat{\boldsymbol{c}}$. So the output of the M-element UCA is given by

$$\boldsymbol{y}(t) = \boldsymbol{CAs}(t) + \boldsymbol{n}(t) \tag{3}$$

where

$$\boldsymbol{y}(t) = [y_1(t), \cdots, y_M(t)]^{\mathrm{T}},$$
$$\boldsymbol{n}(t) = [n_1(t), \cdots, n_M(t)]^{\mathrm{T}},$$
$$\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)]^{\mathrm{T}},$$
$$\boldsymbol{A} = [\boldsymbol{\alpha}(\theta_1), \cdots, \boldsymbol{\alpha}(\theta_K)],$$
$$\boldsymbol{\alpha}(\theta_k) = [1, \nu_k^1, \nu_k^2, \cdots, \nu_k^{M-1}]^{\mathrm{T}},$$
$$\nu_k = \exp(-\mathrm{j}\pi\sin(\theta_k)), k = 1, 2, \cdots, K.$$

For simplicity, the NUCA discussed here is not an arbitrary nonuniform circular array. The NUCA is an array with N ($N < M$) sensors equipped at any N places of the M uniform positions of the circle. The nth sensor in the NUCA is placed on the mth position in the UCA, where $n=\text{ind}\{m\}$ and ind{ } is a map function. So the output of the NUCA is given by

$$\boldsymbol{x}(t) = \boldsymbol{F}'\boldsymbol{CFAs}(t) + \boldsymbol{F}'\boldsymbol{n}(t) = \tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}\boldsymbol{s}(t) + \boldsymbol{F}'\boldsymbol{n}(t) \tag{4}$$

where $\tilde{\boldsymbol{C}} = \boldsymbol{F}'\boldsymbol{C}$, $\tilde{\boldsymbol{A}} = \boldsymbol{FA}$,

$$[\boldsymbol{F}']_{ij} = \begin{cases} 1, & i = n, j = m \\ 0, & \text{otherwise} \end{cases}$$ is a $N \times M$ matrix,

$$[\boldsymbol{F}]_{ij} = \begin{cases} 1, & i = j = m \\ 0, & \text{otherwise} \end{cases}$$ is a $M \times M$ matrix.

For example, when $M=5$ and $\{1,3,4,5\}$th sensor is selected, we obtain

$$\boldsymbol{F}' = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Notice that when $N = M$, the array becomes a UCA.

Define the FOC matrix of the output signals to be $\boldsymbol{C}_x$, where the element of $\{(k_1-1)N+k_2\}$th row and $\{(k_3-1)N+k_4\}$th column can be given by

$$\boldsymbol{C}_x((k_1-1)N+k_2, (k_3-1)N+k_4) = \text{cum}\{x_{k_1}, x_{k_2}^*, x_{k_3}^*, x_{k_4}\} = E\{x_{k_1}, x_{k_2}^*, x_{k_3}^*, x_{k_4}\} - E\{x_{k_1}, x_{k_2}^*\} \cdot E\{x_{k_3}^*, x_{k_4}\} - E\{x_{k_1}, x_{k_3}^*\} \cdot E\{x_{k_2}^*, x_{k_4}\} - E\{x_{k_1}, x_{k_4}\} \cdot E\{x_{k_2}^*, x_{k_3}^*\}$$

Since the FOC of the Gaussian noise is identically zero and the signals are statistically independent, $\boldsymbol{C}_x$ can be written as

$$\boldsymbol{C}_x = [(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}) \odot (\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}})^*]\boldsymbol{C}_s[(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}}) \odot (\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}})^*]^H = \sum_{i=1}^{K} \{[(\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta_i)) \otimes (\tilde{\boldsymbol{C}}^* \boldsymbol{F\alpha}^*(\theta_i))] \cdot r_i[(\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta_i)) \otimes (\tilde{\boldsymbol{C}}^* \boldsymbol{F\alpha}^*(\theta_i))]^H\} \tag{5}$$

where $\boldsymbol{C}_s = \text{diag}\{r_1, r_2, \cdots, r_K\}$, $r_i = \text{cum}\{s_i, s_i^*, s_i^*, s_i\}$ is the FOC of the signal source s_i, $\odot$ denotes Khatri-Rao product, $\otimes$ denotes Kronecker product, and $[\cdot]^*$ and $[\cdot]^H$ represent complex conjugation and conjugate transposition respectively.

The eigendecomposition of $\boldsymbol{C}_x$ can be written as

$$\boldsymbol{C}_x = \boldsymbol{U}_s\boldsymbol{\Sigma}_s\boldsymbol{U}_s^H + \boldsymbol{U}_n\boldsymbol{\Sigma}_n\boldsymbol{U}_n^H \tag{6}$$

where $\boldsymbol{U}_s$ is a $N^2 \times K$ unitary matrix whose columns are the eigenvectors corresponding to the K nonzero eigenvalues, while the columns of $\boldsymbol{U}_n$ are the eigenvectors corresponding to the $N^2 - K$ zero eigenvalues. The columns of $\boldsymbol{U}_s$ span the signal subspace, and the signal subspace is orthogonal to the noise subspace spanned by the columns of $\boldsymbol{U}_n$. When the MCM $\boldsymbol{C}$ is known, the DOAs can be estimated directly by the following formulation (FOC-MUSIC)[12]. Thus, the spatial spectrum function of MUSIC algorithm should be

$$P(\theta) = \frac{1}{\| \boldsymbol{U}_n^H(\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta)) \otimes (\tilde{\boldsymbol{C}}^* \boldsymbol{F\alpha}^*(\theta)) \|^2} \tag{7}$$

where $\| \cdot \|_F$ denotes a Frobenius norm. The peaks of the $P(\theta)$ will correspond to the true DOAs.

2 Coupling calibration

Since the MCM $\boldsymbol{C}$ is unknown, the DOAs of the signals can not be estimated by Eq. (7). Using an exhaustive multidimensional search with respect to θ and $\boldsymbol{C}$ to estimate the DOAs is impractical to realize. Firstly, we can directly utilize Lemma 2 in Ref. [6]

$$\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta) = \boldsymbol{F}'\boldsymbol{T}[\theta]\boldsymbol{c} \tag{8}$$

where $\boldsymbol{c}=[c_0,\cdots,c_t]^{\mathrm{T}}$, and the $M\times(t+1)$ matrix $\boldsymbol{T}[\theta]=\boldsymbol{T}_1+\boldsymbol{T}_2+\boldsymbol{T}_3+\boldsymbol{T}_4$ with

$$[\boldsymbol{T}_1]_{i,j} = \begin{cases} [\boldsymbol{F\alpha}(\theta)]_{i+j-1}, & i+j\leqslant N+1 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

$$[\boldsymbol{T}_2]_{i,j} = \begin{cases} [\boldsymbol{F\alpha}(\theta)]_{i-j+1}, & i\geqslant j\geqslant 2 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

$$[\boldsymbol{T}_3]_{i,j} = \begin{cases} [\boldsymbol{F\alpha}(\theta)]_{N+1+i-j}, & i< j\leqslant p \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

$$[\boldsymbol{T}_4]_{i,j} = \begin{cases} [\boldsymbol{F\alpha}(\theta)]_{i+j-N-1}, & 2\leqslant j\leqslant p,\ i+j\geqslant N+2 \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

where $p=M/2$ when M is even or $(M+1)/2$ when M is odd. The signal subspace is orthogonal to the noise subspace, therefore, from Eq. (8), we have

$$\begin{aligned}
&[(\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta))\otimes(\tilde{\boldsymbol{C}}^{*}\boldsymbol{F\alpha}^{*}(\theta))]^{H}\boldsymbol{U}_n\cdot \\
&\boldsymbol{U}_n^{H}[(\tilde{\boldsymbol{C}}\boldsymbol{F\alpha}(\theta))\otimes(\tilde{\boldsymbol{C}}^{*}\boldsymbol{F\alpha}^{*}(\theta))] = \\
&[(\boldsymbol{F}'\boldsymbol{T}[\theta]\boldsymbol{c})\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta]\boldsymbol{c}^{*})]^{H}\boldsymbol{U}_n\cdot \\
&\boldsymbol{U}_n^{H}[(\boldsymbol{F}'\boldsymbol{T}[\theta]\boldsymbol{c})\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta]\boldsymbol{c}^{*})] = \\
&(\boldsymbol{c}\otimes\boldsymbol{c}^{*})^{H}[(\boldsymbol{F}'\boldsymbol{T}[\theta])\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta])]^{H}\boldsymbol{U}_n\cdot \\
&\boldsymbol{U}_n^{H}[(\boldsymbol{F}'\boldsymbol{T}[\theta])\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta])](\boldsymbol{c}\otimes\boldsymbol{c}^{*}) = 0
\end{aligned} \tag{13}$$

Further, defining an $(t+1)^2\times(t+1)^2$ matrix

$$\begin{aligned}\boldsymbol{Q}(\theta) = &[(\boldsymbol{F}'\boldsymbol{T}[\theta])\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta])]^{H}\boldsymbol{U}_n\cdot \\ &\boldsymbol{U}_n{}^{H}[(\boldsymbol{F}'\boldsymbol{T}[\theta])\otimes(\boldsymbol{F}'\boldsymbol{T}^{*}[\theta])]\end{aligned} \tag{14}$$

then Eq. (13) can be rewritten as

$$(\boldsymbol{c}\otimes\boldsymbol{c}^{*})^{H}\boldsymbol{Q}(\theta)(\boldsymbol{c}\otimes\boldsymbol{c}^{*}) = 0 \tag{15}$$

Similar to the Refs. [9~11], when $(t+1)^2\leqslant N^2-K$, the rank reduction of $\boldsymbol{Q}(\theta)$ will take place on true DOAs of signals. Hence, we can obtain the DOAs by searching the peaks of the following formulation

$$\mathrm{d}(\theta) = 1/\det\{\boldsymbol{Q}(\theta)\} \tag{16}$$

and we obtain

$$\boldsymbol{c}\otimes\boldsymbol{c}^{*} = \boldsymbol{v}_{\min}\{\boldsymbol{Q}(\theta_k)\} \tag{17}$$

where det{ } is the determinant and $\boldsymbol{v}_{\min}\{\ \}$ is the eigenvector corresponding to the smallest eigenvalue.

However, it is important to note that the equation

$$\det(\boldsymbol{Q}(\theta)) = 0 \tag{18}$$

may hold true when $\theta\neq\theta_i\,(i=1,2,\cdots,K)$. This problem also exists in the SOS-based method[9~11]. Assume there are L directions $\{\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_K,\cdots,\hat{\theta}_L\}$ satisfying Eq. (18). Without loss of generality, define the first K directions $\{\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_K\}$ as the true DOAs and the remaining $L-K$ directions $\{\hat{\theta}_{K+1},\hat{\theta}_{K+2},\cdots,\hat{\theta}_L\}$ as the pseudo DOAs. Corresponding to each $\hat{\theta}_l\,(l=1,2,\cdots,L)$, we can obtain a mutual coupling coefficient vector $\boldsymbol{c}_l$. It is easy to see

$$\boldsymbol{U}_n^{H}(\boldsymbol{F}'\boldsymbol{C}_l\boldsymbol{F\alpha}(\hat{\theta}_l))\otimes(\boldsymbol{F}'\boldsymbol{C}_l^{*}\boldsymbol{F\alpha}^{*}(\hat{\theta}_l)) = \begin{cases} 0, & l=1,2,\cdots,K \\ \neq 0, & l=K+1,K+2,\cdots,L \end{cases} \tag{19}$$

where the matrix $\boldsymbol{C}_l$ is MCM constructed by $\boldsymbol{c}_l$. Using Eq. (19), we can select the true DOAs, and then the final mutual coupling coefficient can be obtained by

$$\boldsymbol{c}\otimes\boldsymbol{c}^{*} = \boldsymbol{v}_{\min}\{\sum_{k=1}^{K}\boldsymbol{Q}(\theta_k)\} \tag{20}$$

Finally, we discuss the maximal number of signals that can be estimated by our method. As aforementioned, our method needs $(t+1)^2\leqslant N^2-K$ compared with $t+1\leqslant N-K$ in Ref. [9]. From Ref. [19], we obtain

$$(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}})\odot(\tilde{\boldsymbol{C}}\tilde{\boldsymbol{A}})^{*} = (\tilde{\boldsymbol{C}}\otimes\tilde{\boldsymbol{C}}^{*})(\tilde{\boldsymbol{A}}\odot\tilde{\boldsymbol{A}}^{*}) \tag{21}$$

Since the matrix $\tilde{\boldsymbol{A}}\odot\tilde{\boldsymbol{A}}^{*}$ has N linearly dependent rows, the matrix is of full column rank only if $K\leqslant N^2-N+1$. Then the maximal number of estimated signals $K_{\max}$ is

$$\begin{aligned}K_{\max} &= \min\{N^2-(t+1)^2, N^2-N+1\} = \\ &N^2-\max\{(t+1)^2, N-1\}\end{aligned} \tag{22}$$

Hence, the maximal number of signals that can be estimated is much greater than that in Ref. [9].

3 Simulation results

Assume that all signals are of equal power ($\sigma_1^2=\sigma_2^2=\cdots=\sigma_K^2$) and the signal-to-noise ratio (SNR) is defined as $10\log_{10}(\sigma_K^2/\sigma^2)$.

Simulation 1 is designed to test the maximum number of signals that can be estimated in our algorithm. An NUCA composed of $N=4$ sensors, which are equipped at the $\{1,2,4,6\}$ places of the $M=6$ uniform positions of a circle with $r=0.48\lambda$. The mutual coupling coefficient vector $\boldsymbol{c}$ is $[1,0.2+0i]^{\mathrm{T}}$. When $N=4, t=1$, at most $K_{\max}=12$ signals can be estimated. However, in practical situations the maximum number of signals that can be estimated will be affected by the aperture of the array, the number of snapshots, SNR, among others. For clarity, We consider an ideal scenario, where the FOC matrix of the received signal is constructed directly by Eq. (5), where $r_i=\mathrm{cum}\{s_i, s_i^*, s_i^*, s_i\}=1$, and $K=12$ statistically independent signals come from $\theta_i\in[30°, 70°, 90°, 110°, 150°, 170°, 190°, 230°, 250°, 270°, 300°, 320°]$. Fig. 1 shows the spatial spectrum function of Eq. (16). Obviously, there are some pseudo peaks in the figure. Searching all the peaks of the spatial spectrum in Fig. 1, we will obtain these angles as [0°, 30°, 40.2°, 70°, 90°, 107.9°, 110°, 139.8°, 150°, 170°, 180°, 190°, 220.2°, 230°, 238.1°, 250°, 270°, 300°, 320°]. Then the corresponding mutual coupling coefficients vector can also be obtained. Using Eq. (19), we can select the true DOAs.

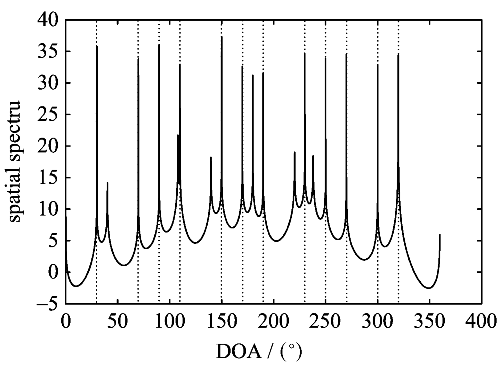

Fig. 1 The maximum number of signals that can be estimated by our methods in ideal scenario

Actually, when the number of signals become small, the dimension of the noise subspace increase, then the pseudo DOAs may be reduced or even eliminated in general. Simulation 2 gives the scenario where only 5 signals are considered from $\theta_i\in[60°, 110°, 150°, 200°, 270°]$ at the same condition as simulation 1. Fig. 2 shows the above result, in which no pseudo peaks exist.

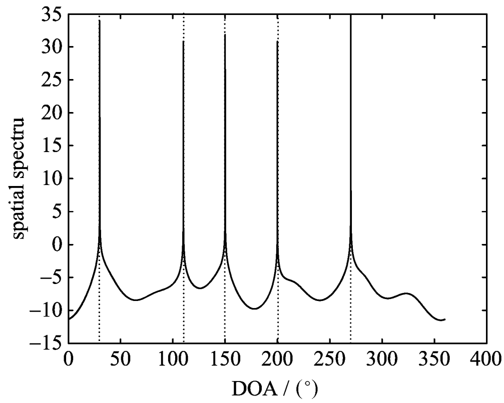

Fig. 2 5 signals can be estimated without pseudo DOA in the ideal scenario

In simulation 3, we assess the performance of the proposed method with respect to several parameters including SNR and the number of snapshots. We compare our method with FOC-MUSIC algorithm, as well as CRBs. Assume the scenario is the same as simulation 1, except that $K=3$ signals arrive at [60°, 140°, 250°]. It is clear that the method in Ref. [9] can not deal with this scenario, because of the general geometry of the circular array and the large number of signals. Fig. 3 shows the root mean square error (RMSE) of DOAs estimate versus input SNR computed via 200 Monte Carlo runs for each SNR and 4 000 snapshots of data for each run. Fig. 4 shows the RMSE of DOAs estimate versus the number of

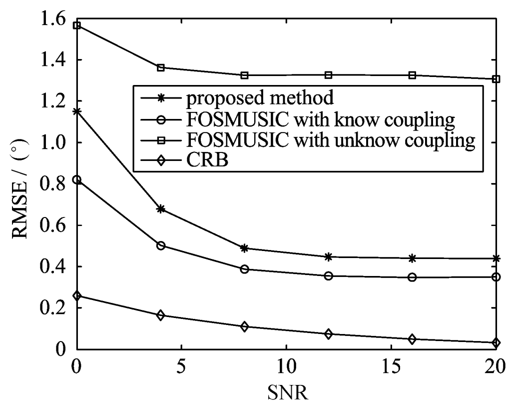

Fig. 3 RMSE of DOA estimates vs. SNR with a Monte Carlo experiment of 200 runs for snapshots=4 000

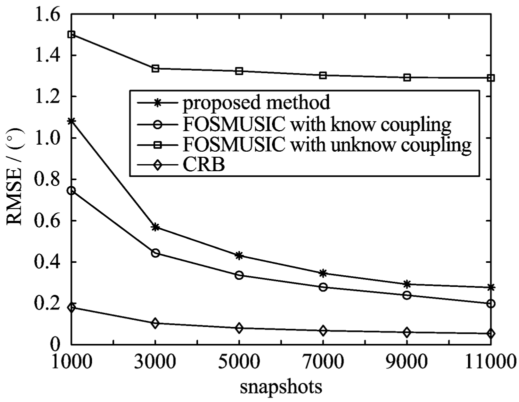

Fig. 4 RMSE of DOA estimates vs. snapshots with a Monte Carlo experiment of 200 runs for SNR=10 dB

snapshots (with SNR=10 dB) computed via 200 Monte Carlo runs for each number of snapshots. The results illustrate that our method achieves performance similar to FOC-MUSIC with known coupling.

4 Conclusion

Based on the special structure of coupling matrix, we proposed a DOA estimation algorithm in the presence of unknown mutual coupling for NUCA. It is analogous to the method developed in Ref. [11], but the geometry of the circular array is more general. Compared with our method, the method in Ref. [11] is a particular case ($M=N$). Validation and performance were illustrated by simulations.

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