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NEW APPROACH IN THE EVALUATION OF SOIL STIFFNESS COEFFICIENTS

BY

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General numerical methods based on the discretization of the contact surface between structure and soil in finite elements are developed and, by their help, the stiffness matrix is determined. There are considered the medium model of linear elastic, homogeneous and isotropic half-space and anisotropic half-space, respectively. In case of rigid structures or with rigid base, it is essentially for their response evaluation to know the stiffness coefficients.

1. Introduction

The response of a great category of structures subjected to dynamic loads strongly depends on soil deformation, or, generally, on support medium deformation. In order to determine this important influence, the medium (soil) stiffness coefficients must be evaluated.

Winkler's model and the elastic, isotropic and homogeneous half-space model are the most common models adopted in the soil deformation analysis, but the admitted assumptions are not valid for all actual situations. That is way a relative general method for soil stiffness coefficients evaluation is developed in the present paper.

2. Principles for Stiffness Coefficients Evaluation

This method uses efficient numerical techniques that can be programmable and takes into account the size and shape of the contact surface between the structural system and ground.

In the analysis, the method of interface finite elements is used. For some bearing support models this method becomes a variant of the boundary finite element method.

The contact surface between the foundation and the ground is divided into disjunctive domains of rectangular or other shape that are called interface finite elements.

Due to the stiffness of the base, in particular of the foundation block, the surface of the ground after deformation remains still plane and to a general displacement, a system of pressures on the ground corresponds.

A global stiffness coefficient of the ground, C_g, can by defined as

$$C_g = \frac{P}{\delta}, \tag{1}$$

where P is the global action, equivalent with the sum of the elastic forces acting on the contact surface and δ – the global displacement of the contact surface.

The action, P, is a force or a couple, while δ is a linear displacement or an angular displacement (rotation). For $\delta = 1$, the equality between the global stiffness coefficient and the pressure over the support surface results,

$$C_g = P_{(\delta=1)}. \tag{2}$$

The flexibility matrix of the support surface is defined according to its discretization into interface finite elements. The displacements of the finite element centres are expressed as functions of pressure resultants acting on each element.

The assumption of uniform pressure on the finite surface yields to enough accurate results when the domains, the support surface has been discretized in, create a relative dense grid.

Briefly, it can be expressed that the vector of the domains centres displacements is proportional to the vector of the resultants of pressures acting on these elements. This proportionality is stated by the stiffness matrix of the support, in particular of ground.

A more refined discretization yields to a better approximation of the ground stiffness matrix. In case of using finite elements of greater size the accuracy of the method can be improved by adopting polynomial functions for the distribution of the pressures over the component domains and also by using adequate interpolation techniques.

The stiffness matrix for the ground enables us to determine the resultants of the pressures acting on the domains the contact surface was divided in. Knowing these forces, their resultant can be computed and then, the global elastic coefficient of the ground, C_g, is evaluated.

3. Uniform Displacement of the Contact Surface

The contact surface between the building and ground is divided into rectangular domains (Fig. 1). It is presumed that the structural system is translated along the vertical direction, the ground displacements (settlements) at the contact surface being uniform. These displacements are produced by the resultants of the uniform pressure acting on them.

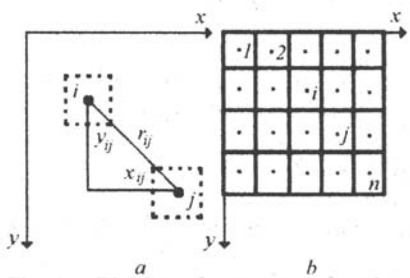

Fig. 1.– Division of contact surface into finite elements.

The mentioned displacements, denoted by w, can be expressed as

$$w_i = \sum_{j=1}^{n} b_{ij} Z_j. \tag{3}$$

Relations (3), n in number, can be written in vectorial form

$$\{w_i\} = [b]\{Z\}, \tag{4}$$

where $\{w\} = \{w_1\, w_2 \dots w_n\}^T$ is the vector of vertical displacements; $\{Z\} = \{Z_1\, Z_2 \dots Z_n\}^T$ – the vector of pressure resultants acting on the finite element surfaces;

$$[b] = \begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1n} \\ b_{21} & b_{22} & . & . & . & b_{2n} \\ . & . & . & . & . & . \\ . & . & . & b_{ij} & . & . \\ . & . & . & . & . & . \\ b_{n1} & b_{n2} & . & . & . & b_{nn} \end{bmatrix}; \tag{5}$$

the matrix from relation (5) represents the flexibility matrix of the support medium. The coefficient b_{ij}, $(i \neq j)$, represents the vertical displacement at the centre of element i, produced by a unit force applied at the centre of the finite element j while b_{ii} is the displacement at the centre of the finite element i produced by a uniform load that equals as intensity the inverse of element i area.

The vector $\{Z\}$ can be obtained, namely

$$\{Z\} = [b]^{-1}\{w\} = [K_S]\{w\}, \tag{6}$$

where $[K_S]$ is the stiffness matrix of the support (ground), that can be written in an extended shape

$$[K_S] = \begin{bmatrix} K_{11} & K_{12} & . & . & . & K_{1n} \\ K_{21} & K_{22} & . & . & . & K_{2n} \\ . & . & . & . & . & . \\ . & . & . & K_{ij} & . & . \\ . & . & . & . & . & . \\ K_{n1} & K_{n2} & . & . & . & K_{nn} \end{bmatrix}. \tag{7}$$

A vertical uniform translation means $w_1 = w_2 = \dots = w_i = \dots = w_n$, so that the vector $\{w\}$ has all components equal in value and when this value equals unity, it results

$$\{Z\}_{w=1} = \{\overline{Z}\} = [K_S]\{1\}. \tag{8}$$

The components of vector $\{\overline{Z}\}$ are $\overline{Z}_1, \overline{Z}_2, ..., \overline{Z}_i, ..., \overline{Z}_n$, where

$$\overline{Z}_i = K_{i1} + K_{i2} + ... + K_{ij} + ... + K_{in} = \sum_{j=1}^{n} K_{ij}. \tag{9}$$

If i cross all values from 1 to n, all the elements of vector $\{Z\}$ are obtained. According to relation (2), the global stiffness coefficient in translation is

$$C_{gZ} = P_{(\delta=1)} = P_{(w=1)}. \tag{10}$$

Obviously $P_{(w=1)}$ is the sum of the resultants of pressures acting on all interface elements

$$C_{gZ} = \overline{Z}_1 + \overline{Z}_2 + ... + \overline{Z}_i + ... + \overline{Z}_n = \sum_{j=1}^{n} \overline{Z}_i. \tag{11}$$

Taking into account (9) the coefficient C_{gZ} is obtained by adding the elements of the stiffness matrix

$$C_{gZ} = \sum_{i=1}^{n}\sum_{j=1}^{n} K_{ij}. \tag{12}$$

For practical applications it is often used the stiffness coefficient of the ground divided by the area of the contact surface (stiffness coefficient per unit area)

$$c_Z = \frac{C_{gZ}}{\Omega}, \tag{13}$$

where Ω is the area of the contact surface.

From the above obtained results it can be concluded that for the stiffness coefficient per unit area, c_Z, or the global stiffness coefficient, C_{gZ}, evaluation, the flexibility matrix of the ground must be determined and then, by inverting it, the stiffness matrix, K_S, can be obtained.

4. The Model of Linear Elastic Isotropic and Homogeneous Half-Space

For the elastic, isotropic and homogeneous half-space the procedure is shown below. The elements of the flexibility matrix are derived starting from Boussinesq relation

$$w = \frac{1-\nu_0^2}{\pi E_0} \cdot \frac{P}{r}, \tag{14}$$

where: P is the normal force to the half-space surface; r – the distance from the force P point of application to the point where w is determined; E_0 – the longitudinal modulus of elasticity; ν – Poisson's ratio of the support (ground).

This relation can be directly applied for the calculus of coefficients $b_{ij}.(i \neq j)$. The errors are small when the ratio of the interface finite element sides equals the unity.

For coefficient b_{ij} the relation (14) permits to write the relation

$$b_{ij} = \frac{1-\nu_0^2}{\pi E_0} \cdot \frac{1}{r_{ij}}. \tag{15}$$

where r_{ij} is the distance between the geometric centres of the elements i and j, that can be expressed in terms of the relative coordinates $x_{ij} = x_j - x_i$ and $y_{ij} = y_j - y_i$ (Fig. 1), that is

$$r_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}\,. \tag{16}$$

The coefficients b_{ij} can be computed with the following known relation

$$b_{ij} = \frac{1-\nu_0^2}{\pi E_0} \cdot \frac{2}{a}\left(\frac{a}{b}\text{arg sh}\frac{b}{a} + \text{arg sh}\frac{a}{b}\right) \cdot \tag{17}$$

where a and b are the sides of the rectangular elements.

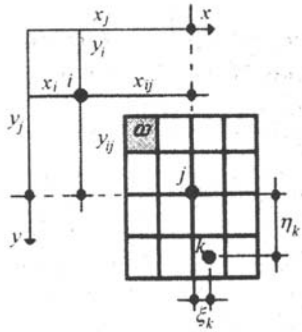

Fig. 2.– Finite element j and point i coordinates.

A numerical evaluation can be performed for the vertical displacements, $\overline{w}_i$, generated by forces $Z_j = 1$ (including for $i = j$), representing the resultants of uniformly distributed pressures $p_z = 1/\Omega_j$, where Ω_j is the corresponding surface to point j.

The surface Ω_j is divided at its turn in e elements of equal surface, $\omega = \Omega_j/e$, so that the forces corresponding to an element of surface ω will be equal to ω/Ω_j.

According to the division shown in Fig. 2, at point $i(x_i, y_j)$, the displacement $\overline{w}_i$, given by the uniformly distributed force acting on the surface Ω_j, having the resultant $Z_j = 1$, is

$$\frac{2\pi E_0}{1+\nu_0}\overline{\omega}_{i(z=1)} = 2\frac{\omega}{\Omega_j}(1-\nu_0)\sum_{k=1}^{e}\frac{1}{\sqrt{(x_{ij} \pm \xi_{jk})^2 + (y_{ij} \pm \eta_{jk})^2}}. \tag{18}$$

In Fig. 2, the geometrical elements involved in relation (18) are represented.

5. The Model of Linear Elastic Anisotropic Half-Space

When the soil is modelled as a linear elastic anisotropic half-space, the vertical displacements at point i, produced by the uniformly distributed load on the finite element j, that has the resultant $Z_j = 1$, is determined as

$$(19) \quad \overline{w}_{(z=1)} = \frac{\omega}{\Omega_j} \cdot \frac{1}{2\pi}\sqrt{\frac{A}{L}}\frac{\left[(AC+L)^2-(F+L)^2\right]^{1/2}}{AC-F^2}\sum_{k=1}^{e}\frac{1}{\sqrt{(x_{ij}\pm\xi_{jk})^2+(y_{ij}\pm\eta_{jk})^2}},$$

where

$$A = \frac{E_H}{\phi}(1+\nu_1\nu_2), \quad C = \frac{E_V}{\phi}(1-\nu_1^2),$$

$$F = \frac{E_V}{\phi}(1-\nu_1), \quad L = G_1 = G_2 = G,$$

$$\phi = (1+\nu_0)(1-\nu_1-2\nu_2\nu_3),$$

E_H, E_V are the soil longitudinal moduli of elasticity in horizontal and vertical direction, respectively, ν_1, ν_2, ν_3 – the Poisson's ratios corresponding to the anisotropy of three principals directions; G_1, G_2 – the soil shear moduli in the horizontal planes and

$$G_3 = \frac{E_H}{2(1+\nu_1)}$$

– the soil shear modulus in the vertical plane.

6. The Stiffness Coefficient of the Ground for the System Rotation with Respect to a Principal Horizontal Axis

The rotation of the structural system is produced by a couple located in a vertical plane.

The global elastic coefficient, $C_{g\varphi}$, is evaluated for the rotation (slope) of the section from the bottom of the system around a principal axis. Let us admit this axis to be y-axis. The pressures that occur on the contact surface produce a moment with respect to this axis, that is proportional to the section rotation, the proportionality coefficient being $C_{g\varphi}$.

The displacements at the centres of the interface finite elements are obtained with the following relations:

$$(20) \qquad w_i = x_i\theta = \sum_{j=1}^{n} b_{ij}Z_j, \quad (i = 1,2,...,n),$$

or

$$(21) \qquad \theta = \frac{1}{x_i}\sum_{j=1}^{n} b_{ij}Z_j, \quad (i = 1,2,...,n).$$

For $\theta = 1$ the system of equation can be put under the following shape:

$$(22) \qquad \left[\frac{1}{x}\right][b]\{Z\}_{\theta=1} = \{1\}.$$

From equation (22) it is obtained

$$Z_{\theta=1} = [b]^{-1}\left[\frac{1}{x}\right]^{-1}\{1\} = [b]^{-1}[x]\{1\}. \tag{23}$$

As $[b]^{-1} = [K_S]$ and $[x]\{1\} = \{x\}$ it results:

$$\{Z\}_{\theta=1} = [K_S]\{x\}. \tag{24}$$

By considering the moment of forces $\{Z\}_{\theta=1}$ with respect to Oy-axis, the global coefficient, $C_{g\varphi}$, of the ground is obtained, namely

$$C_{g\varphi} = \{x\}^T\{Z\}_{\theta=1} = x_1 Z_1 + ... + x_n Z_n \tag{25}$$

and taking into account the previous relations it can be written that:

$$C_{g\varphi} = \{x\}^T[K_S]\{x\}. \tag{26}$$

Consequently knowing the ground stiffness matrix, the coefficient $C_{g\varphi}$ can be immediately determined. It is also defined the coefficient c_φ that results from the relation

$$c_\varphi = \frac{C_{g\varphi}}{I_y} = \frac{1}{I_y}\{x\}^T[K_S]\{x\}, \tag{27}$$

where I_y is the moment of inertia of the contact surface with respect to Oy-axis.

A similar procedure must be followed in order to determine the same coefficient, but with respect to Ox-axis.

7. Conclusions

From the previously analysis it results that the global elastic coefficient depends on the soil nature, on the size and shape of the contact surface between the structure and the ground. These coefficients don't have constant values for a certain soil. They can be better considered as parameters that must be determined for each practical case.

The experimental test results show that the global coefficients computed for contact surfaces of rectangular shape are very close to those obtained by using other methods mentioned in literature.

The theoretical and practical procedures, proposed in the present work, have a more general application but in the same time, the particularities of the structure and ground can be pointed out.

The experimental tests, especially on machinery foundations, but also on other rigid elements subjected to low dynamic actions, have values in good accordance with those obtained by considering for the ground the half-space model.

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O NOUĂ METODĂ DE EVALUARE A COEFICIENŢILOR DE RIGIDITATE AI TERENULUI DE FUNDARE

(Rezumat)

Se dezvoltă metode numerice generale bazate pe discretizarea suprafeţei de contact dintre structură şi teren, în elemente finite de interfaţă, cu ajutorul cărora se determină matricea de complianţă a terenului şi apoi matricea de rigiditate. Pentru mediul de rezemare se adoptă modelul semispaţiului liniar elastic, omogen şi izotrop, cât şi anizotrop. Această procedură permite determinarea coeficienţilor elastici globali şi specifici, care au un rol esenţial în evaluarea răspunsului structurilor rigide sau cu baza rigidă.