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完全导电圆锥并矢 Green 函数* **

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摘要　本文运用算子法和并矢运算的分布理论法，求解了完全导电圆锥并矢 Green 函数，且证明解是满足电磁基本方程的.

关键词　电磁理论；并矢 Green 函数；完全导电圆锥

一、引　言

并矢 Green 函数是电磁理论中的一个重要概念，广泛应用于求解天线和微波技术中的辐射和散射等问题[1,2]. 本文利用文献 [3—7] 的方法和技巧求解了完全导电圆锥并矢 Green 函数（简称锥 DGF），且证明解是满足并矢 Maxwell 方程和电流连续性方程的.

二、锥 DGF 的计算

为了求解电型和磁型锥 DGF，引入矢量位型锥 DGF $\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}')$，它满足方程

$$\nabla^2\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}') + k^2\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}') = -\bar{\boldsymbol{I}}\delta(\boldsymbol{R}-\boldsymbol{R}') \tag{1}$$

在导电锥边界上，$\hat{\boldsymbol{n}}\times\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}')=0$；当 $\boldsymbol{R}\to\infty$ 时，$\bar{\boldsymbol{G}}_A(R/R')$ 满足 Sommerfeld 的辐射条件.

$\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}')$ 与电型和磁型锥 DGF $\bar{\boldsymbol{G}}_e(\boldsymbol{R}/\boldsymbol{R}')$，$\bar{\boldsymbol{G}}_m(\boldsymbol{R}/\boldsymbol{R}')$ 之间的关系式是

$$\bar{\boldsymbol{G}}_e(\boldsymbol{R}/\boldsymbol{R}') = \left(\bar{\boldsymbol{I}} + \frac{1}{k^2}\nabla\nabla\right)\cdot\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}') \tag{2a}$$

$$\bar{\boldsymbol{G}}_m(\boldsymbol{R}/\boldsymbol{R}') = \nabla\times\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}') \tag{2b}$$

圆锥坐标系中的矢量波函数是[1]

$$\begin{aligned}\boldsymbol{L}_{{}^{e}_{o}m^{\mu}_{\lambda}}(K) &= \nabla[\varphi_{{}^{e}_{o}m^{\mu}_{\lambda}}(K)]\\ &= \left(\frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}}\right)\frac{j^{\mu}_{\lambda}(Kr)}{r}\boldsymbol{B}_{{}^{e}_{o}m^{\mu}_{\lambda}}(\Omega) + \frac{dj^{\mu}_{\lambda}(Kr)}{dr}\boldsymbol{D}_{{}^{e}_{o}m^{\mu}_{\lambda}}(\Omega)\end{aligned} \tag{3a}$$

$$\begin{aligned}\boldsymbol{M}_{{}^{e}_{o}m^{\mu}_{\lambda}}(K) &= \nabla\times[\varphi_{{}^{e}_{o}m^{\mu}_{\lambda}}(K)\hat{\boldsymbol{r}}]\\ &= \left(\frac{\sqrt{\mu(\mu+1)}}{\sqrt{\lambda(\lambda+1)}}\right)j^{\mu}_{\lambda}(Kr)\boldsymbol{C}_{{}^{e}_{o}m^{\mu}_{\lambda}}(\Omega)\end{aligned} \tag{3b}$$

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$$\boldsymbol{N}_{\substack{e\\o}m_\lambda^\mu}(K) = \frac{1}{k}\nabla\times\nabla\times[\varphi_{\substack{e\\o}m_\lambda^\mu}(K)\hat{\boldsymbol{r}}]$$

$$= \begin{pmatrix}\sqrt{\mu(\mu+1)}\\ \sqrt{\lambda(\lambda+1)}\end{pmatrix}\left[\frac{j_\lambda^\mu(Kr)}{Kr} + \frac{dj_\lambda^\mu(Kr)}{d(Kr)}\right]\boldsymbol{B}_{\substack{e\\o}m_\lambda^\mu}(\Omega)$$

$$+ \begin{pmatrix}\sqrt{\mu(\mu+1)}\\ \sqrt{\lambda(\lambda+1)}\end{pmatrix}\frac{j_\lambda^\mu(Kr)}{Kr}\boldsymbol{D}_{\substack{e\\o}m_\lambda^\mu}(\Omega) \tag{3c}$$

式中

$$\varphi_{\substack{e\\o}m_\mu^\lambda}(k) = j_\lambda^\mu(Kr)P_\lambda^m(\cos\theta)\frac{\cos}{\sin}m\phi = j_\lambda^\mu(Kr)Y_{\substack{e\\o}m_\lambda^\mu}(\Omega) \tag{4}$$

$$\boldsymbol{B}_{\substack{e\\o}m_\lambda^\mu}(\Omega) = \frac{r}{\begin{pmatrix}\sqrt{\mu(\mu+1)}\\ \sqrt{\lambda(\lambda+1)}\end{pmatrix}}\nabla[Y_{\substack{e\\o}m_\lambda^\mu}(\Omega)] \tag{5a}$$

$$\boldsymbol{C}_{\substack{e\\o}m_\lambda^\mu}(\Omega) = -\hat{\boldsymbol{r}}\times\boldsymbol{B}_{\substack{e\\o}m_\lambda^\mu}(\Omega) \tag{5b}$$

$$\boldsymbol{D}_{\substack{e\\o}m_\lambda^\mu}(\Omega) = \hat{\boldsymbol{r}}Y_{\substack{e\\o}m_\lambda^\mu}(\Omega) \tag{5c}$$

式中，$\hat{\boldsymbol{r}}$ 是径向单位矢量，$j_\lambda^\mu(Kr)$是 μ 或 λ 阶球 Bessel 函数，$P_\lambda^m(\cos\theta)$ 是 m 阶 μ 或 λ 次连带 Legendre 函数.

由(3)—(5)式可知,就所涉及的函数结构条件而言,圆锥矢量波函数与球矢量波函数是相似的,因而我们可用类似的方法和步骤证明圆锥矢量本征函数 $\boldsymbol{L}$, $\boldsymbol{M}$ 和 $\boldsymbol{N}$ 的正交性及归一化系数.

由边界条件,对单圆锥有

$$P_\mu^m(\cos\theta)|_{\theta=\theta_0} = 0 \tag{6a}$$

$$\left.\frac{dP_\lambda^m(\cos\theta)}{d(\cos\theta)}\right|_{\theta=\theta_0} = 0 \tag{6b}$$

对双圆锥有

$$P_\mu^m(\cos\theta)|_{\theta=\theta_1} = P_\mu^m(\cos\theta)|_{\theta=\theta_2} = 0 \tag{7a}$$

$$\left.\frac{dP_\lambda^m(\cos\theta)}{d(\cos\theta)}\right|_{\theta=\theta_1} = \left.\frac{dP_\lambda^m(\cos\theta)}{d(\cos\theta)}\right|_{\theta=\theta_2} = 0 \tag{7b}$$

式中 θ_0, θ_1 和 θ_2 为锥体的张角.

由并矢算子谱理论[4],并矢波动方程(1)式的一般解为

$$\bar{\boldsymbol{U}}(\boldsymbol{R}/\boldsymbol{R}') = (\mathscr{L} - k^2)^{-1}\bar{\boldsymbol{F}}(\boldsymbol{R}/\boldsymbol{R}') = \sum_\nu \frac{\boldsymbol{U}_\nu(\boldsymbol{R})\boldsymbol{A}_\nu(\boldsymbol{R}')}{\lambda_\nu - k^2} \tag{8}$$

式中,$\sum$表示对离散谱求和,对连续谱积分.

首先将(1)式中的广义函数 $\bar{\boldsymbol{I}}\delta(\boldsymbol{R}-\boldsymbol{R}')$ 按圆锥矢量本征函数展开,得

$$\bar{\boldsymbol{I}}\delta(\boldsymbol{R}-\boldsymbol{R}') = \int_0^\infty K^2\sum_m\left\{\sum_\mu C_{m\mu}\left[\frac{\mu(\mu+1)}{K^2}\boldsymbol{L}_{\substack{e\\o}m\mu}(K)\boldsymbol{L}'_{\substack{e\\o}m\mu}(K)\right.\right.$$

$$\left.\left.+ \boldsymbol{N}_{\substack{e\\o}m\mu}(K)\boldsymbol{N}'_{\substack{e\\o}m\mu}(K)\right] + \sum_\lambda C_{m\lambda}\boldsymbol{M}_{\substack{e\\o}m\lambda}(K)\boldsymbol{M}'_{\substack{e\\o}m\lambda}(K)\right\}dK \tag{9}$$

式中

$$C_{m\mu}=\frac{2-\delta_0}{\pi^2\mu(\mu+1)I_{m\mu}} \tag{10a}$$

$$I_{m\mu}=\int_{\theta_0}^{\pi}[P_\mu^m(\cos\theta)]^2\sin\theta d\theta$$

$$C_{m\lambda}=\frac{2-\delta_0}{\pi^2\lambda(\lambda+1)I_{m\lambda}} \tag{10b}$$

$$I_{m\lambda}=\int_{\theta_0}^{\pi}[P_\lambda^m(\cos\theta)]^2\sin\theta d\theta$$

式中

$$\delta_0=\begin{cases}0, & m\neq 0;\\ 1, & m=0.\end{cases}$$

利用(8),(9)式,则可得到方程(1)的解为

$$G_A(R/R')=\int_0^\infty\frac{K^2}{K^2-k^2}\sum_m\left\{\sum_\mu C_{m\mu}\left[\frac{\mu(\mu+1)}{K^2}L_{\substack{e\\o}m\mu}(K)L'_{\substack{e\\o}m\mu}(K)\right.\right.$$
$$\left.\left.+N_{\substack{e\\o}m\mu}(K)N'_{\substack{e\\o}m\mu}(K)\right]+\sum_\lambda C_{m\lambda}M_{\substack{e\\o}m\lambda}(K)M'_{\substack{e\\o}m\lambda}(K)\right\}dK \tag{11}$$

由球 Bessel 函数的递推公式和留数定理,消去(11)式中对连续谱K的积分,得

$$G_A(R/R')=\sum_m\frac{i\pi k}{2}\left\{\left[\sum_\mu C_{m\mu}\left(\frac{\mu(\mu+1)}{k^2}L^{(1)}_{\substack{e\\o}mn}(k)L'_{\substack{e\\o}m\mu}(k)\right.\right.\right.$$
$$\left.\left.+N^{(1)}_{\substack{e\\o}m\mu}(k)N'_{\substack{e\\o}m\mu}(k)\right)+\sum_\lambda C_{m\lambda}M^{(1)}_{\substack{e\\o}m\lambda}(k)M'_{\substack{e\\o}m\lambda}(k)\right]\mu(r-r')$$
$$+\left[\sum_\mu C_{m\mu}\left(\frac{\mu(\mu+1)}{k^2}L_{\substack{e\\o}m\mu}(k)L'^{(1)}_{\substack{e\\o}m\mu}(k)+N_{\substack{e\\o}m\mu}(k)N'^{(1)}_{\substack{e\\o}m\mu}(k)\right)\right.$$
$$\left.\left.+\sum_\lambda C_{m\lambda}M_{\substack{e\\o}m\lambda}(k)M'^{(1)}_{\substack{e\\o}m\lambda}(k)\right]\mu(r'-r)\right\} \tag{12}$$

式中矢量本征函数$L^{(1)}_{\substack{e\\o}m\mu}(K)$, $M^{(1)}_{\substack{e\\o}m\lambda}(k)$和$N^{(1)}_{\substack{e\\o}m\mu}(k)$是用第一类球 Hankel 函数$h^{(1)}_{\substack{\mu\\\lambda}}(kr)$替换(3)式中球 Bessel 函数$j_{\substack{\mu\\\lambda}}(Kr)$得到; $\mu(r-r')$是 Heaviside 函数,定义为

$$\mu(r-r')=\begin{cases}0, & r<r'\\ 1, & r\geq r'\end{cases}$$

由分布理论中的关系式$\frac{d\mu(x-x')}{dx}=\delta(x-x')$和广义函数的求导法则,我们有

$$\frac{d}{dr}\left[h^{(1)}_{\substack{\mu\\\lambda}}(kr)\frac{dj_{\substack{\mu\\\lambda}}(kr')}{dr'}\mu(r-r')+j_{\substack{\mu\\\lambda}}(kr)\frac{dh^{(1)}_{\substack{\mu\\\lambda}}(kr')}{dr'}\mu(r'-r)\right]$$
$$=\frac{dh^{(1)}_{\substack{\mu\\\lambda}}(kr)}{dr}\frac{dj_{\substack{\mu\\\lambda}}(kr')}{dr'}\mu(r-r')+\frac{dj_{\substack{\mu\\\lambda}}(kr)}{dr}\frac{dh^{(1)}_{\substack{\mu\\\lambda}}(kr')}{dr'}\mu(r'-r)$$
$$+A\delta(r-r') \tag{13}$$

式中

$$A = h^{(1)}_{\mu_\lambda}(kr)\frac{dj_{\mu_\lambda}(kr')}{dr'} - j_{\mu_\lambda}(kr)\frac{dh^{(1)}_{\mu_\lambda}(kr')}{dr'} \tag{14}$$

由球类 Bessel 函数 ϕ_l 的递推关系式

$$l\phi_{l-1} - (l+1)\phi_{l+1} = (2l+1)\phi_l' \tag{15a}$$

和 Bessel 函数的关系式

$$Y_{n-1}J_n - Y_nJ_{n+1} = \frac{2}{\pi x} \tag{15b}$$

得

$$A = -\frac{i}{kr^2} \tag{16}$$

利用(13)式和矢量恒等式

$$\nabla \times [\varphi(r)\boldsymbol{A}(\boldsymbol{R})] = \varphi(r)\nabla \times \boldsymbol{A}(\boldsymbol{R}) + \hat{\boldsymbol{r}} \times \left[\boldsymbol{A}(\boldsymbol{R})\frac{d\varphi(r)}{dr}\right] \tag{17}$$

及

$$\nabla\nabla\cdot \boldsymbol{L}_{{}^e_o m\mu}(k) = -k^2\boldsymbol{L}_{{}^e_o m\mu}(k) \tag{18a}$$

$$\nabla \times \boldsymbol{M}_{{}^e_o m\lambda}(k) = k\boldsymbol{N}_{{}^e_o m\lambda}(k) \tag{18b}$$

$$\nabla \times \boldsymbol{N}_{{}^e_o m\mu}(k) = k\boldsymbol{M}_{{}^e_o m\mu}(k) \tag{18c}$$

得

$$\begin{aligned}\nabla \times [\boldsymbol{L}^{(1)}_{{}^e_o m\mu}(k)\boldsymbol{L}'_{{}^e_o m\mu}(k)\mu(r-r') + \boldsymbol{L}_{{}^e_o m\mu}(k)\boldsymbol{L}'^{(1)}_{{}^e_o m\mu}(k)\mu(r'-r)] \\ = \sqrt{\mu(\mu+1)}\,\hat{\boldsymbol{r}} \times \frac{\boldsymbol{B}_{{}^e_o m\mu}(\Omega)}{r}\boldsymbol{D}_{{}^e_o m\mu}(\Omega')\left(\frac{-i}{kr^2}\right)\delta(r-r')\end{aligned} \tag{19a}$$

$$\begin{aligned}\nabla \times \{[\boldsymbol{M}^{(1)}_{{}^e_o m\lambda}(k)\boldsymbol{M}'_{{}^e_o m\lambda}(k) + \boldsymbol{N}^{(1)}_{{}^e_o m\mu}(k)\boldsymbol{N}'_{{}^e_o m\mu}(k)]\mu(r-r') \\ + [\boldsymbol{M}_{{}^e_o m\lambda}(k)\boldsymbol{M}'^{(1)}_{{}^e_o m\lambda}(k) + \boldsymbol{N}_{{}^e_o m\mu}(k)\boldsymbol{N}'^{(1)}_{{}^e_o m\mu}(k)]\mu(r'-r)\} \\ = k\{[\boldsymbol{N}^{(1)}_{{}^e_o m\lambda}(k)\boldsymbol{M}'_{{}^e_o m\lambda}(k) + \boldsymbol{M}^{(1)}_{{}^e_o m\mu}(k)\boldsymbol{N}'_{{}^e_o m\mu}(k)]\mu(r-r') \\ + \boldsymbol{N}_{{}^e_o m\lambda}(k)\boldsymbol{M}'^{(1)}_{{}^e_o m\lambda}(k) + \boldsymbol{M}_{{}^e_o m\mu}(k)\boldsymbol{N}'^{(1)}_{{}^e_o m\mu}(k)]\mu(r'-r)\} \\ + (\mu(\mu+1))^{\frac{3}{2}}\hat{\boldsymbol{r}} \times \frac{\boldsymbol{B}_{{}^e_o m\mu}(\Omega)}{kr}\boldsymbol{D}_{{}^e_o m\mu}(\Omega')\left(\frac{i}{k^2r^2}\right)\delta(r-r')\end{aligned} \tag{19b}$$

和

$$\begin{aligned}\nabla\nabla\cdot[\boldsymbol{L}^{(1)}_{{}^e_o m\mu}(k)\boldsymbol{L}'_{{}^e_o m\mu}(k)\mu(r-r') + \boldsymbol{L}_{{}^e_o m\mu}(k)\boldsymbol{L}'^{(1)}_{{}^e_o m\mu}(k)\mu(r'-r)] \\ = -k^2[\boldsymbol{L}^{(1)}_{{}^e_o m\mu}(k)\boldsymbol{L}'_{{}^e_o m\mu}(k)\mu(r-r') + \boldsymbol{L}_{{}^e_o m\mu}(k)\boldsymbol{L}'^{(1)}_{{}^e_o m\mu}(k)\mu(r'-r)] \\ + \nabla\left[\sqrt{\mu(\mu+1)}\,\hat{\boldsymbol{r}}\cdot\boldsymbol{D}_{{}^e_o m\mu}(\Omega)\frac{\boldsymbol{B}_{{}^e_o m\mu}(\Omega')}{r'}\left(\frac{i}{kr^2}\right)\delta(r-r')\right] \\ - k^2\left(\frac{-i}{kr^2}\right)\boldsymbol{D}_{{}^e_o m\mu}(\Omega)\boldsymbol{D}_{{}^e_o m\mu}(\Omega')\delta(r-r')\end{aligned} \tag{20a}$$

$$\nabla\nabla\cdot\{[\boldsymbol{M}^{(1)}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'_{\substack{e\\o}m\lambda}(k)+\boldsymbol{N}^{(1)}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'_{\substack{e\\o}m\mu}(k)]\mu(r-r')$$
$$+\boldsymbol{M}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'^{(1)}_{\substack{e\\o}m\lambda}(k)+\boldsymbol{N}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'^{(1)}_{\substack{e\\o}m\mu}(k)]\mu(r'-r)\}$$
$$=\nabla\left[(\mu(\mu+1))^{\frac{3}{2}}\boldsymbol{r}\cdot\boldsymbol{D}_{\substack{e\\o}m\mu}(\Omega)\frac{\boldsymbol{B}_{\substack{e\\o}m\mu}(\Omega')}{kr'}\left(\frac{-i}{k^2r^2}\right)\delta(r-r')\right] \tag{20b}$$

将 $\bar{\boldsymbol{G}}_A(\boldsymbol{R}/\boldsymbol{R}')$ 代入(2)式,利用(19),(20)式得电型锥 DGF

$$\bar{\boldsymbol{G}}_e(\boldsymbol{R}/\boldsymbol{R}')=\sum_m\frac{i\pi k}{2}\left\{\left[\sum_\lambda C_{m\lambda}\boldsymbol{M}^{(1)}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'_{\substack{e\\o}m\lambda}(k)+\sum_\mu C_{m\mu}\boldsymbol{N}^{(1)}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'_{\substack{e\\o}m\mu}(k)\right]\mu(r-r')\right.$$
$$\left.+\left[\sum_\lambda C_{m\lambda}\boldsymbol{M}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'^{(1)}_{\substack{e\\o}m\lambda}(k)+\sum_\mu C_{m\mu}\boldsymbol{N}_{\substack{e\\o}m\lambda}(k)\boldsymbol{N}'_{\substack{e\\o}m\lambda}(k)\right]\mu(r'-r)\right\}$$
$$-\sum_m\sum_\mu\left(\frac{\pi C_{m\mu}}{2}\right)\frac{\mu(\mu+1)}{k^2r^2}\boldsymbol{D}_{\substack{e\\o}m\mu}(\Omega)\boldsymbol{D}_{\substack{e\\o}m\mu}(\Omega')\delta(r-r') \tag{21a}$$

和磁型锥 DGF

$$\bar{\boldsymbol{G}}_m(\boldsymbol{R}/\boldsymbol{R}')=\sum_m\frac{i\pi k^2}{2}\left\{\left[\sum_\mu C_{m\mu}\boldsymbol{M}^{(1)}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'_{\substack{e\\o}m\mu}(k)\right.\right.$$
$$\left.+\sum_\lambda C_{m\lambda}\boldsymbol{N}^{(1)}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'_{\substack{e\\o}m\lambda}(k)\right]\mu(r-r')$$
$$+\left[\sum_\mu C_{m\mu}\boldsymbol{M}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'^{(1)}_{\substack{e\\o}m\mu}(k)+\sum_\lambda C_{m\lambda}\boldsymbol{N}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'^{(1)}_{\substack{e\\o}m\lambda}(k)\right]\mu(r'-r) \tag{21b}$$

利用分布理论中的关系式

$$\sum_m\sum_\mu\frac{2-\delta_0}{2\pi I_{m\mu}}Y_{\substack{e\\o}m\mu}(\Omega)Y_{\substack{e\\o}m\mu}(\Omega')=\frac{\delta(\theta-\theta')\delta(\phi-\phi')}{\sin\theta} \tag{22}$$

则 (21a) 式中的具有 δ 函数的奇异项可改写为

$$\bar{\boldsymbol{G}}_{eL}(\boldsymbol{R}/\boldsymbol{R}')=-\frac{1}{k^2}\hat{\boldsymbol{r}}\hat{\boldsymbol{r}}\frac{\delta(\theta-\theta')\delta(\phi-\phi')\delta(r-r')}{r^2\sin\theta} \tag{23}$$

三、解的验证

对(21)式取旋度,利用(19)式得

$$\nabla\times\bar{\boldsymbol{G}}_e(\boldsymbol{R}/\boldsymbol{R}')=\sum_m\frac{i\pi k^2}{2}\left\{\left[\sum_\mu C_{m\mu}\boldsymbol{M}^{(1)}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'_{\substack{e\\o}m\mu}(k)\right.\right.$$
$$\left.+\sum_\lambda C_{m\lambda}\boldsymbol{N}^{(1)}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'_{\substack{e\\o}m\lambda}(k)\right]\mu(r-r')$$
$$+\left[\sum_\mu C_{m\mu}\boldsymbol{M}_{\substack{e\\o}m\mu}(k)\boldsymbol{N}'_{\substack{e\\o}m\mu}(k)\right.$$
$$\left.\left.+\sum_\lambda C_{m\lambda}\boldsymbol{N}_{\substack{e\\o}m\lambda}(k)\boldsymbol{M}'^{(1)}_{\substack{e\\o}m\lambda}(k)\right]\mu(r'-r)\right\}$$

$$+(\mu(\mu+1))^{3/2}\hat{r}\times\frac{B_{\substack{e\\o}m\mu}(\Omega)}{kr}D_{\substack{e\\o}m\mu}(\Omega')\left(\frac{i}{k^2r^2}\right)\delta(r-r')$$
$$-\frac{\mu(\mu+1)}{k^2r^2}\nabla\times D_{\substack{e\\o}m\mu}(\Omega)D_{\substack{e\\o}m\mu}(\Omega')\delta(r-r')$$
$$=\bar{G}_m(R/R') \tag{24a}$$

和

$$\nabla\times G_m(R/R')=\nabla\times\nabla\times\bar{G}_A(R/R')=(-\nabla^2+\nabla\nabla\cdot)\bar{G}_A(R/R')$$
$$=[k^2\bar{G}_A(R/R')+\bar{I}\delta(R-R')]$$
$$+\sum_m\sum_\mu\left(\frac{-i\pi k}{2}\right)[\mu(\mu+1)(L^{(1)}_{\substack{e\\o}m\mu}(k)L'_{\substack{e\\o}m\mu}(k)\mu(r-r')$$
$$+L_{\substack{e\\o}m\mu}(k)L'^{(1)}_{\substack{e\\o}m\mu}(k)\mu(r'-r))]-\hat{r}\hat{r}\delta(R-R')$$
$$=\sum_m\frac{i\pi k^3}{2}\left\{\left[\sum_\lambda C_{m\lambda}M^{(1)}_{\substack{e\\o}m\lambda}(k)M'_{\substack{e\\o}m\lambda}(k)\right.\right.$$
$$\left.+\sum_\mu C_{m\mu}N^{(1)}_{\substack{e\\o}m\mu}(k)N'_{\substack{e\\o}m\mu}(k)\right]\mu(r-r')$$
$$+\left[\sum_\lambda C_{m\lambda}M_{\substack{e\\o}m\lambda}(k)M'^{(1)}_{\substack{e\\o}m\lambda}(k)\right.$$
$$\left.\left.+\sum_\mu C_{m\mu}N_{\substack{e\\o}m\mu}(k)N'^{(1)}_{\substack{e\\o}m\mu}(k)\right]\mu(r'-r)\right\}$$
$$-\hat{r}\hat{r}\delta(R-R')+\bar{I}\delta(R-R')$$
$$=k^2\bar{G}_c(R/R')+\bar{I}\delta(R-R') \tag{24b}$$

对 (21a) 式取散度，利用(20)式得

$$\nabla\cdot\bar{G}_c(R/R')=-\sum_m\sum_\mu\frac{i\pi k}{2}\mu(\mu+1)\hat{r}\cdot D_{\substack{e\\o}m\mu}(\Omega)B_{\substack{e\\o}m\mu}(\Omega')\left(\frac{i}{k^2r^2}\right)\delta(r-r')$$
$$-\nabla\cdot\frac{1}{k^2}\hat{r}\hat{r}\delta(R-R')$$
$$=-\frac{1}{k^2}\nabla\cdot\bar{I}_t\delta(R-R')-\frac{1}{k^2}\nabla\cdot\hat{r}\hat{r}\delta(R-R')$$
$$=-\frac{1}{k^2}\nabla\cdot\bar{I}\delta(R-R') \tag{25}$$

(24)和(25)式表明本文给出的电型和磁型锥 DGF 是满足并矢 Maxwell 方程和电流连续性原理的。

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DYADIC GREEN'S FUNCTIONS FOR PERFECTLY CONDUCTING CIRCULAR CONES

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Abstract The dyadic Green's functions for perfectly conducting circular cones are derived by means of the operator method and the theory of distribution of dyadic operation. It is shown that the electric and magnetic dyadic Green's functions given satisfy the basic dyadic equations.

Key words Electromagnetic field theory; Perfectly conducting circular cone; Dyadic Green's function