An Extended Adiabatic Approach for Nuclear Reactions

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Abstract

In this letter, an alternative approximation is developed for the treatment of breakup process in nuclear reactions, which presents a way of evaluating non-adiabatic corrections to the adiabatic model three-body wavefunction by introducing a more consistent prescription for the continuum n-p center of mass energy. Although the present formalism goes beyond the quasi-adiabatic approach, due to the use of exact continuum channel energies, its application to deuteron involving reactions at medium energies shows that such an approach can not be used in its present form. A possible reason behind this failure is discussed.

Introduction

In nuclear reactions between composite particles, those involving a loosely bound particle, such as the deuteron, break-up of the particle takes place as a real or virtual process. The process have attracted the attention of many researchers and a number of investigations have been made [1].

Such studies started with those on the deuteron break-up. The deuteron break-up process are at least a three-body system composed of a proton (p), a neutron (n) and the target nucleus in its ground state. Since the resultant theoretical three-body Hamiltonian is too complicated for practical purposes, most of the studies have employed a phenomenological three-body Hamiltonian in which the nucleon-nucleus interaction is represented by the optical potentials as half the deuteron incident energy, and an effective nucleon-nucleon potential is for the n-p interaction.

Several different approximate methods for solving the pnenomenological three-body Hamiltonian were proposed for theoretical treatment of the process before the adiabatic approximation was introduced by Johnson and Soper [2] for deuteron involving reactions. The approximation was based on the observation that the internal motion of the n-p

system, with a weakly bound ground state, is much slower than its center-of-mass motion. It was shown that, with this approach, the breakup process greatly affects the transition matrix elements of these reactions. The approximation has since been used widely for theoretical analysis of the breakup processes and for the analysis of the experimental data, not only on reactions involving a deuteron, but also on reactions induced by ^{6}Li ions, and has established the importance of the breakup process in such reactions [1].

The application of adiabatic ideas [2] to nuclear breakup effects played a key role in the development of models for the treatments of quantum mechanical three-body systems. In spite of the efficiency and success of the adiabatic approach, however, the high quality spin-dependent experimental transfer reaction data [3] clarified that some physical contributions are missing from the calculation of the reaction amplitude. As the adiabatic approximation is formulated under the assumption of low energy n-p breakup, its treatment of such energetic contributions is naturally suspect. An improved treatment of the higher energy breakup configurations is thus required.

Recent calculations [4] using the quasi-adiabatic approximation, where the n-p center of mass energy in breakup contributions is assigned some average breakup energy, have yielded significant corrections to the adiabatic model [3] in such reactions in the description of experimental data for large neutron angular momentum (d,p) transfer reactions at medium energies.

The quasi-adiabatic model is simple enough to clarify many aspects of reactions involving deuteron and weakly bound composite nuclei and can be used for the practical analysis of experimental data. The model avoids the complications of the more powerful but computationally more expensive CDCC technique [1]. In particular, discritization is avoided. However the quasi-adiabatic theory depends implicitly on an average excitation energy which represents excitation energy of the n-p pair in the continuum. The theory itself does not automatically provide a unique or obvious prescription for the mean breakup energy. There is a certain flexibility as to the correct choice of the center-of-mass energy of the broken n-p pair. This is shortcoming of the quasi-adiabatic approximation. Nevertheless, the model, together with a reasonable prescription is capable of reproducing many of the features of the more exact calculations using a small fraction of the computer resources [4].

Following the success of the quasi-adiabatic model calculation, it appears that it is worthwhile to devote more theoretical effort to introduce an alternative approach for a quantum mechanical treatment of three-body systems, which is the main motivation behind the work presented in this letter.

Here, an alternative approximation is developed for the treatment of the breakup process, which presents a way of evaluating non-adiabatic corrections to the adiabatic model three-body wavefunction using **exact** breakup energies, unlike the adiabatic and quasi-adiabatic theories.

Under the restriction to S-wave relative n-p configurations, we start with the exact Schrödinger Equation,

$$[E - Hnp - T_R - U(r, \bar{R})]\Psi(r, \bar{R}) = 0, \tag{1}$$

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where $H_{np} = (T_r + V_{np})$ is the n-p Hamiltonian, T_R the center of mass kinetic energy operator and $U(r, \bar{R})$ is the effective total nucleon target interactions including the Coulomb interaction which is assumed to act on the n-p center of mass.

Adding and substracting the deuteron binding energy ϵ_d in Eq (1), one gets

$$\left[E_{c.m.} - T_R - U(r, \bar{R}) \right] \Psi(r, \bar{R}) = (H_{np} + \epsilon_d)\Psi(r, \bar{R}) \tag{2}$$

where $E_{c.m.} = E + \epsilon_d$. By decomposing the projectile-target three-body wavefunction, $\Psi(r, \bar{R})$, into the adiabatic wavefunction plus the correction, i.e.,

$$\Psi(r, \bar{R}) = \Psi^{AD}(r, \bar{R}) + \Delta\Psi(r, \bar{R}), \tag{3}$$

Eq. (2) is reduced to the form

$$\left[E_{c.m.} - T_R - U(r, \bar{R}) \right] \Delta\Psi(r, \bar{R}) = (H_{np} + \epsilon_d)\Psi^{BU}(r, \bar{R}), \tag{4}$$

since

$$\left[E_{cm} - T_R - U(r, \bar{R}) \right] \Psi^{AD}(r, \bar{R}) = 0, (H_{np} + \epsilon_d)\Psi^{EL}(r, \bar{R}) = 0. \tag{5}$$

In the above equations, the correction term $\Delta\Psi$ has only outgoing waves and accounts for non-adiabatic corrections to both the breakup and elastic channels.

It is clear from the formalism that for an improved solution of $\Delta\Psi$, one should use an iterative scheme that leads to more accurate calculations. The zeroth order iteration starts with use of the adiabatic model breakup wavefunctions on the RHS of Eq. (4). As $\Delta\Psi = \Delta\Psi^{EL} + \Delta\Psi^{BU}$, in each iteration elastic and breakup pieces of the correction term are projected out [5] and are added up to the related wavefunctions,

$$\Psi^{BU} = \Psi^{AD,BU} + \Delta\Psi^{BU}, \quad \Psi^{EL} = \Psi^{AD,EL} + \Delta\Psi^{EL} \tag{6}$$

The iteration procedure is repeated until the calculations converge.

However, the RHS of Eq. (4) has an infinite range due to the incorrect behavior of the adiabatic breakup wavefunctions in the asymptotic region of R [2, 4]. To get a physical solution for $\Delta\Psi$ in Eq. (4), one thus needs to have a finite source term. To do that, instead of following the mathematical treatment given in Ref. [6], we approach the problem in a different way discussed below.

It is known that the assumed degeneracy of the n-p center-of-mass energy in all breakup configurations in the adiabatic model $(H_{np\rightarrow} - \epsilon_d)$ results in a lack of phase averaging and, in particular an overestimation of the breakup component of the entrance channel wavefunction near the nuclear surface, a concern of most importance to transfer reactions. The lack of phase averaging is revealed by the constancy of continuum channel partial wave in the adiabatic model, $\chi_{JL}^{AD,BU}(r, R)$, at beyond the nuclear surface. To simulate the effect of phase averaging in this region in a simple way, Coley and Tostevin [5] introduced a smooth radial cut-off in the breakup component according to

$$\overset{\sim}{\chi}_{JL}^{AD,BU}(r,R) = \left\{ 1 + exp\left[\frac{(R-R_0)}{a} \right] \right\}^{-1} \chi_{JL}^{AD,BU}(r,R). \tag{7}$$

This inadequacy of the breakup wavefunction was then studied quantitatively and they showed that the properties of the wavefunction in the extreme nuclear surface play a very minor role for the breakup contributions to the (d,p) channel at medium energies. Therefore, the work of Coley and Tostevin allows us to suppress the breakup wavefunction Ψ^{BU} that appears on the RHS of Eq. (4). In Fig. 1, we show the modulus of the breakup components (full curve) of the full adiabatic wavefunction as the zeroth order solution for an arbitrary partial wave for $d+^{116}$Sn system at 79 MeV, together with the suppressed breakup componend (broken curve) for the choice of $R_0 = 10$ fm and $a = 0.5$ fm. The reader is referred to Ref. [5] for the choice of R_0 and a values.

Supressed Breakup Wavefunction
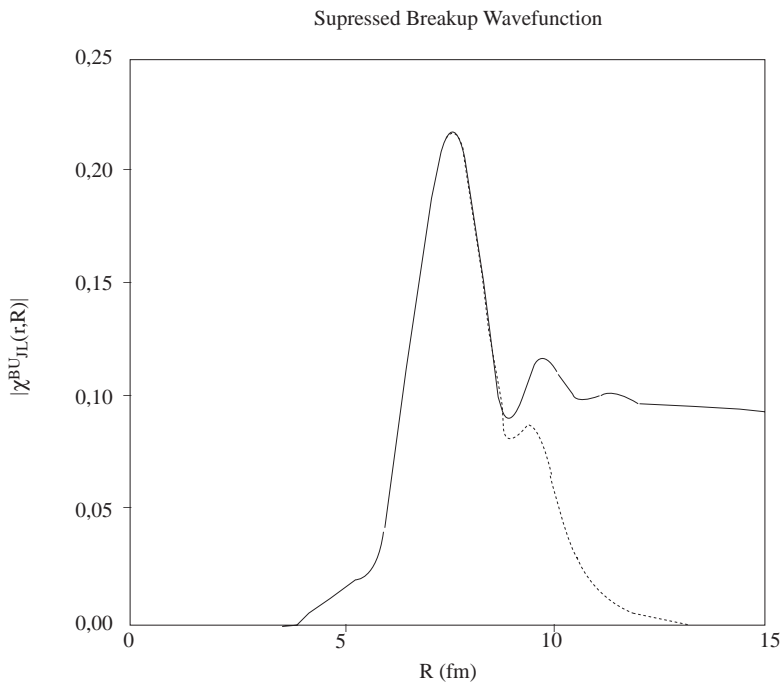
Figure 1. Modulus of the breakup (solid curve) component of the full adiabatic wavefunction for the L=18 partial wave for the $d+^{116}$Sn system at 79 MeV (for simplicity, the nucleon spin-orbit forces are neglected). The dotted curve shows the suppressed breakup component for the choice $R_0 = 10fm, a = 0.5fm$.

One readily sees that Eq. (4) may be solved replacing H_{np} by $(T_r + V_{np})$. However, in order to convince the reader that the present formalism goes beyond the adiabatic and quasi-adiabatic approaches, due to the use of exact continuum channel energies, we start

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with an exact definition of the continuum channel breakup wavefunction in partial wave form:

$$\Psi_{JL}^{BU}(r, R) = \int_0^\infty \phi_k(r)\chi_{JLK}(R)dk, \tag{8}$$

where $\phi_k(r)$ is a triplet n-p scattering state with asymptotic normalization

$$\phi_k(r) \rightarrow \left(\sqrt{\frac{2}{\pi}}\right)\frac{sin(kr + \delta_0)}{r}, \tag{9}$$

such that $\int_0^\infty dr r^2 \phi_k(r)\phi_k'(r) = \delta(k - k')$.

We need to evaluate the χ_{JLK} by integration via

$$\chi_{JLK}(R) = \int_0^\infty dr r^2 \phi_k(r)\Psi_{JL}^{BU}(r, R), \tag{10}$$

and the substitution of Eq. (8) into Eq. (4) yields

$$[E_{c.m.} - T_R - U(r, \bar{R})]\Delta\Psi(r, \bar{R})] = \int_0^\infty dk(\epsilon_k + \epsilon_d)\phi_k(r)\chi_{JLK}(R), \tag{11}$$

where $H_{np}\phi_k(r) = \epsilon_k \phi_k(r)$ in which $\epsilon_k = \frac{\hbar^2 k^2}{2\mu_{np}}$.

A major problem in the application of an exact method to the breakup process is that the breakup states are continuous and one has to deal with a continuum and infinite number of coupled channels.

The formalism presented here treats the continuous channels of the S-wave breakup states directly in a single channel as the solution of a simple differential equation. The new formalism clearly reduces to the adiabatic approximation if ϵ_k in Eq. (11) is replaced by $-\epsilon_d$, and also collapses back to the standard quasi-adiabatic model equation if ϵ_k is replaced by mean energy $\bar{\epsilon}(R)$ [4]. For the clarification of this point, one may start from Eq. (2) replacing the RHS by its latest form, e.g.,

$$[E_{c.m.} - T_R - U(r, \bar{R})]\Psi(r, \bar{R})] = \int_0^\infty dk(\epsilon_k + \epsilon_d)\phi_k(r)\chi_{JLK}(R), \tag{12}$$

The above equation can be reduced to the form

$$[E_{c.m.} - T_R - U(r, \bar{R})]\Psi^{BU}(r, \bar{R}) = \left[U(r, \bar{R} - U^{opt}(\bar{R})\right]\Psi^{EL}(r, \bar{R})$$
$$+ \int_0^\infty dk(\epsilon_k + \epsilon_d)\phi_k(r)\chi_{JLK}(R), \tag{13}$$

since $\Psi = \Psi^{BU} + \Psi^{EL}$. It is clear from the above equation if the last term on the RHS of the equation is zero, which means that $\epsilon_k \approx -\epsilon_d$, and the exact continuum channel wavefunction Ψ^{BU} approximates to $\Psi^{AD,BU}$. Using the same analogy, if ϵ_k were replaced

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by a mean breakup energy such as $\bar{\epsilon}(R)$, as in the quasi-adiabatic model, the formalism would return the standard quasi-adiabatic ones. This serves as a test of the numerical calculations. Thus, one easily sees that the last term on the RHS of Eq. (13) is the correction term to the adiabatic and quasi-adiabatic approaches, and therefore the new formalism appears to provide an improved description of the three-body wavefunction.

It also incorporates some effects of closed channels, which have breakup energies $\epsilon_k > E$. This is expected to cause sensitive modifications of stripping cross-sections. Phase relations in the construction of the cross-section are particularly and strongly affected by closed channel contributions.

Although the technique developed seems mathematically convenient, the iterated calculations in describing the three-body wavefunction for the dt^{116}Sn system at 79 MeV, and also its application to some (d,p) reactions, did not converged. We have investigated possible reasons for the non-convergence of the calculations. The detailed analyses of the calculations have led to the conclusion that the essentially exact quantum mechanical solution developed here cannot be used in its present form in the treatment of the breakup process in reactions involving deuterons. Since the continuum channel breakup wavefunction produced in the adiabatic model is not expected to give an accurate representation of the continuum channel wavefunction, for values for r outside the range of n-p potential, then use of the adiabatic wavefunction in the source term of the zeroth order calculations beyond such ranges is suspect, which causes non-converged results.

Nevertheless, the extended adiabatic models can be seen as an important tool for those hoping to understand the mechanism of transfer reactions involving loosely bound systems, and is worthy of further investigation. In particular, such theories should find application when the rapidly increasing data based on reactions induced by light neutron-rich radioactive nuclei.

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