Delay-dependent stability analysis for discrete singular systems with time-varying delays

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Abstract This paper studies the problem of stability analysis for discrete singular time delay systems. Without resorting to the decomposition and equivalent transformation of the considered system, some new delay-dependent criteria are established for the considered systems to be regular, causal and stable in terms of linear matrix inequality (LMI) approach. The obtained criteria are less conservative, because the technique used in this paper makes more use of the information on the involved time-varying delays than the existing techniques do. A numerical example is given to illustrate the effectiveness and the benefits of the proposed methods.

Key words Discrete singular systems, time-varying delays, delay-dependent stability, linear matrix inequality (LMI)

In the past few decades, singular systems have received intensive interest because they appear frequently in several applications including large-scale systems, power systems, economic systems, and so on [1,2]. Many fundamental notions and results based on regular systems have been extended to singular systems. It should be mentioned that the stability problem for singular systems is much more complicated than that for regular systems because it requires to consider not only stability, but also regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems) simultaneously, the latter two do not arise in the regular systems [2]. For more details on singular systems, we refer the readers to [1,2] and the references therein. On the other hand, it is well known that time delays exist commonly in many practical systems, which have been generally regarded as the main source of instability and poor performance. Therefore, much attention has been devoted to the problem of stability analysis for singular systems with time delay. Both delay-independent stability results [2,3] and delay-dependent stability results [4]-[12] were obtained for the singular time delay systems in terms of linear matrix inequality (LMI) approach. Generally speaking, when the time delay is small enough, the delay-dependent results are generally less conservative than the delay-independent ones.

But all the above-mentioned works were developed in the context of continuous singular time delay systems. Recently, the stability problem for discrete singular systems with time delay was discussed in [13]-[19], where several delay-dependent conditions were established for the considered systems to be regular, causal and stable in terms of linear matrix inequality (LMI) approach. But the involved time delays of [13]-[19] are all time invariant, which limits the scope of applications of the established stability results. In the case where time-varying delays appear in the discrete singular systems, [20]-[22] made use of the free-weighting matrices method [23]-[25] to establish some delay-dependent stability conditions in terms of LMI approach. However, it should be mentioned that the decomposition and equivalent model transformation of the original system matrices are involved in [20] and the established condition is in terms of the coefficient matrices of the decomposed systems, which makes the stability analysis procedure indirect and complicated. It should also be pointed out that some useful terms are ignored in the Lyapunov functionals reported in [20]-[22]. The ignorance of these terms may lead to conservatism to some extent. Therefore, it is important and necessary to further improve the stability results on discrete singular systems with time-varying delay.

In this paper, the problem of delay-dependent stability is discussed for discrete singular systems with time-varying delays in terms of LMI approach. Some new delay-dependent sufficient conditions are established for the discrete singular systems to be regular, causal and stable. Unlike the condition in [20], the derived conditions in this paper are in terms of all the coefficient matrices of the original system, which avoids the decomposition and transformation of the given singular system. Furthermore, because the Lyapunov functional and the technique reported in this paper make more use of the information on the considered time-varying delays than those in [20]-[22], our results have less conservatism, which will be demonstrated by a numerical example.

1 Problem Formulation

Consider discrete singular systems with time-varying delays described by

$$\begin{aligned} E\boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d(k)), \\ \boldsymbol{x}(k) &= \boldsymbol{\phi}(k),\ k\in[-d_2,0], \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k)\in\mathbf{R}^n$ is the state vector and $d(k)$ is a time-varying delay satisfying $d_1\leqslant d(k)\leqslant d_2$, where d_1 and d_2 are prescribed positive integers representing the lower and upper bounds of the time delay, respectively. $\boldsymbol{\phi}(k)$ is the compatible initial condition. The matrix $E\in\mathbf{R}^{n\times n}$ may be singular and it is assumed that $\mathrm{rank}\,(E)=r\leqslant n$. A and A_d are known real constant matrices with appropriate dimensions.

The following definition will be used in the proof of the main results.

Definition 1 [15,20].

1. For given integers $d_1>0$, $d_2>0$, the discrete singular time delay system (1) is said to be regular and causal for any time delay $d(k)$ satisfying $d_1\leqslant d(k)\leqslant d_2$, if the pair (E,A) is regular and causal,
2. The discrete singular time delay system (1) is said to be stable if for any scalar $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$ such that, for any compatible initial conditions $\phi(k)$ satisfying $\sup_{-d_2\leqslant k\leqslant 0}\|\boldsymbol{\phi}(k)\|\leqslant\delta(\varepsilon)$, the solution $x(k)$ of system (1) satisfies $\|\boldsymbol{x}(k)\|\leqslant\varepsilon$ for any $k\geqslant 0$, moreover $\lim_{k\to\infty}\boldsymbol{x}(k)=0$.

In this paper, we analyze the stability of system (1), and new stability criteria that are less conservative than the existing ones will be proposed.

2 Main results

Theorem 1: For given integers $d_1>0$ and $d_2>0$, the discrete singular time delay system (1) is regular, causal and stable for any time-varying delay $d(k)$ satisfying $d_1\leqslant d(k)\leqslant d_2$, if there exist symmetric positive-definite matrices P, Q_i $(i=1,2,3)$, Z_l $(l=1,2)$, and matrices S_j

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$(j = 1, 2)$ such that

$$\Xi = \begin{bmatrix} \Phi_{11} & \Phi_{12}^{\mathrm{T}} P & d_1 \Phi_{13}^{\mathrm{T}} Z_1 & d_{12} \Phi_{13}^{\mathrm{T}} Z_2 \\ * & -P & 0 & 0 \\ * & * & -Z_1 & 0 \\ * & * & * & -Z_2 \end{bmatrix} < 0, \quad (2)$$

where $d_{12} = d_2 - d_1$, $R \in \mathbf{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E^{\mathrm{T}} R = 0$, and

$$\Phi_{11} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & E^{\mathrm{T}} Z_1 E \\ * & \Xi_{22} & E^{\mathrm{T}} Z_2 E & E^{\mathrm{T}} Z_2 E \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44} \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} =& -E^{\mathrm{T}} P E + Q_1 + (d_{12}+1) Q_2 + Q_3 \\
& - E^{\mathrm{T}} Z_1 E + S_1 R^{\mathrm{T}} A + A^{\mathrm{T}} R S_1^{\mathrm{T}}, \\
\Xi_{12} =& S_1 R^{\mathrm{T}} A_d + A^{\mathrm{T}} R S_2^{\mathrm{T}}, \\
\Xi_{22} =& -Q_2 - 2E^{\mathrm{T}} Z_2 E + S_2 R^{\mathrm{T}} A_d + A_d^{\mathrm{T}} R S_2^{\mathrm{T}}, \\
\Xi_{33} =& -Q_3 - E^{\mathrm{T}} Z_2 E, \\
\Xi_{44} =& -Q_1 - E^{\mathrm{T}} Z_1 E - E^{T} Z_2 E, \\
\Phi_{12} =& \begin{bmatrix} A & A_d & 0 & 0 \end{bmatrix}, \\
\Phi_{13} =& \begin{bmatrix} A - E & A_d & 0 & 0 \end{bmatrix}.
\end{aligned}$$

Proof: We first show that the system (1) is regular and causal for any time-varying delay $d(k)$ satisfying $d_1 \leqslant d(k) \leqslant d_2$. To this end, we choose two nonsingular matrices G and H such that

$$GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}.$$

Set

$$GAH = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \; H^{\mathrm{T}} S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \; G^{-\mathrm{T}} R = \begin{bmatrix} 0 \\ I \end{bmatrix} U,$$

where $U \in \mathbf{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix. Then, Pre-multiplying and post-multiplying $\Xi_{11} < 0$ by H^{T} and H, respectively, we have $S_2 U^{\mathrm{T}} A_4 + A_4^{\mathrm{T}} U S_2^{\mathrm{T}} < 0$, which implies A_4 is nonsingular. Thus, the pair (E, A) is regular and causal [1]. According to Definition 1, the system (1) is regular and causal for any time-varying delay satisfying $d_1 \leqslant d(k) \leqslant d_2$.

Next we will show that system (1) is stable for any time-varying delay satisfying $d_1 \leqslant d(k) \leqslant d_2$. To this end, we define $\boldsymbol{\eta}(k) = \boldsymbol{x}(k+1) - \boldsymbol{x}(k)$ and consider the following Lyapunov functional candidate for system (1):

$$V(k) = V_1(k) + V_2(k), \quad (3)$$

where

$$\begin{aligned}
V_1(k) =& \boldsymbol{x}(k)^{\mathrm{T}} E^{\mathrm{T}} P E \boldsymbol{x}(k) + \sum_{i=k-d_1}^{k-1} \boldsymbol{x}(i)^{\mathrm{T}} Q_1 \boldsymbol{x}(i) \\
& + \sum_{i=k-d_2}^{k-1} \boldsymbol{x}(i)^{\mathrm{T}} Q_3 \boldsymbol{x}(i) \\
& + \sum_{j=-d_2+1}^{-d_1+1} \sum_{i=k-1+j}^{k-1} \boldsymbol{x}(i)^{\mathrm{T}} Q_2 \boldsymbol{x}(i), \\
V_2(k) =& d_1 \sum_{j=-d_1}^{-1} \sum_{i=k+j}^{k-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} Z_1 E \boldsymbol{\eta}(i) \\
& + d_{12} \sum_{j=-d_2}^{-d_1-1} \sum_{i=k+j}^{k-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} Z_2 E \boldsymbol{\eta}(i)
\end{aligned}$$

Define $\Delta V(k) = V(k+1) - V(k)$. Then, along the solution of system (1), we have

$$\begin{aligned}
\Delta V_1(k) \leqslant& \boldsymbol{x}(k+1)^{\mathrm{T}} E^{\mathrm{T}} P E \boldsymbol{x}(k+1) \\
& - \boldsymbol{x}(k)^{\mathrm{T}} E^{\mathrm{T}} P E \boldsymbol{x}(k) \\
& + \boldsymbol{x}(k)^{\mathrm{T}} Q_1 \boldsymbol{x}(k) - \boldsymbol{x}(k-d_1)^{\mathrm{T}} Q_1 \boldsymbol{x}(k-d_1) \\
& + \boldsymbol{x}(k)^{\mathrm{T}} Q_3 \boldsymbol{x}(k) - \boldsymbol{x}(k-d_2)^{\mathrm{T}} Q_3 \boldsymbol{x}(k-d_2) \\
& + (d_{12}+1) \boldsymbol{x}(k)^{\mathrm{T}} Q_2 \boldsymbol{x}(k) \\
& - \boldsymbol{x}(k-d(k))^{\mathrm{T}} Q_2 \boldsymbol{x}(k-d(k)) \\
& + 2 \left[\boldsymbol{x}(k)^{\mathrm{T}} S_1 R^{\mathrm{T}} + \boldsymbol{x}(k-d(k))^{\mathrm{T}} S_2 R^{\mathrm{T}} \right] \\
& \times E \boldsymbol{x}(k+1)
\end{aligned} \quad (4)$$

and

$$\begin{aligned}
\Delta V_2(k) =& d_1^2 \boldsymbol{\eta}(k)^{\mathrm{T}} E^{\mathrm{T}} Z_1 E \boldsymbol{\eta}(k) \\
& - d_1 \sum_{i=k-d_1}^{k-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} Z_1 E \boldsymbol{\eta}(i) \\
& + d_{12}^2 \boldsymbol{\eta}(k)^{\mathrm{T}} E^{\mathrm{T}} Z_2 E \boldsymbol{\eta}(k) \\
& - d_{12} \sum_{i=k-d_2}^{k-d_1-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} Z_2 E \boldsymbol{\eta}(i)
\end{aligned} \quad (5)$$

It is easy to get that

$$\begin{aligned}
& - d_1 \sum_{i=k-d_1}^{k-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} Z_1 E \boldsymbol{\eta}(i) \\
\leqslant& - \left[\sum_{i=k-d_1}^{k-1} \boldsymbol{\eta}(i)^{\mathrm{T}} E^{\mathrm{T}} \right] Z_1 \left[E \sum_{i=k-d_1}^{k-1} \boldsymbol{\eta}(i) \right] \\
=& \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k-d_1) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -E^{\mathrm{T}} Z_1 E & E^{\mathrm{T}} Z_1 E \\ * & -E^{\mathrm{T}} Z_1 E \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{x}(k-d_1) \end{bmatrix},
\end{aligned} \quad (6)$$

and

$$
\begin{aligned}
&- d_{12}\sum_{i=k-d_2}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
=&- d_{12}\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&- d_{12}\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-(d(k)-d_1)\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&-(d_2-d(k))\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-\left[\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}\right]Z_2\left[E\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)\right]\\
&-\left[\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}\right]Z_2\left[E\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)\right]\\
=&\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_1)\\ \boldsymbol{x}(k-d_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-2E^{\mathrm{T}}Z_2E & E^{\mathrm{T}}Z_2E & E^{\mathrm{T}}Z_2E\\ * & -E^{\mathrm{T}}Z_2E & 0\\ * & * & -E^{\mathrm{T}}Z_2E\end{bmatrix}\\
&\times\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_1)\\ \boldsymbol{x}(k-d_2)\end{bmatrix}.
\end{aligned}
\tag{7}
$$

Thus,

$$\Delta V(k)\leqslant\boldsymbol{\xi}(k)^{\mathrm{T}}\Theta\boldsymbol{\xi}(k),\tag{8}$$

where

$$
\begin{aligned}
&\Theta=\Phi_{11}+\Phi_{12}^{\mathrm{T}}P\Phi_{12}+\Phi_{13}^{\mathrm{T}}(d_1^2Z_1+d_{12}^2Z_2)\Phi_{13},\\
&\boldsymbol{\xi}(k)=\begin{bmatrix}\boldsymbol{x}(k)^{\mathrm{T}} & \boldsymbol{x}(k-d(k))^{\mathrm{T}} & \boldsymbol{x}(k-d_2)^{\mathrm{T}} & \boldsymbol{x}(k-d_1)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}.
\end{aligned}
$$

According to Schur complements, $\Theta<0$ is equivalent to (2). Hence, there exists a scalar $\alpha>0$ such that

$$\Delta V(k)\leqslant-\alpha\|\boldsymbol{x}(k)\|^2.\tag{9}$$

Using the similar method of [15], (9) implies that the system (1) is stable. This completes the proof. □

It is noted that in (7), the terms

$$-d_{12}\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)$$

and

$$-d_{12}\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)$$

are expanded as

$$-(d(k)-d_1)\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)$$

and

$$-(d_2-d(k))\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i),$$

respectively. It is obvious that such treatment may lead to conservatism, so there is room for further investigation. Next, an improved stability criterion of Theorem 1 can be developed as follows.

Theorem 2: For given integers $d_1>0$ and $d_2>0$, the discrete singular time delay system (1) is regular, causal and stable for any time-varying delay $d(k)$ satisfying $d_1\leqslant d(k)\leqslant d_2$, if there exist symmetric positive-definite matrices P, Q_i $(i=1,2,3)$, Z_l $(l=1,2)$, and matrices S_j $(j=1,2)$ such that

$$\Xi-\Delta_f^{\mathrm{T}}E^{\mathrm{T}}Z_2E\Delta_f<0,\ f=1,2,\tag{10}$$

where Ξ follows the same definition as the one in Theorem 1 and

$$
\begin{aligned}
\Delta_1&=\begin{bmatrix}0 & I & 0 & -I & 0 & 0 & 0\end{bmatrix},\\
\Delta_2&=\begin{bmatrix}0 & I & -I & 0 & 0 & 0 & 0\end{bmatrix}.
\end{aligned}
$$

Proof Because (7) can be rewritten as

$$
\begin{aligned}
&- d_{12}\sum_{i=k-d_2}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
=&- d_{12}\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&- d_{12}\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
=&-(d(k)-d_1)\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&-(d_2-d(k))\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&-(d_2-d(k))\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
&-(d(k)-d_1)\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i).
\end{aligned}
\tag{11}
$$

Let $g(k)=\frac{d(k)-d_1}{d_{12}}$, we can get $0\leqslant g(k)\leqslant1$ and $d_2-d(k)=$

$(1-g(k))d_{12}$. Thus,

$$\begin{aligned}
&-(d_2-d(k))\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
=&-(1-g(k))d_{12}\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-(1-g(k))(d(k)-d_1)\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-(1-g(k))\begin{bmatrix}\sum_{i=k-d(k)}^{k-d_1-1}\eta(i)^{\mathrm{T}}E^{\mathrm{T}}\end{bmatrix}Z_2\begin{bmatrix}E\sum_{i=k-d(k)}^{k-d_1-1}\boldsymbol{\eta}(i)\end{bmatrix}\\
=&-(1-g(k))\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_1)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}E^{\mathrm{T}}Z_2E & -E^{\mathrm{T}}Z_2E\\ * & E^{\mathrm{T}}Z_2E\end{bmatrix}\\
&\times\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_1)\end{bmatrix},
\end{aligned}\tag{12}$$

and

$$\begin{aligned}
&-(d(k)-d_1)\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
=&-g(k)d_{12}\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-g(k)(d_2-d(k))\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)^{\mathrm{T}}E^{\mathrm{T}}Z_2E\boldsymbol{\eta}(i)\\
\leqslant&-g(k)\begin{bmatrix}\sum_{i=k-d_2}^{k-d(k)-1}\eta(i)^{\mathrm{T}}E^{\mathrm{T}}\end{bmatrix}Z_2\begin{bmatrix}E\sum_{i=k-d_2}^{k-d(k)-1}\boldsymbol{\eta}(i)\end{bmatrix}\\
=&-g(k)\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_2)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}E^{\mathrm{T}}Z_2E & -E^{\mathrm{T}}Z_2E\\ * & E^{\mathrm{T}}Z_2E\end{bmatrix}\\
&\times\begin{bmatrix}\boldsymbol{x}(k-d(k))\\ \boldsymbol{x}(k-d_2)\end{bmatrix}.
\end{aligned}\tag{13}$$

Thus, we can get from (4), (5), (6) and (13) that

$$\begin{aligned}\Delta V(k)\leqslant\boldsymbol{\xi}(k)^T((1-g(k))(\Theta-\Psi_1^{\mathrm{T}}E^{\mathrm{T}}Z_2E\Psi_1)\\ +g(k)(\Theta-\Psi_2^{\mathrm{T}}E^{\mathrm{T}}Z_2E\Psi_2))\boldsymbol{\xi}(k),\end{aligned}$$

where $\boldsymbol{\xi}(k)$ and Θ follow the same definitions as those in (8), and

$$\begin{aligned}\Psi_1&=\begin{bmatrix}0 & I & 0 & -I\end{bmatrix},\\ \Psi_2&=\begin{bmatrix}0 & I & -I & 0\end{bmatrix}.\end{aligned}$$

According to Schur complements, $\Theta-\Psi_1^{\mathrm{T}}E^{\mathrm{T}}Z_2E\Psi_1<0$ and $\Theta-\Psi_2^{\mathrm{T}}E^{\mathrm{T}}Z_2E\Psi_2<0$ is equivalent to (10). Hence, there exists a scalar $\alpha>0$ such that

$$\Delta V(k)\leqslant-\alpha\|\boldsymbol{x}(k)\|^2,\tag{14}$$

which implies that the system (1) is stable. This completes the proof. □

3 Numerical Example

Example 1: Consider discrete singular time delay system (1) with

$$E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ A=\begin{bmatrix}0.8 & 0\\ 0.05 & 0.9\end{bmatrix},\ A_d=\begin{bmatrix}-0.1 & 0\\ -0.2 & -0.1\end{bmatrix}.$$

Using the methods of [20]-[22] and the results reported in this paper, the allowable maximum values of d_2 with various d_1 ensuring the regularity, causality and stability of system (1) are presented in Table 1, which shows our conditions give better results than those in [20]-[22] and Theorem 2 has less conservatism than Theorem 1. In addition, the stability criteria proposed by [13]-[19] are invalid for the above-mentioned system. Fig. 1 gives the simulation results of $x_1(k)$ and $x_2(k)$ when $d(k)=10+6\sin(k\pi/2)$ and the initial function is $\boldsymbol{\phi}(k)=\begin{bmatrix}4\\ 0.75\end{bmatrix}$, $k\in[-16,0]$. From Fig. 1, we can see that the states $x_1(k)$ and $x_2(k)$ asymptotically converge to zero.

4 Conclusion

In this paper, the delay-dependent stability problem of discrete singular systems with time-varying delay is investigated. In terms of LMI approach, several new delay-dependent criteria are derived ensuring the considered system to be regular, causal and stable without resorting to the decomposition and equivalent transformation of the considered system. The newly obtained results are less conservative than the existing ones. Numerical example is given to illustrate the effectiveness of the presented criteria and their improvement over the existing results.

Table 1 Comparison of the allowable maximum values of d_2 for various d_1

d_1	0	3	6	9	12	15
[20]	7	8	10	13	15	18
[21]	12	14	14	15	17	20
[22]	12	13	14	15	17	20
Theorem 1	13	15	18	20	23	26
Theorem 2	15	16	19	22	25	28

References

1 Dai L, *Singular Control Systems.* Berlin: Springer, 1989.
2 Xu S, Lam, J. *Robust Control and Filtering of Singular Systems.* Berlin: Springer, 2006.
3 Xu S, Dooren PV, Stefan R, Lam J. Robust stability analysis and stabilization for singular systems with state delay and parameter uncertainty. *IEEE Transactions on Automatic Control*, 2002, **47**(7): 1122-1128
4 Yue D, Han Q. Delay-dependent robust H_∞ controller design for uncertain descriptor systems with time-varying discrete and distributed delays. *IEE Proceedings-Control Theory & Applications*, 2005, **152**(6): 628-638
5 Yang F, Zhang Q. Delay-dependent H_∞ control for linear descriptor systems with delay in state. *Journal of Control Theory and Applications*, 2005, **3**(1): 76-84
6 Zhu S, Zhang C, Cheng Z, Feng J. Delay-dependent robust stability criteria for two classes of uncertain singular time-delay systems. *IEEE Transactions on Automatic Control*, 2007, **52**(5): 880-885

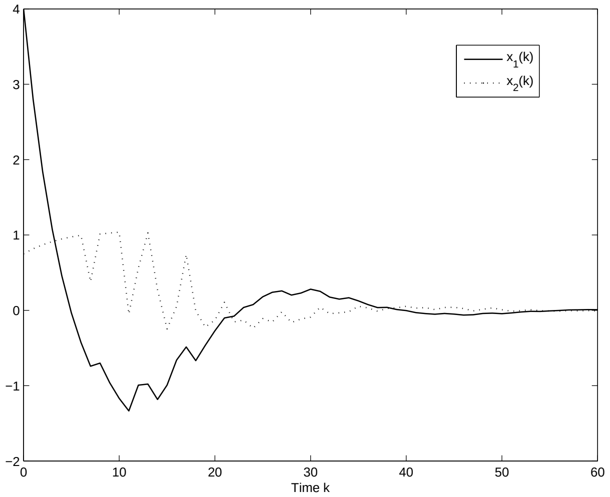

Fig. 1 State responses of the discrete singular time delay system (1).

7 Wu Z, Zhou W. Delay-dependent robust H_∞ control for uncertain singular time-delay systems. *IET Control Theory & Applications*, 2007, **1**(5): 1234-1241

8 Xu S, Lam J, Zou Y. An improved characterization of bounded realness for singular delay systems and its applications. *International Journal of Robust and Nonlinear Control*, 2008, **18**(3): 263-277

9 Wang H, Xue A, Lu R. Absolute stability criteria for a class of nonlinear singular systems with time delay. *Nonlinear Analysis: Theory, Methods & Applications*, 2009, **70**(2): 621-630

10 Lu R, Dai X, Su H, Chu J, Xue A. Delay-dependant robust stability and stabilization conditions for a class of Lur'e singular time-delay systems. *Asian Journal of Control*, 2008, **10**(4): 462-469

11 Du Z, Zhang Q, Liu L. Delay-dependent stabilization of uncertain singular systems with multiple state delays. *International Journal of Innovative Computing, Information and Control*, 2009, **5**(6): 1655-1664

12 Xie Y, Gui W, Jiang Z. Delay-dependent stabilization of singular systems with multiple internal and external incommensurate constant point delays. *International Journal of Control, Automation, and Systems*, 2008, **6**(4): 515-525

13 Ji X, Su H, Chu J. An LMI approach to robust stability of uncertain discrete singular time-delay systems. *Asian Journal of Control*, 2006, **8**(1): 56-62

14 Mahmoud M S, Ismail A. Uncertain discrete delay singular systems: delay-dependent stability and stabilization. *Applied Mathematical Sciences*, 2007, **1**(10): 491-505

15 Wu Z, Su H, Chu J. Robust stabilization for uncertain discrete singular systems with state delay. *International Journal of Robust and Nonlinear Control*, 2008, **18**(16): 1532-1550

16 Ma S, Zheng C, Cheng Z. Delay-dependent robust H_∞ control for uncertain discrete-time singular systems with time-delays. *Journal of Computational and Applied Mathematics*, 2008, **217**(1): 194-211

17 Pan S, Chen C. The robust D-stability analysis of uncertain discrete-delay descriptor systems via genetic algorithms. *International Journal of General Systems*, 2009, **37**(3): 305-317

18 Ma S, Zheng C, Cheng Z. Delay-dependent robust stability and stabilization for uncertain discrete singular systems with delays. *Asian Journal of Control*, 2009, **11**(3): 309-318

19 Chen N, Zhai G, Gui W. Robust decentralized H_∞ control of multi-channel discrete-time descriptor systems with time-delay. *International Journal of Innovative Computing, Information and Control*, 2009, **5**(4): 305-317

20 Ma S, Cheng Z, Zheng C. Delay-dependent robust stability and stabilisation for uncertain discrete singular systems with time-varying delays. *IET Control Theory and Applications*, 2007, **1**(5): 1086-1095

21 Wang H, Xue A, Lu R, Chen Y. Delay-dependent robust H_∞ control for uncertain discrete singular time-varying delay systems based on a finite sum inequality. In: Proceedings of the 26th Chinese Control Conference. Zhangjiajie, China: IEEE, 2007. 595-599

22 Du Z, Qiu Z, Zhang Q, Liu L. New delay-dependent robust stability of discrete singular systems with time-varying delay. In: Proceedings of the 7th World Congress on Intelligent Control and Automation. Chongqing, China: IEEE, 2008. 6359-6364

23 Wu M, He Y, She J, Liu G. Delay-dependent criteria for robust stability of time-varying delay systems. *Automatica*, 2004, **40**(8): 1435-1439

24 He Y, Wang Q, Lin C. An improved H_∞ filter design for systems with time-varying interval delay. *IEEE Transactions on Circuits and Systems Part II-Express Briefs*, 2006, **53**(11): 1235-1239

25 He Y, Wu M, Liu G, She J, Output feedback stabilization for a discrete-time system with a time-varying delay. *IEEE Transactions Automatic Control*, 2008, **53**(10): 2327-2377

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