Modified $F(R)$ Hořava-Lifshitz gravity: a way to accelerating FRW cosmology

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We propose a general approach for the construction of modified gravity which is invariant under foliation-preserving diffeomorphisms. Special attention is paid to the formulation of modified $F(R)$ Hořava-Lifshitz gravity, whose Hamiltonian structure is studied. The consistency of spatially-flat FRW equations is demonstrated. The analysis of de Sitter solutions for several versions of this theory indicates that the unification of the early-time inflation with the late-time acceleration is possible. It is shown that a special choice of parameters for such a theory leads to the same spatially-flat FRW equations as in the case of traditional $F(R)$-gravity. Finally, an essentially most general modified Hořava-Lifshitz gravity is proposed, motivated by its fully diffeomorphism-invariant counterpart, with the restriction that the action does not contain derivatives higher than the second order with respect to the time coordinate.

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I. INTRODUCTION

Recent observational data clearly indicates that our universe is currently expanding with an accelerating rate, apparently due to Dark Energy. The early universe has also undergone a period of accelerated expansion (inflation). The modified gravity approach (for a general review, see [1]) suggests that such accelerated expansion is caused by a modification of gravity at the early/late-time universe. A number of modified theories of gravity, which successfully describe the unification of early-time inflation with late-time acceleration and which are cosmologically and observationally viable, has been proposed (for a review, see [1]). Despite some indications [2] that such alternative theories of gravity may emerge from string/M-theory, they are still mostly phenomenological theories that are not yet related to a fundamental theory.

Recently the so-called Hořava-Lifshitz quantum gravity [3] has been proposed. This theory appears to be power-counting renormalizable in 3+1 dimensions. One of the key elements of such a formulation is to abandon the local Lorentz invariance so that it is restored as an approximate symmetry at low energies. Despite its partial success as a candidate for a fundamental theory of gravity, there are a number of unresolved problems (see refs. [4–9]) related with the detailed balance and the projectability conditions (see section II for definitions), strong couplings, an extra propagating degree of freedom and the GR (infrared) limit, the relation with other modified theories of gravity etc. Moreover, study of the spatially-flat FRW cosmology in the Hořava-Lifshitz gravity indicates that its background cosmology [10] is almost the same as in the usual GR. Hence, it seems that there is no natural way (without extra fields) to obtain an accelerating universe from Hořava-Lifshitz gravity, let alone a unified description of the early-time inflation with the late-time acceleration. Therefore it is natural to search for a generalization of the Hořava-Lifshitz theory that could be easily related to a traditional modified theory of gravity. On the one hand, it may be very useful for the study of the low-energy limit of such a generalized Hořava-Lifshitz theory due to the fact that a number of modified theories of gravity are cosmologically viable and pass the local tests. On the other hand, it is expected that such a generalized Hořava-Lifshitz gravity may have a much richer cosmological structure, including the possibility of a unification of the early-time inflation with the late-time acceleration. Finally, within a more general theory one may hope to formulate the dynamical scenario for the Lorentz symmetry violation/restoration caused by the expansion of the universe.

In the present work we propose such a general modified Hořava-Lifshitz gravity. We mainly consider modified $F(R)$ Hořava-Lifshitz gravity which is shown to coincide with the traditional $F(R)$-gravity on the spatially-flat FRW background for a special choice of parameters. Another limit of our model leads to the degenerate $F(R)$ Hořava-Lifshitz gravity proposed in ref. [11]. The Hamiltonian analysis of the modified $F(R)$ Hořava-Lifshitz theory is presented. The preliminary investigation of the FRW equations for models from this class indicates a rich cosmological structure and

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a natural possibility for the unification of the early-time inflation with the Dark Energy epoch. Finally, we propose the most general modification of Hořava-Lifshitz-like theory of gravity. Our formulation ensures that the spatially-flat FRW cosmology of any modified Hořava-Lifshitz gravity (for a special choice of parameters) coincides with the one of its traditional modified gravity counterpart.

II. MODIFIED $F(R)$ HOŘAVA-LIFSHITZ GRAVITY

In this section we propose a new extended action for $F(R)$ Hořava-Lifshitz gravity. The FRW equations for this theory are also formulated. The action of the standard $F(R)$-gravity is given by

$$S_{F(R)} = \int \mathrm{d}^4x\sqrt{-g}F(R)\,. \tag{1}$$

Here F is a function of the scalar curvature R. By using the ADM decomposition [12] (for reviews and mathematical background see [13, 14]), we can write the metric in the following form:

$$ds^2 = -N^2\mathrm{d}t^2 + g^{(3)}_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)\,, \quad i=1,2,3\,. \tag{2}$$

Here N is called the lapse variable and N^i's are the shift variables. Then the scalar curvature R has the following form:

$$R = K^{ij}K_{ij} - K^2 + R^{(3)} + 2\nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right) \tag{3}$$

and $\sqrt{-g} = \sqrt{g^{(3)}}N$. Here $R^{(3)}$ is the three-dimensional scalar curvature defined by the metric $g^{(3)}_{ij}$ and K_{ij} is the extrinsic curvature defined by

$$K_{ij} = \frac{1}{2N}\left(\dot{g}^{(3)}_{ij} - \nabla^{(3)}_i N_j - \nabla^{(3)}_j N_i\right)\,, \quad K = K^i{}_i\,. \tag{4}$$

n^μ is a unit vector perpendicular to the three-dimensional hypersurface Σ_t defined by $t=$ constant and $\nabla^{(3)}_i$ expresses the covariant derivative on the hypersurface Σ_t.

Recently an extension of $F(R)$-gravity to a Hořava-Lifshitz type theory [3] has been proposed [11], by introducing the action

$$S_{F_{\rm HL}(R)} = \int \mathrm{d}^4x\sqrt{g^{(3)}}NF(R_{\rm HL})\,, \quad R_{\rm HL} \equiv K^{ij}K_{ij} - \lambda K^2 - E^{ij}\mathcal{G}_{ijkl}E^{kl}\,. \tag{5}$$

Here λ is a real constant in the "generalized De Witt metric" or "super-metric" ("metric of the space of metric"),

$$\mathcal{G}^{ijkl} = \frac{1}{2}\left(g^{(3)ik}g^{(3)jl} + g^{(3)il}g^{(3)jk}\right) - \lambda g^{(3)ij}g^{(3)kl}\,, \tag{6}$$

defined on the three-dimensional hypersurface Σ_t, E^{ij} can be defined by the so called *detailed balance condition* by using an action $W[g^{(3)}_{kl}]$ on the hypersurface Σ_t

$$\sqrt{g^{(3)}}E^{ij} = \frac{\delta W[g^{(3)}_{kl}]}{\delta g_{ij}}\,, \tag{7}$$

and the inverse of $\mathcal{G}^{ijkl}$ is written as

$$\mathcal{G}_{ijkl} = \frac{1}{2}\left(g^{(3)}_{ik}g^{(3)}_{jl} + g^{(3)}_{il}g^{(3)}_{jk}\right) - \tilde{\lambda}g^{(3)}_{ij}g^{(3)}_{kl}\,, \quad \tilde{\lambda} = \frac{\lambda}{3\lambda - 1}\,. \tag{8}$$

The action $W[g^{(3)}_{kl}]$ is assumed to be defined by the metric and the covariant derivatives on the hypersurface Σ_t. The original motivation for the detailed balance condition is its ability to simplify the quantum behaviour and renormalization properties of theories that respect it. Otherwise there is no a priori physical reason to restrict E^{ij} to be defined by (7). There is an anisotropy between space and time in the Hořava-Lifshitz gravity. In the ultraviolet (high energy) region, the time coordinate and the spatial coordinates are assumed to behave as

$$\boldsymbol{x} \to b\boldsymbol{x}\,, \quad t \to b^z t\,, \quad z = 2,3,\cdots\,, \tag{9}$$

under the scale transformation. In [3], $W[g^{(3)}_{kl}]$ is explicitly given for the case $z = 2$,

$$W = \frac{1}{\kappa_W^2} \int \mathrm{d}^3\boldsymbol{x}\, \sqrt{g^{(3)}}(R - 2\Lambda_W)\,, \tag{10}$$

and for the case $z = 3$,

$$W = \frac{1}{w^2} \int_{\Sigma_t} \omega_3(\Gamma)\,. \tag{11}$$

Here κ_W in (10) is a coupling constant of dimension $-1/2$ and w^2 in (11) is the dimensionless coupling constant. $\omega_3(\Gamma)$ in (11) is given by

$$\omega_3(\Gamma) = \mathrm{Tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) \equiv \varepsilon^{ijk}\left(\Gamma^m_{il}\partial_j\Gamma^l_{km} + \frac{2}{3}\Gamma^n_{il}\Gamma^l_{jm}\Gamma^m_{kn}\right)\mathrm{d}^3\boldsymbol{x}\,. \tag{12}$$

A general E^{ij} consist of all contributions to W up to the chosen value z.

In the Hořava-Lifshitz-like $F(R)$-gravity, we assume that N can only depend on the time coordinate t, which is called the *projectability condition*. The reason is that the Hořava-Lifshitz gravity does not have the full diffeomorphism invariance, but is invariant only under "foliation-preserving" diffeomorphisms, i.e. under the transformations

$$\delta x^i = \zeta^i(t, \boldsymbol{x})\,, \quad \delta t = f(t)\,. \tag{13}$$

If N depended on the spatial coordinates, we could not fix N to be unity ($N = 1$) by using the foliation-preserving diffeomorphisms. There exists a version of Hořava-Lifshitz gravity without the projectability condition, but it is suspected to possess few additional consistency problems [5, 9]. Therefore we prefer to assume that N depends only on the time coordinate t.

Let us consider the FRW universe with a flat spatial part,

$$ds^2 = -N^2\mathrm{d}t^2 + a(t)^2 \sum_{i=1,2,3}\left(\mathrm{d}x^i\right)^2\,. \tag{14}$$

Then, it is clear from the explicit expressions in (10) and (11) that $W[g^{(3)}_{kl}]$ vanishes identically if $\Lambda_W = 0$, which we assume since a non-vanishing Λ_W gives a cosmological constant. Then one can obtain

$$R = \frac{12H^2}{N^2} + \frac{6}{N}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H}{N}\right) = -\frac{6H^2}{N} + \frac{6}{a^3N}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Ha^3}{N}\right)\,, \quad R_{\mathrm{HL}} = \frac{(3-9\lambda)\,H^2}{N^2}\,. \tag{15}$$

Here the Hubble rate H is defined by $H \equiv \dot{a}/a$. In the case of the Einstein gravity, the second term in the last expression for R becomes a total derivative:

$$\int \mathrm{d}^4x\sqrt{-g}R = \int \mathrm{d}^4x\; a^3N\left\{-\frac{6H^2}{N} + \frac{6}{a^3N}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Ha^3}{N}\right)\right\} = \int \mathrm{d}^4x\left\{-6H^2a^3 + 6\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Ha^3}{N}\right)\right\}\,. \tag{16}$$

Therefore, this term can be dropped in the Einstein gravity. The total derivative term comes from the last term $2\nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right)$ in (3), which is dropped in the usual Hořava-Lifshitz gravity. In the $F(R)$-gravity, however, this term cannot be dropped due to the non-linearity. Then if we consider the FRW cosmology with the flat spatial part, there is almost no qualitative difference between the Einstein gravity and the Hořava-Lifshitz gravity, except that there could appear an effective dark matter as a kind of a constant of integration in the Hořava-Lifshitz gravity [15]. The effective dark matter appears since the constraint given by the variation over N becomes global in the projectable Hořava-Lifshitz gravity.

Now we propose a new and very general Hořava-Lifshitz-like $F(R)$-gravity by

$$S_{F(\tilde{R})} = \int \mathrm{d}^4x\sqrt{g^{(3)}}NF(\tilde{R})\,, \quad \tilde{R} \equiv K^{ij}K_{ij} - \lambda K^2 + 2\mu\nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right) - E^{ij}\mathcal{G}_{ijkl}E^{kl}\,. \tag{17}$$

In the FRW universe with the flat spatial part, $\tilde{R}$ has the following form:

$$\tilde{R} = \frac{(3-9\lambda)\,H^2}{N^2} + \frac{6\mu}{a^3N}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Ha^3}{N}\right) = \frac{(3-9\lambda+18\mu)\,H^2}{N^2} + \frac{6\mu}{N}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H}{N}\right)\,. \tag{18}$$

The case one obtains with the choice of parameters $\lambda = \mu = 1$ corresponds to the usual $F(R)$-gravity as long as we consider spatially-flat FRW cosmology, since $\tilde{R}$ reduces to R in (15). On the other hand, in the case of $\mu = 0$, $\tilde{R}$ reduces to R_{HL} in (15) and therefore the action (17) becomes identical with the action (5) of the Hořava-Lifshitz-like $F(R)$-gravity in [11]. Hence, the $\mu = 0$ version corresponds to some degenerate limit of the above general Hořava-Lifshitz $F(R)$-gravity.

For the action (17), the FRW equation given by the variation over $g^{(3)}_{ij}$ has the following form after assuming the FRW space-time (14) and setting $N = 1$:

$$0 = F\left(\tilde{R}\right) - 2\left(1 - 3\lambda + 3\mu\right)\left(\dot{H} + 3H^2\right)F'\left(\tilde{R}\right) - 2\left(1 - 3\lambda\right)H\frac{dF'\left(\tilde{R}\right)}{\mathrm{d}t} + 2\mu\frac{\mathrm{d}^2F'\left(\tilde{R}\right)}{\mathrm{d}t^2} + p\,, \tag{19}$$

where F' denotes the derivative of F with respect to its argument. Here, the matter contribution (the pressure p) is included. On the other hand, the variation over N gives the global constraint:

$$0 = \int \mathrm{d}^3\boldsymbol{x}\left[F\left(\tilde{R}\right) - 6\left\{\left(1 - 3\lambda + 3\mu\right)H^2 + \mu\dot{H}\right\}F'\left(\tilde{R}\right) + 6\mu H\frac{\mathrm{d}F'\left(\tilde{R}\right)}{\mathrm{d}t} - \rho\right]\,, \tag{20}$$

after setting $N = 1$. Here ρ is the energy density of matter. Since N only depends on t, but does not depend on the spatial coordinates, we only obtain the global constraint given by the integration. If the standard conservation law is used,

$$0 = \dot{\rho} + 3H\left(\rho + p\right)\,, \tag{21}$$

Eq. (19) can be integrated to give

$$0 = F\left(\tilde{R}\right) - 6\left\{\left(1 - 3\lambda + 3\mu\right)H^2 + \mu\dot{H}\right\}F'\left(\tilde{R}\right) + 6\mu H\frac{\mathrm{d}F'\left(\tilde{R}\right)}{\mathrm{d}t} - \rho - \frac{C}{a^3}\,. \tag{22}$$

Here C is the integration constant. Using (20), one finds $C = 0$. In [15], however, it has been claimed that C need not always vanish in a local region, since (20) needs to be satisfied in the whole universe. In the region $C > 0$, the Ca^{-3} term in (22) may be regarded as dark matter.

Note that Eq. (22) corresponds to the first FRW equation and (19) to the second one. Specifically, if we choose $\lambda = \mu = 1$ and $C = 0$, Eq. (22) reduces to

$$\begin{aligned} 0 &= F\left(\tilde{R}\right) - 6\left(H^2 + \dot{H}\right)F'\left(\tilde{R}\right) + 6H\frac{\mathrm{d}F'\left(\tilde{R}\right)}{\mathrm{d}t} - \rho \\ &= F\left(\tilde{R}\right) - 6\left(H^2 + \dot{H}\right)F'\left(\tilde{R}\right) + 36\left(4H^2\dot{H} + \ddot{H}\right)F''\left(\tilde{R}\right) - \rho\,, \end{aligned} \tag{23}$$

which is identical to the corresponding equation in the standard $F(R)$-gravity (see Eq. (2) in [16] where a reconstruction of the theory has been made).

We should note that in the degenerate $\mu = 0$ case [11], the action (17) or (5) does not contain any term with second derivatives with respect to the coordinates, which appears in the usual $F(R)$-gravity. The existence of the second derivatives in the usual $F(R)$-gravity induces the third and fourth derivatives in the FRW equation as in (19). Due to such higher derivatives, there appears an extra scalar mode, which is often called the scalaron in the usual $F(R)$-gravity. This scalar mode often affects the correction to the Newton law as well as other solar tests. Therefore, such a scalar mode does not appear in the Hořava-Lifshitz-like $F(R)$-gravity with $\mu = 0$. Hence, we have formulated a general Hořava-Lifshitz $F(R)$-gravity which describes the standard $F(R)$-gravity or its non-degenerate Hořava-Lifshitz extension in a consistent way.

III. HAMILTONIAN FORMALISM

Let us present some elements of the Hamiltonian analysis of our proposal (for Hamiltonian analysis of constrained systems, and their quantization, see [17]). By introducing two auxiliary fields A and B we can write the action (17) into a form that is linear in $\tilde{R}$:

$$S_{F(\tilde{R})} = \int \mathrm{d}^4x\sqrt{g^{(3)}}N\left[B(\tilde{R} - A) + F(A)\right]\,. \tag{24}$$

Variation with respect to B yields $\tilde{R} = A$ that can be inserted back into the action (24) in order to produce the original action (17). The variation with respect to A yields $B = F'(A)$.

First we rewrite $\tilde{R}$ in (24) into a more explicit and useful form (see (17) for the definition of $\tilde{R}$). The unit normal n^μ to the hypersurface Σ_t in space-time can be written in terms of the lapse and the shift vector as $n^\mu = (n^0, n^i) = \left(\frac{1}{N}, -\frac{N^i}{N}\right)$. The corresponding one-form is $n_\mu = -N\nabla_\mu t = (-N, 0, 0, 0)$. The term in (17) that involves the unit normal can be written

$$\nabla_\mu \left(n^\mu \nabla_\nu n^\nu - n^\nu \nabla_\nu n^\mu\right) = \nabla_\mu \left(n^\mu K\right) - \frac{1}{N} g^{(3)ij} \nabla_i^{(3)} \nabla_j^{(3)} N \,. \tag{25}$$

Thus we can rewrite $\tilde{R}$ as

$$\tilde{R} = K_{ij}\mathcal{G}^{ijkl}K_{kl} + 2\mu\nabla_\mu \left(n^\mu K\right) - \frac{2\mu}{N}\nabla_i^{(3)}\nabla^{(3)i}N - E^{ij}\mathcal{G}_{ijkl}E^{kl} \;. \tag{26}$$

Introducing (26) into (24) and performing integrations by parts yields the action

$$\begin{aligned} S_{F(\tilde{R})} \;=\; & \int \mathrm{d}t\mathrm{d}^3\boldsymbol{x}\sqrt{g^{(3)}}\Big\{N\left[B\left(K_{ij}\mathcal{G}^{ijkl}K_{kl} - E^{ij}\mathcal{G}_{ijkl}E^{kl} - A\right) + F(A)\right] \\ & -2\mu K\left(\dot{B} - N^i\partial_i B\right) - 2\mu N g^{(3)ij}\nabla_i^{(3)}\nabla_j^{(3)}B\Big\} \;, \end{aligned} \tag{27}$$

where the integral is taken over the union $\mathcal{U}$ of the $t =$ constant hypersurfaces Σ_t with t over some interval in $\mathbb{R}$, and we have written $Nn^\mu\nabla_\mu B = \dot{B} - N^i\partial_i B$. We assume that the boundary integrals on $\partial\mathcal{U}$ and $\partial\Sigma_t$ vanish.

In the Hamiltonian formalism the field variables g_{ij}, N, N_i, A and B have the canonically conjugated momenta π^{ij}, π_N, π^i, π_A and π_B, respectively. For the spatial metric and the field B we have the momenta

$$\pi^{ij} \;=\; \frac{\delta S_{F(\tilde{R})}}{\delta \dot{g}_{ij}} = \sqrt{g^{(3)}}\left[B\mathcal{G}^{ijkl}K_{kl} - \frac{\mu}{N}g^{(3)ij}\left(\dot{B} - N^i\partial_i B\right)\right] \;, \tag{28}$$

$$\pi_B \;=\; \frac{\delta S_{F(\tilde{R})}}{\delta \dot{B}} = -2\mu\sqrt{g^{(3)}}K \,. \tag{29}$$

Because the action does not depend on the time derivative of N, N^i or A, the rest of the momenta form the set of primary constraints:

$$\pi_N \approx 0 \,, \quad \pi^i(\boldsymbol{x}) \approx 0 \,, \quad \pi_A(\boldsymbol{x}) \approx 0 \,. \tag{30}$$

We consider N to be projectable, i.e. $N = N(t)$, and therefore also the momentum $\pi_N = \pi_N(t)$ is constant on Σ_t for each t. The Poisson brackets are postulated in the form (equal time t is understood)

$$\begin{aligned} &\{g_{ij}^{(3)}(\boldsymbol{x}), \pi^{kl}(\boldsymbol{y})\} = \frac{1}{2}\left(\delta_i^k\delta_j^l + \delta_i^l\delta_j^k\right)\delta(\boldsymbol{x}-\boldsymbol{y}) \,, \\ &\{N, \pi_N\} = 1 \,, \quad \{N_i(\boldsymbol{x}), \pi^j(\boldsymbol{y})\} = \delta_i^j\delta(\boldsymbol{x}-\boldsymbol{y}) \,, \\ &\{A(\boldsymbol{x}), \pi_A(\boldsymbol{y})\} = \delta(\boldsymbol{x}-\boldsymbol{y}) \,, \quad \{B(\boldsymbol{x}), \pi_B(\boldsymbol{y})\} = \delta(\boldsymbol{x}-\boldsymbol{y}) \,, \end{aligned} \tag{31}$$

with all the other Poisson brackets vanishing. We shall continue to omit the argument $(\boldsymbol{x})$ of the fields when there is no risk of confusion. In order to obtain the Hamiltonian, we first solve (28)–(29) for K_{ij} and $\dot{B}$,

$$\begin{aligned} K_{ij} \;&=\; \frac{1}{\sqrt{g^{(3)}}B}\mathcal{G}_{ijkl}\left[\pi^{kl} - \frac{1}{3}g^{(3)kl}\left(g_{mn}^{(3)}\pi^{mn} + \frac{1-3\lambda}{2\mu}B\pi_B\right)\right] \\ &=\; \frac{1}{\sqrt{g^{(3)}}B}\left(g_{ik}^{(3)}g_{jl}^{(3)}\pi^{kl} - \frac{1}{3}g_{ij}^{(3)}g_{kl}^{(3)}\pi^{kl}\right) - \frac{1}{6\mu\sqrt{g^{(3)}}}g_{ij}^{(3)}\pi_B \,, \\ \dot{B} \;&=\; N^i\partial_i B - \frac{N}{3\mu\sqrt{g^{(3)}}}\left(g_{ij}^{(3)}\pi^{ij} + \frac{1-3\lambda}{2\mu}B\pi_B\right) \,, \end{aligned} \tag{32}$$

and further obtain $\dot{g}_{ij}^{(3)} = 2NK_{ij} + \nabla_i^{(3)}N_j + \nabla_j^{(3)}N_i$. Therefore both $g_{ij}^{(3)}$ and B are dynamical variables and no more primary constraints are needed. The Hamiltonian is then defined

$$H = \int \mathrm{d}^3\boldsymbol{x}\left(\pi^{ij}\dot{g}_{ij}^{(3)} + \pi_B\dot{B}\right) - L = \int \mathrm{d}^3\boldsymbol{x}\left(N\mathcal{H}_0 + N_i\mathcal{H}^i\right) \,, \tag{33}$$

where the Lagrangian L is defined by the action (27), $S_{F(\tilde{R})} = \int \mathrm{d}tL$, and the so called Hamiltonian constraint and the momentum constraint are found to be

$$\begin{aligned}
\mathcal{H}_0 &= \frac{1}{\sqrt{g^{(3)}}}\left[\frac{1}{B}\left(g^{(3)}_{ik}g^{(3)}_{jl}\pi^{ij}\pi^{kl} - \frac{1}{3}\left(g^{(3)}_{ij}\pi^{ij}\right)^2\right) - \frac{1}{3\mu}g^{(3)}_{ij}\pi^{ij}\pi_B - \frac{1-3\lambda}{12\mu^2}B\pi_B^2\right] \\
&\quad + \sqrt{g^{(3)}}\left[B\left(E^{ij}\mathcal{G}_{ijkl}E^{kl} + A\right) - F(A) + 2\mu g^{(3)ij}\nabla^{(3)}_i\nabla^{(3)}_j B\right], \\
\mathcal{H}^i &= -2\nabla^{(3)}_j\pi^{ij} + g^{(3)ij}\partial_j B\pi_B \\
&= -2\partial_j\pi^{ij} - g^{(3)ij}\left(2\partial_k g^{(3)}_{jl} - \partial_j g^{(3)}_{kl}\right)\pi^{kl} + g^{(3)ij}\partial_j B\pi_B\,,
\end{aligned} \tag{34}$$

respectively. Again we assume that the boundary term resulting from an integration by parts vanishes. We define the total Hamiltonian by

$$H_T = H + \lambda_N\pi_N + \int \mathrm{d}^3\boldsymbol{x}\left(\lambda_i\pi^i + \lambda_A\pi_A\right), \tag{35}$$

where the primary constraints (30) are multiplied by the Lagrange multipliers λ_N, λ_i, λ_A. Note that there is no space integral over the product $\lambda_N\pi_N$ since they depend only on the time coordinate t due to the projectability of N.

The primary constraints (30) have to be preserved under time evolution of the system:

$$\begin{aligned}
\dot{\pi}_N &= \{\pi_N, H_T\} = -\int \mathrm{d}^3\boldsymbol{x}\mathcal{H}_0\,, \\
\dot{\pi}^i &= \{\pi^i, H_T\} = -\mathcal{H}^i\,, \\
\dot{\pi}_A &= \{\pi_A, H_T\} = \sqrt{g^{(3)}}N\left(-B + F'(A)\right).
\end{aligned} \tag{36}$$

Therefore we impose the secondary constraints:

$$\begin{aligned}
\Phi_0 &\equiv \int \mathrm{d}^3\boldsymbol{x}\mathcal{H}_0 \approx 0\,, \\
\Phi^i_S(\boldsymbol{x}) &\equiv \mathcal{H}^i(\boldsymbol{x}) \approx 0\,, \\
\Phi_A(\boldsymbol{x}) &\equiv B(\boldsymbol{x}) - F'(A(\boldsymbol{x})) \approx 0\,.
\end{aligned} \tag{37}$$

Here the Hamiltonian constraint Φ_0 is global and the other two, the momentum constraint $\Phi^i_S(\boldsymbol{x})$ and the constraint $\Phi_A(\boldsymbol{x})$, are local. It is convenient to introduce a globalised version of the momentum constraints Φ^i_S:

$$\Phi_S(\xi_i) \equiv \int \mathrm{d}^3\boldsymbol{x}\xi_i\mathcal{H}^i \approx 0\,, \tag{38}$$

where $\xi_i, i = 1,2,3$ are three arbitrary smearing functions — the choices $\xi_i = \delta^j_i\delta(\boldsymbol{x}-\boldsymbol{y})$ will produce the three local constraints $\mathcal{H}^j$ which in turn imply the smeared one.

The total Hamiltonian (35) can be written in terms of the constraints as

$$H_T = N\Phi_0 + \Phi_S(N_i) + \lambda_N\pi_N + \int \mathrm{d}^3\boldsymbol{x}\left(\lambda_i\pi^i + \lambda_A\pi_A\right). \tag{39}$$

The consistency of the system requires that also the secondary constraints Φ_0, $\Phi_S(\xi_i)$ and $\Phi_A(\boldsymbol{x})$ have to be preserved under time evolution:

$$\begin{aligned}
\dot{\Phi}_0 &= \{\Phi_0, H_T\} = N\{\Phi_0,\Phi_0\} + \{\Phi_0, \Phi_S(N_i)\} + \int \mathrm{d}^3\boldsymbol{x}\lambda_A(\boldsymbol{x})\{\Phi_0, \pi_A(\boldsymbol{x})\} \approx 0\,, \\
\dot{\Phi}_S(\xi_i) &= \{\Phi_S(\xi_i), H_T\} = N\{\Phi_S(\xi_i), \Phi_0\} + \{\Phi_S(\xi_i), \Phi_S(N_i)\} \approx 0 \\
\dot{\Phi}_A(\boldsymbol{x}) &= \{\Phi_A(\boldsymbol{x}), H_T\} = N\{\Phi_A(\boldsymbol{x}), \Phi_0\} + \{\Phi_A(\boldsymbol{x}), \Phi_S(N_i)\} + \int \mathrm{d}^3\boldsymbol{y}\lambda_A(\boldsymbol{y})\{\Phi_A(\boldsymbol{x}), \pi_A(\boldsymbol{y})\} \approx 0\,,
\end{aligned} \tag{40}$$

where we have used the fact that the constraints π_N and π^i have strongly vanishing Poisson brackets with every constraint. We need to calculate the rest of the algebra of the constraints under the Poisson bracket. The Poisson brackets between the constraint $\Phi_S(\xi_i)$ and the canonical variables are

$$\begin{aligned}
\{\Phi_S(\xi_i), B\} &= -\xi^i\partial_i B\,, \\
\{\Phi_S(\xi_i), \pi_B\} &= -\partial_i\left(\xi^i\pi_B\right)\,, \\
\{\Phi_S(\xi_k), g^{(3)}_{ij}\} &= -\xi^k\partial_k g^{(3)}_{ij} - g^{(3)}_{ik}\partial_j\xi^k - g^{(3)}_{jk}\partial_i\xi^k\,, \\
\{\Phi_S(\xi_k), \pi^{ij}\} &= -\partial_k\left(\xi^k\pi^{ij}\right) + \pi^{ik}\partial_k\xi^j + \pi^{jk}\partial_k\xi^i\,,
\end{aligned} \tag{41}$$

where $\xi^i = g^{(3)ij}\xi_j$, and trivially zero for A and π_A,

$$\{\Phi_S(\xi_i), A\} = 0\,, \quad \{\Phi_S(\xi_i), \pi_A\} = 0\,. \tag{42}$$

Thus $\Phi_S(\xi_i)$ generates the spatial diffeomorphisms for the variables $B, \pi_B, g^{(3)}_{ij}, \pi^{ij}$, and consequently for any function or functional constructed from these variables, and treates the variables A, π_A as constants. By using this result (41)–(42) we obtain the Poisson brackets for the constraints Φ_0 and $\Phi_S(\xi_i)$:

$$\{\Phi_0, \Phi_0\} = 0\,, \quad \{\Phi_S(\xi_i), \Phi_0\} = 0\,, \quad \{\Phi_S(\xi_i), \Phi_S(\eta_i)\} = \Phi_S(\xi^j\partial_j\eta_i - \eta^j\partial_j\xi_i) \approx 0\,. \tag{43}$$

For the constraints π_A and $\Phi_A(\boldsymbol{x})$ the Poisson brackets that do not vanishing strongly are:

$$\begin{aligned}
&\{\pi_A(\boldsymbol{x}), \Phi_0\} = -\sqrt{g^{(3)}}\Phi_A(\boldsymbol{x}) \approx 0\,, \quad \{\pi_A(\boldsymbol{x}), \Phi_A(\boldsymbol{y})\} = F''(A(\boldsymbol{x}))\delta(\boldsymbol{x}-\boldsymbol{y}) \\
&\{\Phi_0, \Phi_A(\boldsymbol{x})\} = \frac{1}{3\mu\sqrt{g^{(3)}}}\left(g^{(3)}_{ij}\pi^{ij} + \frac{1-3\lambda}{2\mu}B\pi_B\right)\,, \quad \{\Phi_S(\xi_i), \Phi_A(\boldsymbol{x})\} = -\xi^i\partial_i B\,.
\end{aligned} \tag{44}$$

Thus, in order to satisfy the consistency conditions (40), we have to impose the tertiary constraint

$$\Phi_{\rm ter} \equiv N^i\partial_i B - \frac{N}{3\mu\sqrt{g^{(3)}}}\left(g^{(3)}_{ij}\pi^{ij} + \frac{1-3\lambda}{2\mu}B\pi_B\right) - \lambda_A F''(A) \approx 0\,. \tag{45}$$

Since $F''(A) = 0$ would essentially reproduce the original projectable Hořava-Lifshitz gravity, we assume that $F''(A) \neq 0$. The first two terms in (45), i.e. the expression for $\dot{B}$ in (32), does not vanish due to the established constraints (30) and (37). Therefore (45) is a restriction on the Lagrange multiplier λ_A, and we can solve it from $\Phi_{\rm ter} = 0$:

$$\lambda_A = \frac{1}{F''(A)}\left(N^i\partial_i B - \frac{N}{3\mu\sqrt{g^{(3)}}}\left(g^{(3)}_{ij}\pi^{ij} + \frac{1-3\lambda}{2\mu}B\pi_B\right)\right)\,. \tag{46}$$

Introducing (46) into the Hamiltonian (39) ensures that now all the constraints of the system are consistent.

According to the Poisson brackets (43)–(44) between the constraints, we can set the second-class constraints $\pi_A(\boldsymbol{x})$ and $\Phi_A(\boldsymbol{x})$ to vanish strongly, and as a result turn the Hamiltonian constraint Φ_0 and the momentum constraint $\Phi_S(\xi_i)$ into first-class constraints. For this end, we replace the Poisson bracket with the Dirac bracket, which is given by

$$\{f(\boldsymbol{x}), h(\boldsymbol{y})\}_{\rm DB} = \{f(\boldsymbol{x}), h(\boldsymbol{y})\} + \int d^3\boldsymbol{z}\frac{1}{F''(A(\boldsymbol{z}))}\left(\{f(\boldsymbol{x}), \pi_A(\boldsymbol{z})\}\{\Phi_A(\boldsymbol{z}), h(\boldsymbol{y})\} - \{f(\boldsymbol{x}), \Phi_A(\boldsymbol{z})\}\{\pi_A(\boldsymbol{z}), h(\boldsymbol{y})\}\right)\,, \tag{47}$$

where f and h are any functions of the canonical variables. Assuming we can solve the constraint $\Phi_A(\boldsymbol{x}) = 0$, i.e. $B = F'(A)$, for $A = \tilde{A}(B)$, where $\tilde{A}$ is the inverse of the function F', we can eliminate the variables A and π_A. Thus the final variables of the system are $g^{(3)}_{ij}, \pi^{ij}, B, \pi_B$. The lapse N and the shift vector N_i, together with λ_N and λ_i, are non-dynamical multipliers. Then since every dynamical variable has a vanishing Poisson bracket with the constraint π_A, the Dirac bracket (47) reduces to the Poisson bracket,

$$\{f(\boldsymbol{x}), h(\boldsymbol{y})\}_{\rm DB} = \{f(\boldsymbol{x}), h(\boldsymbol{y})\}\,. \tag{48}$$

Finally the total Hamiltonian is the sum of the first-class constraints

$$H_T = N\Phi_0 + \Phi_S(N_i) + \lambda_N\pi_N + \int d^3\boldsymbol{x}\lambda_i\pi^i\,. \tag{49}$$

It defines the equations of motion for every function $f(\boldsymbol{x})$ (or functional f) of the canonical variables

$$\dot{f}(\boldsymbol{x}) = \{f(\boldsymbol{x}), H_T\} = N\{f(\boldsymbol{x}), \Phi_0\} + \{f(\boldsymbol{x}), \Phi_S(N_i)\} + \lambda_N\{f(\boldsymbol{x}), \pi_N\} + \int \mathrm{d}^3\boldsymbol{y}\lambda_i(\boldsymbol{y})\{f(\boldsymbol{x}), \pi^i(\boldsymbol{y}) \, . \tag{50}$$

We have calculated the Hamitonian (33)–(34) of the proposed modified Hořava-Lifshitz $F(R)$-gravity and established the preservation of the primary constraints (30) by imposing the required secondary constraints (37), including the Hamiltonian constraint and the momentum constraint. In order to ensure the consistency of the secondary constraints we introduced the tertiary constraint (45) that was used to fix the Lagrange multiplier λ_A of the primary constraint π_A. Finally, we eliminated the pair of variables A, π_A by imposing the second-class constraints π_A, Φ_A and introducing the Dirac bracket (47), and obtained the total Hamiltonian in its final form (49) as a sum of the four remaining first-class constraints. We conclude that the proposed action (17) of this modified Hořava-Lifshitz $F(R)$-gravity, which obeys the projectability condition, defines a consistent theory. This result agrees with the recent analysis presented in ref. [18].

IV. FRW COSMOLOGY FOR SOME VERSIONS OF MODIFIED HOŘAVA-LIFSHITZ $F(R)$-GRAVITY.

This section is devoted to the study of the FRW Eqs. (19) and (20) which admit a de Sitter universe solution. We now neglect the matter contribution by putting $p = \rho = 0$. Then by assuming $H = H_0$, both of Eq. (19) and (20) lead to the same equation

$$0 = F\left(3\left(1 - 3\lambda + 6\mu\right)H_0^2\right) - 6\left(1 - 3\lambda + 3\mu\right)H_0^2 F'\left(3\left(1 - 3\lambda + 6\mu\right)H_0^2\right) \, . \tag{51}$$

Eqs. (19) and (20) are consistent with each other as long as $C = 0$.

First we consider the popular case that

$$F\left(\tilde{R}\right) \propto \tilde{R} + \beta\tilde{R}^2 \, . \tag{52}$$

Then Eq. (51) gives

$$0 = H_0^2\left\{1 - 3\lambda + 9\beta\left(1 - 3\lambda + 6\mu\right)\left(1 - 3\lambda + 2\mu\right)H_0^2\right\} \, . \tag{53}$$

In the case of usual $F(R)$-gravity, where $\lambda = \mu = 1$ and therefore $1 - 3\lambda + 2\mu = 0$, there is only the trivial solution $H_0^2 = 0$, although the R^2-term could generate the inflation when more gravitational terms, like $R_{\mu\nu}R^{\mu\nu}$ etc., are added. For our general case, however, there exists the non-trivial solution

$$H_0^2 = -\frac{1 - 3\lambda}{\beta\left(1 - 3\lambda + 6\mu\right)\left(1 - 3\lambda + 2\mu\right)} \, , \tag{54}$$

as long as the r.h.s. of (54) is positive. If the magnitude of this non-trivial solution is small enough, this solution might correspond to the accelerating expansion in the present universe. Hence, the R^2-term may generate the late-time acceleration. On the other hand, the above solution may serve as an inflationary solution for the early universe (with the corresponding choice of parameters).

Instead of (52) one may consider the following model:

$$F\left(\tilde{R}\right) \propto \tilde{R} + \beta\tilde{R}^2 + \gamma\tilde{R}^3 \, . \tag{55}$$

Then Eq. (51) becomes

$$0 = H_0^2\left\{1 - 3\lambda + 9\beta\left(1 - 3\lambda + 6\mu\right)\left(1 - 3\lambda + 2\mu\right)H_0^2 + 9\gamma\left(1 - 3\lambda + 6\mu\right)^2\left(5 - 15\lambda + 12\mu\right)H_0^4\right\} \, , \tag{56}$$

which has the following two non-trivial solutions,

$$H_0^2 = -\frac{\left(1 - 3\lambda + 2\mu\right)\beta}{2\left(1 - 3\lambda + 6\mu\right)\left(5 - 15\lambda + 12\mu\right)\gamma}\left(1 \pm \sqrt{1 - \frac{4\left(1 - 3\lambda\right)\left(5 - 15\lambda + 12\mu\right)\gamma}{9\left(1 - 3\lambda + 2\mu\right)^2\beta^2}}\right) \, , \tag{57}$$

as long as the r.h.s. is real and positive. If

$$\left|\frac{4\left(1 - 3\lambda\right)\left(5 - 15\lambda + 12\mu\right)\gamma}{9\left(1 - 3\lambda + 2\mu\right)^2\beta^2}\right| \ll 1 \, , \tag{58}$$

one of the two solutions is much smaller than the other solution. Then one may regard that the larger solution corresponds to the inflation in the early universe and the smaller one to the late-time acceleration, similarly to the modified gravity model [19], where such unification has been first proposed. Hence, we have proved the qualitative possibility to unify the early-time inflation with late-time acceleration in the modified Hořava-Lifshitz $F(R)$-gravity.

V. MORE GENERAL ACTION

In the formulation of $F(R)$ Hořava-Lifshitz-like gravity, we do not require full diffeomorphism-invariance, but only invariance under "foliation-preserving" diffeomorphisms (13). Therefore there are many invariants or covariant quantities made from the metric like K, K_{ij}, $\nabla^{(3)}_i K_{jk}$, $\cdots$, $\nabla^{(3)}_{i_1}\nabla^{(3)}_{i_2}\cdots\nabla^{(3)}_{i_n}K_{jk}$, $R^{(3)}$, $R^{(3)}_{ij}$, $R^{(3)}_{ijkl}$, $\nabla^{(3)}_i R^{(3)}_{jk}$, $\cdots$, $\nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right)$, $\cdots$, etc. Then the action composed of such invariants as

$$\begin{aligned} S_{\rm gHL} \ = \ & \int d^4x\sqrt{g^{(3)}}NF\left(g^{(3)}_{ij}, K, K_{ij}, \nabla^{(3)}_i K_{jk}, \cdots, \nabla^{(3)}_{i_1}\nabla^{(3)}_{i_2}\cdots\nabla^{(3)}_{i_n}K_{jk},\right. \\ & \left.\cdots, R^{(3)}, R^{(3)}_{ij}, R^{(3)}_{ijkl}, \nabla^{(3)}_i R^{(3)}_{jk}, \cdots, \nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right)\right) , \end{aligned} \tag{59}$$

could be a rather general action for the generalized Hořava-Lifshitz gravity. Note that one can also include the (cosmological) constant in the above action. Here it has been assumed that the action does not contain derivatives higher than the second order with respect to the time coordinate t. In the usual $F(R)$-gravity, there appears the extra scalar mode since the equations given by the variation over the metric tensor contain the fourth derivative. Now we avoid such extra modes except the one scalar mode.

In the FRW space-time (14) with the flat spatial part and non-trivial $N = N(t)$, we find

$$\begin{aligned} & \Gamma^0_{00} = \frac{\dot N}{N}\, , \quad \Gamma^0_{ij} = \frac{a^2H}{N^2}\delta_{ij}\, , \quad \Gamma^i_{j0} = H\delta^i_{\ j} \quad \text{other } \Gamma^\mu_{\nu\rho} = 0\, , \\ & K_{ij} = \frac{a^2H}{N}\delta_{ij}\, , \quad \nabla^{(3)}_i = 0\, , \quad R^{(3)}_{ijkl} = 0\, , \quad \nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right) = \frac{3}{a^3N}\frac{d}{dt}\left(\frac{a^3H}{N}\right)\, . \end{aligned} \tag{60}$$

Then one gets

$$\begin{aligned} & g^{(3)}_{ij} = a^2\delta_{ij}\, , \quad K = \frac{3H}{N}\, , \quad \nabla^{(3)}_i K_{jk} = \cdots = \nabla^{(3)}_{i_1}\nabla^{(3)}_{i_2}\cdots\nabla^{(3)}_{i_n}K_{jk} = \cdots = 0\, , \\ & R^{(3)} = R^{(3)}_{ij} = R^{(3)}_{ijkl} = \nabla^{(3)}_i R^{(3)}_{jk} = \cdots = 0\, , \end{aligned} \tag{61}$$

and since F must be a scalar under the spatial rotation, the action (59) reduces to

$$S_{\rm gHL} \ = \ \int d^4x\sqrt{g^{(3)}}NF\left(\frac{H}{N}, \frac{3}{a^3N}\frac{d}{dt}\left(\frac{a^3H}{N}\right)\right)\, . \tag{62}$$

Therefore, if we consider the FRW cosmology, the function F should depend on only two variables, $\frac{H}{N}$ and $\frac{3}{a^3N}\frac{d}{dt}\left(\frac{a^3H}{N}\right)$. For instance, $\tilde R$ in (18) is given by this combination. As an illustrative example, we may consider the following one:

$$F = f_0\left(K^{ij}K_{ij} - \lambda K^2\right) + f_1\nabla_\mu\left(n^\mu\nabla_\nu n^\nu - n^\nu\nabla_\nu n^\mu\right)^2\, . \tag{63}$$

Then in the FRW space-time (2), by the variation of the scale factor a, we obtain the following equation:

$$0 = 2f_0\left(1-3\lambda\right)\left(H^2 + \dot H\right) + 3f_1\left(27H^4 + 54H^2\dot H + 15\dot H^2 + 18H\ddot H + 2\dddot H\right)\, . \tag{64}$$

If we assume a de Sitter universe $H = H_0$ with constant H_0, Eq. (64) reduces to

$$0 = 2f_0\left(1-3\lambda\right)H^2 + 81f_1H^4\, , \tag{65}$$

which has the non-trivial solution

$$H^2 = -\frac{2f_0\left(1-3\lambda\right)}{81f_1}\, , \tag{66}$$

as long as the r.h.s. is positive. In the same way, a large class of modified Hořava-Lifshitz gravities may be constructed. For instance, one can construct Hořava-Lifshitz-like generalizations of $F(G)$-gravity where the action is the Einstein-Hilbert term plus a function F of the Gauss-Bonnet invariant G, non-local gravity, $F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})$, etc. It is remarkable that some special subclass of such Hořava-Lifshitz-like theories will have the same spatially-flat FRW background dynamics as the corresponding traditional modified gravity.

VI. DISCUSSION

We have suggested a quite general approach for the modification of Hořava-Lifshitz gravity. Concentrating mainly on the $F(R)$-gravity version, the consistency of the spatially-flat FRW field equations has been proven. The Hamiltonian and the corresponding constraints of the system have also been derived and proven to be consistent. It is demonstrated that a degenerate subclass of the proposed general modified $F(R)$ Hořava-Lifshitz gravity corresponds to the earlier proposed $F(R)$ extension of Hořava-Lifshitz gravity. The preliminary study of FRW cosmology indicates a possibility to describe or even to unify the early-time inflation with the late-time acceleration.

Our proposal opens the bridge between the conventional modified gravity and its Hořava-Lifshitz counterpart. Indeed, it is demonstrated that our model with a special choice of parameters ($\lambda = \mu = 1$) leads to the same spatially-flat FRW dynamics as its traditional counterpart, which is fully diffeomorphism-invariant. Moreover, we eventually proposed the most general construction for a modified gravity that is invariant under foliation-preserving diffeomorphisms. In this way, any traditional modified gravity has its counterpart, where the Lorentz symmetry is broken. The explicit construction may be made using the results of Section V. Having in mind that a number of traditional modified theories of gravity are cosmologically viable and pass the local tests, one can expect that it will eventually be possible to realize any accelerating FRW cosmology in this modified Hořava-Lifshitz theory. This will be studied elsewhere.

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