Ghost Dark Matter

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Abstract: We revisit ghost dark matter, the possibility that ghost condensation may serve as an alternative to dark matter. In particular, we investigate the Friedmann-Robertson-Walker (FRW) background evolution and the large-scale structure (LSS) in the ΛGDM universe, i.e. a late-time universe dominated by a cosmological constant and ghost dark matter. The FRW background of the ΛGDM universe is indistinguishable from that of the standard ΛCDM universe if $M \gtrsim 1$ eV, where M is the scale of spontaneous Lorentz breaking. From the LSS we find a stronger bound: $M \gtrsim 10$ eV. For smaller M, ghost dark matter would have non-negligible sound speed after the matter-radiation equality, and thus the matter power spectrum would significantly differ from observation. These bounds are compatible with the phenomenological upper bound $M \lesssim 100$ GeV known in the literature.

Keywords: Dark Energy, Dark Matter, Modified Gravity.

Contents

1. Introduction

Current data of the cosmological observations (e.g., Cosmic Microwave Background (CMB), Large Scale Structure (LSS) and SuperNovae (SNe)) show that our Universe today mostly consists of dark matter responsible for the structure formation and dark energy causing late time accelerated expansion of the Universe [1, 2, 3, 4, 5]. From observational point of view, the paradigm of dark energy and dark matter is very successful to fit the data.

However, from theoretical viewpoint, we do not know what they really are, despite the fact that there are many theoretical models (for review, see e.g. [6] for dark energy and [7] for dark matter). This situation has been a strong motivation for modification of gravity as an alternative to dark energy and dark matter: just changing behavior of gravity at long distance/time scales might be able to explain the observational data without introducing dark energy and dark matter. Indeed, many theories of modification of gravity have been proposed, such as massive gravity [8], DGP model [9, 10], ghost condensate [11, 12] and so on. It is important to investigate cosmological implications of those modified gravity theories toward the future observations.

In this paper, we focus on the ghost condensate scenario and investigate the possibility that it might serve as an alternative to dark matter. This possibility, dubbed *ghost dark matter*, was already pointed out in [11] but has not been investigated in detail. Based on the Friedmann-Robertson-Walker (FRW) background evolution and the large-scale structure of the universe, in the present paper we shall find a lower bound on the scale of spontaneous Lorentz breaking, $M \gtrsim 10$ eV, under the assumption that ghost dark matter is responsible for all dark matter in the universe. Most importantly, this bound is compatible with the phenomenological upper bound $M \lesssim 100$ GeV found in [12].

The rest of this paper is organized as follows. In the next section, we briefly review the ghost condensate scenario, including the basic idea, the low-energy effective theory and the phenomenological upper bound on the scale of spontaneous Lorentz breaking. In Sec. 3, we introduce a simplified description of ghost dark matter and investigate the FRW background evolution. We also clarify the regime of validity of the simplified description. In Sec. 4, we consider effects of ghost dark matter on the large-scale structure of the universe. We discuss how density perturbations evolve in the universe dominated by the cosmological constant and ghost dark matter, i.e. the ΛGDM universe, and then give a constraint on the model from the shape of the matter power spectrum. The final section is devoted to conclusion of this paper and discussions.

2. Review of ghost condensation

2.1 Basic idea

In particle physics it is the so-called Higgs mechanism that modifies force law in the infrared (IR) and that makes it possible to describe the weak interaction in a theoretically controllable way. A non-vanishing vacuum expectation value (vev) of a scalar field spontaneously breaks a part of the gauge symmetry and modifies the IR behavior of the corresponding gauge force from Gauss law to Yukawa law.

Ghost condensation applies the idea of Higgs mechanism to general relativity to modify the IR behavior of gravity [11, 12]. In order to spontaneously break a part of the symmetry of general relativity, i.e. spacetime diffeomorphism invariance, we consider *a non-vanishing vev of derivative of a scalar field*. In addition, we demand that *the vev is timelike* so that only the time reparametrization symmetry is spontaneously broken and the 3-dimensional spatial diffeomorphism invariance remains unbroken. Note that this symmetry breaking pattern is totally consistent with observational fact that our universe averaged over large scales is isotropic. However, unlike usual situations in cosmology with a scalar field, we require that *the vev of derivative should remain non-zero and finite as the universe expands towards maximally symmetric spacetimes*, i.e. Minkowski or de Sitter spacetimes [1]. This is because we would like the IR modification of gravity to persist in Minkowski or de Sitter backgrounds.

Therefore the equation of motion of the scalar field and the Einstein equation must allow a solution with $X \equiv -\partial^\mu\phi\partial_\mu\phi$ constant and positive [2] in Minkowski or de Sitter spacetime. This requirement forbids inclusion of any non-trivial potentials for the scalar field since a potential for an eternally running scalar field would lead to time-dependence in the stress-energy tensor. In other words, we must invoke *the shift symmetry* for the scalar field action, i.e. invariance of the action under constant shift of the scalar field: $\phi \to \phi + c$, where c is an arbitrary constant. With the shift symmetry, the action for the scalar field minimally coupled to gravity should be of the form

$$I_\phi = \int d^4x\sqrt{-g}L_\phi, \quad L_\phi = L_\phi(X, \Box\phi, Y, Z, \cdots), \tag{2.1}$$

where $Y \equiv \nabla^\mu\nabla^\nu\phi\nabla_\mu\nabla_\nu\phi$, $Z \equiv \nabla^\mu\nabla^\nu\phi\partial_\mu\phi\partial_\nu\phi$, and so on. In addition to the shift symmetry, we assume *the Z_2 symmetry* for the scalar field action, i.e. invariance of the action under reflection $\phi \to -\phi$. The Z_2 symmetry is to ensure that the effective theory for excitations of ghost condensate is invariant under simultaneous reflection of the time t and the scalar field perturbation $\delta\phi$: $t \to -t$, $\delta\phi \to -\delta\phi$.

The simple Lagrangian

$$L_\phi = P(X) \tag{2.2}$$

is of the form (2.1). The equation of motion for a homogeneous $\phi(t)$ in the flat Friedmann-Robertson-Walker (FRW) background

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \tag{2.3}$$

is $\partial_t(a^3 P_X \partial_t\phi) = 0$, where $P_X \equiv dP/dX$, and leads to

$$P_X X^{1/2} \propto a^{-3} \to 0 \quad (a \to \infty), \tag{2.4}$$

i.e. either $P_X \to 0$ or $X \to 0$. Note that X is either positive or zero for a homogeneous ϕ. Thus, if $P(X)$ has an extremum at $X = M^4 > 0$, i.e. $P_X(M^4) = 0$, then $X = M^4$ is a dynamical attractor of the system and ghost condensate may be realized automatically.

However, we shall see below that a more general action depending on e.g. $\Box\phi$ (see (2.1)) is needed to describe excitations around the ghost condensate properly.

[1] We do not consider anti-de Sitter spacetimes as backgrounds since we are interested in cosmology. If we nonetheless considered ghost condensation in an anti-de Sitter background then the vev of derivative would be spacelike and excitations around the condensate would not have a healthy kinetic term.

[2] In this paper we adopt the mostly positive sign for the spacetime metric.

2.2 Decoupling limit in Minkowski background

Let us now consider excitations around the extremum $X = M^4 > 0$ of $P(X)$. For simplicity we consider Minkowski background. We shall expand the action (2.2) around $\phi = ct$ and take the limit $c^2 \to M^4$. Note that in Minkowski background the equation of motion for ϕ is satisfied for any c and that this treatment is consistent as far as the backreaction to the geometry is negligible.

For

$$\phi = ct + \pi(t, \vec{x}), \tag{2.5}$$

by expanding $P(X)$ with respect to π we obtain the quadratic Lagrangian for π as

$$L_\pi^{(0)} = \left[2c^2 P_{XX}(c^2) + P_X(c^2)\right](\partial_t\pi)^2 - P_X(c^2)(\vec{\nabla}\pi)^2, \tag{2.6}$$

where $P_{XX} \equiv d^2P/dX^2$. Note that we did not take into account the backreaction of the scalar field to the background geometry nor include metric perturbations. These treatments are justified in the limit $E/M_{\rm Pl} \to 0$, where E represents energy scales of interest and $M_{\rm Pl}$ is the Planck scale. Since we shall later take the limit $c^2 \to M^4$ and we shall see that M sets the cutoff scale of the effective field theory, E can be replaced by M in the regime of validity of the effective field theory. Thus, the decoupling limit is characterized by $M/M_{\rm Pl} \to 0$. In this limit and for energies and momenta sufficiently lower than M, the action (2.6) is valid and the small fluctuation π is stable if

$$2c^2 P_{XX}(c^2) + P_X(c^2) > 0, \quad P_X(c^2) > 0. \tag{2.7}$$

By taking the limit $c^2 \to M^4$, we obtain

$$L_\pi^{(0)} = 2M^4 P_{XX}(M^4)(\partial_t\pi)^2. \tag{2.8}$$

Thus, the excitation π around the attractor $X = M^4$ has a healthy time kinetic term if $P_{XX}(M^4) > 0$, i.e. if $X = M^4$ is a local minimum of the function $P(X)$. However, the spatial gradient term vanishes.

This means that the action (2.2) is too simple to describe excitations around the ghost condensate background. Indeed, while we have assumed the shift symmetry to prevent a non-trivial potential from being generated, there is no way to prevent $\Box\phi$, Y, Z, etc. from appearing in the action. We therefore have to go back to the general action (2.1) and seek leading gradient terms.

It turns out that the leading gradient term is of the form

$$\Delta L_\pi = -\frac{\alpha}{2M^2}(\vec{\nabla}^2\pi)^2 \tag{2.9}$$

where α is a constant of order unity. (The reason why this is indeed the leading term will be made clear in the next subsection.) Combining this with the leading time kinetic term (2.8), we obtain the quadratic action for π

$$I_\pi = \int dt d^3\vec{x} L_\pi, \quad L_\pi = M^4\left\{\frac{1}{2}(\partial_t\pi)^2 - \frac{\alpha}{2M^2}(\vec{\nabla}^2\pi)^2\right\}, \tag{2.10}$$

where we have normalized π and α as $2\sqrt{P_{XX}(M^4)}\pi \to \pi$ and $\alpha \to 4M^4 P_{XX}(M^4)\alpha$. Thus, the dispersion relation for π in the decoupling limit $M/M_{\rm Pl} \to 0$ is

$$\omega^2 = \frac{\alpha}{M^2}\vec{k}^4. \tag{2.11}$$

2.3 Scaling dimension and suppression of extra terms

The general action (2.1) should in principle include any terms consistent with the shift symmetry and the Z_2 symmetry. In terms of π, they are invariance under the constant shift of π ($\pi \to \pi + c$, where c is an arbitrary constant) and invariance under the simultaneous reflection of the time t and π ($t \to -t$, $\pi \to -\pi$), respectively. Thus, there should be infinite number of terms added to the quadratic action (2.10) for π. Nonetheless, one can show that the quadratic action (2.10) is a good description of low energy behavior of the system.

In order to show that those extra terms are irrelevant at low energies, let us first identify the scaling dimensions as the energy E is scaled by $E \to sE$ (or the time interval dt is scaled by $dt \to s^{-1}dt$), where s is some constant. By requiring that the quadratic action (2.10) be invariant under scaling, we can fix the scaling dimensions as

$$\begin{aligned} E &\to sE, \\ dt &\to s^{-1}dt, \\ d\vec{x} &\to s^{-1/2}d\vec{x}, \\ \pi &\to s^{1/4}\pi. \end{aligned} \tag{2.12}$$

Note that the scaling dimensions of dt and $d\vec{x}$ are consistent with the dispersion relation (2.11). With this scaling, one can check that the leading interaction

$$M^4 \int dt d^3\vec{x} (\vec{\nabla}\pi)^2 \partial_t \pi \tag{2.13}$$

scales as $s^{1/4}$. Thus, this term is irrelevant and becomes less and less important at energies and momenta sufficiently lower than M. All other terms are even more irrelevant.

There is one relevant operator, namely $(\vec{\nabla}\pi)^2$. However, as we have already seen, the coefficient of this operator is proportional to P_X and goes to zero as the universe expands.

Therefore, if energies, momenta and the field amplitude are sufficiently lower than M then low energy/momentum/amplitude expansion around the quadratic action (2.10) is under control. This in particular implies that apparent extra modes due to higher time derivative terms have frequencies of order M or higher. Hence there is no ghost in the regime of validity of the effective field theory if we set the cutoff scale slightly below M.

2.4 Jeans-like instability and IR modification of linear gravity

So far, we have considered the decoupling limit $M/M_{\rm Pl} \to 0$ of the theory. For small but finite $M/M_{\rm Pl}$, the dispersion relation (2.11) gets corrected due to mixing with gravity and becomes [11]

$$\omega^2 = \frac{\alpha}{M^2}\vec{k}^4 - \frac{\alpha M^2}{2M_{\rm Pl}^2}\vec{k}^2. \tag{2.14}$$

This dispersion relation exhibits Jeans-like instability for modes with length scales longer than L_c and the corresponding time scale is T_c, where

$$L_c \sim \frac{M_{\rm Pl}}{M^2}, \quad T_c \sim \frac{M_{\rm Pl}^2}{\sqrt{\alpha}M^3}. \tag{2.15}$$

Note that this is an IR instability and has nothing to do with ghost. The Jeans-like instability is the origin of the IR modification of gravity in ghost condensate background [11]. The time scale and the length scale of the modification are T_c and L_c, respectively, and are much longer than the naive scale $1/M$.

In Sec. 4 we shall investigate Jeans instability of ghost dark matter, a component which arises from excitation around ghost condensate and which behaves like dark matter. One should note that the Jeans-like instability in the exact ghost condensate background considered in this subsection is both conceptually and qualitatively different from the Jeans instability of ghost dark matter in Sec. 4.

In the decoupling limit $M/M_{\rm Pl} \to 0$, the timescale T_c diverges (in the unit of $1/M$) and thus the Jeans-like instability disappears.

If we required that the Jeans timescale T_c be longer than the age of the universe then we would end up with the (would-be) upper bound $M \lesssim 10$ MeV [11]. However, we shall see below that nonlinear dynamics becomes important much earlier than T_c, that this (would-be) bound is not necessary and that the current upper bound on M is as weak as $M \lesssim 100$ GeV [12].

2.5 Nonlinear dynamics and upper bound on M

A sightly nonlinear extension of the quadratic action (2.10) coupled to linearized gravity is

$$I_\pi = \int dt d^3\vec{x} L_\pi, \quad L_\pi = M^4\left\{\frac{1}{2}\left[\partial_t\pi - (\vec{\nabla}\pi)^2 - \Phi\right]^2 - \frac{\alpha}{2M^2}(\vec{\nabla}^2\pi)^2\right\}, \tag{2.16}$$

where $\Phi = -\delta g_{00}/2$ is the Newtonian potential [12] in the longitudinal gauge. This is obtained by not dropping the leading nonlinear term and the metric perturbation in going from (2.2) to (2.8) before adding (2.9). Note that $(\partial_t\pi)^2$ has been dropped from the expression in the square bracket since it has a scaling dimension higher than $(\vec{\nabla}\pi)^2$, following the discussions in subsection 2.3.

From the quadratic part of the action (2.16), i.e. (2.10), it is easy to see that the timescale of linear dynamics $T_{\rm Lin}$ is determined by

$$\frac{\pi^2}{T_{\rm Lin}^2} \sim \frac{\pi^2}{M^2L^4}, \tag{2.17}$$

where we have assumed that the length scale of interest L is shorter than the Jeans scale L_J (so that the second term in the right hand side of (2.14) is negligible) and we have set $\alpha = O(1)$. Thus we obtain

$$T_{\rm Lin} \sim ML^2, \tag{2.18}$$

which is consistent with the dispersion relation (2.11). On the other hand, from the nonlinear action (2.16), the timescale of nonlinear dynamics $T_{\rm NL}$ is determined by

$$\frac{\pi}{T_{\rm NL}} \sim \frac{\pi^2}{L^2} \sim \Phi, \tag{2.19}$$

and we obtain

$$T_{\rm NL} \sim \frac{L}{\sqrt{|\Phi|}} \sim \sqrt{\frac{M_{\rm Pl}^2L^3}{M_{\rm src}}}, \tag{2.20}$$

where $M_{\rm src}$ is the mass of the gravitational source. Note that this timescale is nothing but the Kepler time. Therefore, nonlinear dynamics dominates before linear dynamics if $T_{\rm NL} \lesssim T_{\rm Lin}$, i.e.

$$\frac{M}{M_{\rm Pl}} \gtrsim \sqrt{\frac{1}{M_{\rm src}L}}. \tag{2.21}$$

This condition is satisfied in virtually all interesting situations. For example, for the earth's surface gravity, this condition is as weak as

$$M \gtrsim 10^{-9} \text{ eV}. \tag{2.22}$$

From the dispersion relation (2.14) it is easy to see that modes with $L \gtrsim L_J$ grow due to Jeans-like instability in timescale

$$\tau \sim \frac{M_{\rm Pl}}{M}L. \tag{2.23}$$

The nonlinear term $(\vec{\nabla}\pi)^2$ in the squared bracket in the action (2.16) becomes important when it is comparable to $\partial_t\pi$ or larger, i.e. when

$$|\pi| \gtrsim \pi_c \equiv \frac{L^2}{\tau}. \tag{2.24}$$

This can be rewritten as a condition on the energy density $\rho_\pi \sim M^4\partial_t\pi$:

$$\rho_\pi \gtrsim \rho_c \equiv \frac{M^4\pi_c}{\tau} \sim \frac{M^6}{M_{\rm Pl}^2}. \tag{2.25}$$

Hereafter, we assume that nonlinear dynamics cutoff the Jeans-like instability at $|\rho_\pi| \sim \rho_c$. Positive and negative regions can grow to $|\rho_\pi| \sim \rho_c$ within the age of the universe H_0^{-1} if $L \gtrsim L_c$ and $\tau \lesssim H_0^{-1}$, i.e. if

$$L_c \lesssim L \lesssim L_{\rm max} \equiv \frac{M}{M_{\rm Pl} H_0} \sim 10 R_\odot \times \frac{M}{100{\rm GeV}}, \tag{2.26}$$

where $R_\odot$ is the solar radius. Therefore, if M is lower than 100 GeV (we shall indeed see below that M must be lower than 100 GeV) then the typical size of positive and negative regions is smaller than $10R_\odot$. This means that ρ_π averaged over large scales such as galactic scales is almost zero and does not gravitate significantly.

Based on these properties of nonlinear dynamics, several phenomenological upper bounds on M were derived in [12]. The strongest among them is the 'twinkling-from-lensing' bound, which we shall briefly describe here.

Suppose that the universe is filled with regions with $\rho_\pi \sim \pm\rho_c$ of the size $L_c \lesssim L \lesssim L_{\rm max}$ moving relative to the cosmic microwave background (CMB). Each region contributes to weak gravitational lensing with the deflection angle

$$\Delta\theta_{\rm each} \sim \frac{r_g}{b} \sim \frac{M^6 b^2}{M_{\rm Pl}^4} \sim \frac{M^6 L^2}{M_{\rm Pl}^4}, \tag{2.27}$$

where b ($\lesssim L$) is the impact parameter and $r_g \sim \rho_c b^3/M_{\rm Pl}^2$ is the gravitational radius of the mass contained within the impact parameter. In the final expression, we have maximized $\Delta\theta_{\rm each}$ with respect to b. Since a light-ray from the distance d experiences N ($\sim d/L$) lens events, the total deflection angle is

$$\Delta\theta_{\rm tot} \sim \Delta\theta_{\rm each}\sqrt{N} \sim \frac{M^6 d^{1/2} L^{3/2}}{M_{\rm Pl}^4} \sim \frac{M^6 d^{1/2} L_{\rm max}^{3/2}}{M_{\rm Pl}^4}, \tag{2.28}$$

where we have maximized $\Delta\theta_{\rm tot}$ with respect to L in the final expression. We can apply this result to the CMB by setting $d \sim H_0^{-1}$. Requiring that $\Delta\theta_{\rm tot}$ be smaller than the angular resolution of CMB experiments $\sim 10^{-3}$, we obtain the upper bound

$$M \lesssim 100 \text{ GeV}. \tag{2.29}$$

The twinkling timescale for the CMB is

$$T_{\rm twinkle} \sim \frac{L_{\rm max}}{v} \sim \frac{M}{100{\rm GeV}} \cdot \frac{300{\rm km/s}}{v} \times 0.1 \text{ day}, \tag{2.30}$$

where v is the typical velocity of positive and negative regions relative to the CMB rest frame. Thus, if M is close to 100 GeV then the twinkling effect may be detected in future CMB experiments.

3. Ghost dark matter and background evolution

3.1 Simple description of ghost dark matter

Now let us reconsider the simple Lagrangian (2.2) which depends on X only. As we have already seen in subsection 2.2, we need to add the extra term (2.9) to this Lagrangian in order to describe perturbations around the exact ghost condensate background, i.e. a local minimum of the function $P(X)$. In this subsection we shall instead consider a background with non-vanishing P_X and see that, under a certain condition, the Lagrangian $P(X)$ without the additional term (2.9) can properly describe the background and perturbations around it. In this situation, as we shall see below, deviation from the exact ghost condensate behaves like dark matter.

The stress-energy tensor corresponding to the Lagrangian $P(X)$ is

$$T_{\mu\nu} = 2P_X \partial_\mu \phi \partial_\nu \phi + P g_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}, \tag{3.1}$$

where

$$\rho = 2XP_X - P, \quad u_\mu = -\frac{\partial_\mu \phi}{\sqrt{X}}. \tag{3.2}$$

The sound speed squared for perturbation is [13]

$$c_s^2 = \frac{dP/dX}{d\rho/dX} = \frac{P_X}{2XP_{XX} + P_X}. \tag{3.3}$$

This agrees with minus the ratio of the coefficient of the gradient term to the coefficient of the time kinetic term in the quadratic Lagrangian (2.6).

As we have already seen in subsection 2.2, in order to describe perturbations around the exact ghost condensate background, the simple Lagrangian $P(X)$ is not sufficient but we need to add the term (2.9) to it. The reason is that the coefficient of the would-be leading gradient term $(\vec{\nabla}\pi)^2$ vanishes in the exact ghost condensate background. However, if the background is not exact ghost condensate but has $P_X \neq 0$ then the coefficient of $(\vec{\nabla}\pi)^2$ does not vanish as shown in (3.3). Therefore, if c_s^2 is large enough then we do not have to add the extra term (2.9) to the simple Lagrangian $P(X)$. To be more precise, the extra term (2.9) is not needed if

$$c_s^2 \gg \frac{1}{M^2 L^2}, \tag{3.4}$$

where L is the length scale of interest and we have supposed that $\alpha = O(1)$.

We have also seen that a local minimum of $P(X)$ is a dynamical attractor in the expanding universe. Thus, it is rather natural to Taylor expand $P(X)$ around a local minimum $X = M^4$ as

$$P \simeq P(M^4) + \frac{1}{2} P_{XX}(M^4)(X - M^4)^2. \tag{3.5}$$

This expansion of $P(X)$ is valid if and only if $|X - M^4| \ll M^4$. Note that we anyway have to restrict our consideration to this regime; otherwise, higher dimensional operators, which in general depend on not only X but also $\Box\phi$, Y, Z, etc., would be unsuppressed and the system would exit the regime of validity of the low energy effective theory. (See discussions in subsection 2.3.) Correspondingly, we have the following expansions.

$$\rho \simeq -P(M^4) + 2M^4 P_{XX}(M^4)(X - M^4), \tag{3.6}$$

$$c_s^2 \simeq \frac{X - M^4}{2M^4}. \tag{3.7}$$

These expressions are rewritten as

$$\rho \simeq \rho_{\rm gde} + \rho_{\rm gdm}, \quad P \simeq P_{\rm gde} + P_{\rm gdm}, \quad c_s^2 \simeq \frac{\rho_{\rm gdm}}{\bar{M}^4} \tag{3.8}$$

where

$$P_{\rm gde} = -\rho_{\rm gde} = {\rm const.}, \quad P_{\rm gdm} = \frac{\rho_{\rm gdm}^2}{2\bar{M}^4}. \tag{3.9}$$

Here, $\bar{M}^4 \equiv 4M^8 P_{XX}(M^4) \sim M^4$. In the regime $|X - M^4| \ll M^4$, it is intriguing to note that $(\rho_{\rm gde}, P_{\rm gde})$ and $(\rho_{\rm gdm}, P_{\rm gdm})$ behave like dark energy and dark matter, respectively. In particular, we shall call the latter *ghost dark matter* [12].

In terms of ghost dark matter component, the necessary condition (3.4) for the validity of the simple Lagrangian $P(X)$ is written as

$$M \ll L\rho_{\rm gdm}^{1/2} \simeq 10^{15}{\rm GeV} \times \left(\frac{L}{1{\rm Mpc}}\right) \cdot \left(\frac{\rho_{\rm gdm}}{0.3 \times \rho_0}\right)^{1/2}, \tag{3.10}$$

where $\rho_0 \equiv 3M_{\rm Pl}^2 H_0^2$ is the critical density. For example, if $M \ll 10^{15}$ GeV and if we suppose that the ghost dark matter is responsible for all dark matter in the universe ($\rho_{\rm gdm} \simeq 0.3 \times \rho_0$) then (3.10) is satisfied for length scales longer than ~ 1 Mpc. Note, however, that validity of low energy effective theory requires that all associated energies, momenta and amplitudes be sufficiently lower than unity in the unit of M. Thus, (3.8) and (3.9) are valid only if

$$\rho_{\rm gdm} \ll M^4, \quad H \ll M. \tag{3.11}$$

On the other hand, we do not have to require that the radiation temperature be lower than M since interactions between ghost condensate and radiation are highly suppressed (typically by the Planck scale).

3.2 Ghost dark matter production from ghost inflation

In the previous subsection we have seen that the background evolution and the behavior of perturbations of ghost dark matter can be described by (3.8) and (3.9) under the conditions (3.10) and (3.11), where L is the length scale for perturbations of interest and $\bar{M} \sim M$. If (3.10) is not satisfied then the use of the simple Lagrangian of the form $P(X)$ is not justified and we need to take into account effects of the extra term (2.9). This is not a big problem but would make analysis slightly complicated [14]. Fortunately, for the purpose of the present paper, i.e. for understanding of the evolution of the FRW background and the large-scale structure of the universe, we are interested in L of order 1 Mpc or longer. In this case, the condition (3.10) always holds if the phenomenological upper bound $M \lesssim 100$ GeV (see subsection 2.5) is satisfied and if we suppose that a non-trivial fraction of dark matter of the universe is ghost dark matter. On the other hand, if (3.11) is not met then the system exits the regime of validity of the low energy effective theory and we need a ultraviolet (UV) completion [3] to describe the system.

This naturally leads to the question "what happens in the early universe?" The condition (3.11) does not hold in the very early epoch of the universe. In this early epoch the sector including ghost condensation should be governed by a theory more fundamental than what we have been describing since M is the energy scale above which a new physics kicks in. While it is important to seek a UV completion to describe this epoch properly, it is also plausible to consider cosmological scenarios in which all interesting observables are predicted within the regime of validity of the low energy effective theory. As a possible realization of such scenarios let us consider a generation mechanism of ghost dark matter at the end of ghost inflation [21].

In ghost inflation the scalar field ϕ responsible for ghost condensation plays the role of inflaton as well. For example we can consider a hybrid inflation-type implementation. In this case we suppose that the mass squared m_χ^2 of a water-fall field χ depends on ϕ in such a way that $m_\chi^2(\phi)$ is positive and constant for $\phi \ll -\phi_*$, and negative and constant for $\phi \gg \phi_*$, where ϕ_* is a positive constant. This setup is technically natural since the shift symmetry is broken only in the vicinity of the transition region $|\phi| \lesssim \phi_*$ and otherwise exact. We suppose that $\partial_t\phi > 0$ so that the sign of m_χ^2 changes from positive to negative.

For $\phi \ll -\phi_*$, ϕ enjoys the shift symmetry and χ has a positive mass squared. Thus the system has a de Sitter attractor with $\partial_t\phi = M^2$ and $\chi = 0$, where $X = M^4$ is a local minimum of $P(X)$. We suppose that the system settles in this state well before ϕ crosses the transition region. Noting that the fluctuation $\delta\phi$ of ϕ has the scaling dimension 1/4 (see subsection 2.3) and the mass dimension 1, we can easily estimate the amplitude of quantum fluctuations of ϕ as

$$\delta\phi \sim M \left(\frac{H_{\rm inf}}{M}\right)^{1/4}, \tag{3.12}$$

[3]See [15, 16, 17, 18] for some attempts towards possible scenarios of UV completion, and [19, 20] for compatibility with the generalized second law of black hole thermodynamics.

where $H_{\rm inf}$ is the Hubble expansion rate of the de Sitter attractor. As usual, quantum fluctuations of ϕ is eventually converted to temperature anisotropies as

$$\frac{\delta T}{T} \sim \frac{H_{\rm inf}\delta\phi}{\partial_t \phi}. \tag{3.13}$$

Thus, we obtain

$$\frac{\delta T}{T} \sim \left(\frac{H_{\rm inf}}{M}\right)^{5/4}. \tag{3.14}$$

By requiring that this is responsible for the observed amplitude of temperature anisotropies $\delta T/T \sim 10^{-5}$, $H_{\rm inf}$ is determined as

$$H_{\rm inf} \simeq 10^{-4} \times M. \tag{3.15}$$

Under this condition, one can also estimate non-Gaussian features of CMB anisotropies. The shape of the bispectrum is of the equilateral type and the nonlinear parameter f_{NL} is of order ~ 80 if we set all dimensionless parameters to unity [21]. The essential reason for the relatively large non-Gaussianities is that the leading nonlinear operator has the scaling dimension $1/4$ and thus is less suppressed than in usual slow-roll inflation.

The condition (3.15) shows that ghost inflation is well within the regime of validity of the low energy effective theory: $H_{\rm inf} \ll M$. The condition (3.15) also shows that there is a lower bound on M in terms of the reheating temperature $T_{\rm reh}$:

$$M \gtrsim 10^4 \times \frac{T_{\rm reh}^2}{M_{\rm Pl}}. \tag{3.16}$$

Ghost inflation can generate not only temperature anisotropies observed in the CMB but also ghost dark matter. In the hybrid inflation-type implementation, the shift symmetry is broken and a potential for ϕ should be generated by quantum corrections in and only in the vicinity of the transition region $|\phi| \lesssim \phi_*$. Correspondingly, the equation of motion for homogeneous ϕ is

$$\frac{1}{a^3}\partial_t \left[a^3 P_X \partial_t \phi\right] + V'_{\rm gen}(\phi) = 0, \tag{3.17}$$

where $V_{\rm gen}(\phi)$ is the potential for ϕ generated by quantum corrections. Note that it is not $V_{\rm gen}(\phi)$ but a potential for χ that is responsible for most of the potential energy during the inflationary phase:

$$|\Delta V_{\rm gen}| \ll 3M_{\rm Pl}^2 H_{\rm inf}^2, \tag{3.18}$$

where $\Delta V_{\rm gen} \equiv V_{\rm gen}^{\rm before} - V_{\rm gen}^{\rm after}$, and the superscripts 'before' and 'after' represent values before and after the transition. Noting that $\partial_t\phi \simeq M^2$ and ignoring the

Hubble friction term, it is easy to integrate (3.17) over the transition region. The result is

$$P_X^{\mathrm{after}} \simeq \frac{\Delta V_{\mathrm{gen}}}{M^4}. \tag{3.19}$$

This gives the amplitude of ghost dark matter right after the transition as

$$\rho_{\mathrm{gdm}}^{\mathrm{after}} \sim \Delta V_{\mathrm{gen}}. \tag{3.20}$$

Ghost dark matter does not dominate the universe just after the end of ghost inflation as easily seen from (3.18). However, since it evolves more slowly than radiation, it can dominate the late time universe.

In obtaining the estimate (3.19) we have ignored the Hubble friction. This is justified if the transition timescale, $\Delta t \sim \phi_*/\partial_t\phi \simeq \phi_*/M^2$, is short compared with the Hubble timescale $H_{\mathrm{inf}}^{-1} \simeq 10^4/M$, i.e. if

$$\phi_* \ll 10^4 \times M. \tag{3.21}$$

3.3 Phenomenological constraint from background evolution

While it is certainly interesting and important to investigate concrete cosmological scenarios such as one presented in the previous subsection, it is also important to constrain the ghost dark matter in a model independent way. This subsection and the next section are devoted to this subject, assuming that ghost dark matter is responsible for all dark matter in the universe.

We now consider background evolution from the radiation-matter equality to the present time to give a lower bound on M. In this epoch we know from observations that the background dark matter component behaves like pressure-less dust. Thus, if all dark matter in the universe is ghost dark matter then $P_{\mathrm{gdm}}/\rho_{\mathrm{gdm}} \sim \rho_{\mathrm{gdm}}/M^4$ must be sufficiently lower than unity. (This condition incidentally agrees with the first inequality in (3.11).) In this case ρ_{gdm} behaves as

$$\rho_{\mathrm{gdm}} \simeq \Omega_{\mathrm{gdm}}\rho_0 \cdot \left(\frac{a}{a_0}\right)^{-3}. \tag{3.22}$$

We set $\Omega_{\mathrm{gdm}} \simeq 0.3$ by the assumption that ghost dark matter is responsible for all dark matter in the universe. Hence, by requiring that $\rho_{\mathrm{gdm}} \ll M^4$ all the way up the matter-radiation equality $a \simeq 10^{-4}a_0$, we obtain the lower bound

$$M \gtrsim 1 \text{ eV}. \tag{3.23}$$

4. Large-scale structure with ghost dark matter

In this section, we calculate the evolution of density perturbations in the ΛGDM universe, where the sound speed for matter perturbation, c_s, is given by Eq. (3.7) unlike the standard ΛCDM universe. Then, we consider the bound on M in order not to conflict with current observations of large-scale structure.

4.1 Jeans wavenumber for ghost dark matter

In subsection 3.3, we obtained the lower bound on M as $M \gtrsim 1$ eV by demanding that $P_{\rm gdm}/\rho_{\rm gdm} \ll 1$ all the way up to the matter-radiation equality. The condition $M \gtrsim 1$ eV also justifies the use of the simplified description given in subsection 3.1 for the matter dominated era since $P_{\rm gdm}/\rho_{\rm gdm} \ll 1$ is equivalent to $\rho_{\rm gdm} \ll M^4$.

We consider the evolution of the matter perturbations in this situation. In this subsection, before detailed numerical analysis in the next subsection, let us foresee that a bound stronger than $M \gtrsim 1$ eV can be obtained.

For the sound speed given by Eq. (3.7), we can define the Jeans wavenumber as

$$k_J = \sqrt{\frac{3}{2}}\frac{aH}{c_s} \ . \tag{4.1}$$

As is well-known, the matter perturbations do not evolve very much during radiation dominated era and we need not to consider the effects of the Jeans scales. However, after the matter-radiation equality we have to take into account the effects of the Jeans scales, namely, the perturbations with scales shorter than the Jeans scales can not grow. From Eq. (4.1), the Jeans wavenumber evolves as $k_J \propto a$ during the matter dominated era and hence in the matter dominated era the comoving Jeans scale ($\sim k_J^{-1}$) becomes the largest at the matter-radiation equality. The Jeans wavenumber at the matter-radiation equality is given by

$$k_{J,{\rm eq}} \simeq 1 \times \left(\frac{\Omega_{\rm gdm}h^2}{0.11}\right)^{-5/6} \left(\frac{M}{10{\rm eV}}\right)^{4/3} \ {\rm Mpc}^{-1}. \tag{4.2}$$

The matter power spectrum is expected to be significantly suppressed for modes with wavenumbers $k \gtrsim k_{J,{\rm eq}}$.

Almost all current observational data do not indicate any suppression in the matter power spectrum, roughly for the wavenumber $k/h \lesssim 1$ Mpc^{-1}. Hence, from the above expression, we can roughly obtain a constraint on the model parameter M as $M \gtrsim 10$ eV. In the next subsection, we shall confirm this bound by numerically calculating the matter power spectrum.

4.2 Numerical calculation

4.2.1 Field description and background evolution

In the numerical calculation, it will be practically difficult to use Eqs. (3.6)-(3.9) because the sound speed of GDM increases beyond the speed of light as we go back in time. (Of course, this is not a physical problem but just a breakdown of the description.) Therefore, in the following, we shall solve X assuming a working Lagrangian to avoid this difficulty and follow the time evolution in a numerically stable way, instead of solving Eqs. (3.6)-(3.9). Let us consider a model in which

$P(X)$ is given by

$$P(X) = \frac{1}{8M^4}(X - M^4)^2 \ . \tag{4.3}$$

In this case $\bar{M} = M$. The time evolution of X is given by

$$\frac{dX}{d\ln a} = \frac{6X(X - M^4)}{3X - M^4} \ , \tag{4.4}$$

and the energy density, pressure, and sound speed of GDM are given by

$$\rho_{\rm gdm} = \frac{1}{8M^4}(3X^2 - 2XM^4 - M^8) \ , \tag{4.5}$$

$$P_{\rm gdm} = \frac{1}{8M^4}(X - M^4)^2 \ , \tag{4.6}$$

$$c_s^2 = \frac{X - M^4}{3X - M^4} \ , \tag{4.7}$$

respectively. The initial condition for the value of $X (= X_0)$ is fixed at present demanding that GDM component should be accounting for the dark matter component observed today:

$$\Omega_{\rm gdm} = \frac{1}{24M_{\rm Pl}^2 H_0^2 M^4}(3X_0^2 - 2X_0 M^4 - M^8) = 0.3 \tag{4.8}$$

Then the evolution of X can be solved backward in time using Eq.(4.4).

As we have discussed in the previous section, we have to restrict our consideration to the regime $|X - M^4|/M^4 \ll 1$ and in this regime the sound speed evolves as $c_s \propto \sqrt{|X - M^4|} \propto a^{-3/2}$. Hence, if we look back the background evolution in ΛGDM universe, the condition $|X - M^4|/M^4 \ll 1$ might be violated at some point in time. Let us define a critical scale factor when the condition $|X - M^4|/M^4 \ll 1$ is violated as

$$\left|X(a_{\rm cr}) - M^4\right|/M^4 = 2c_s^2(a_{\rm cr}) = 0.1 \ . \tag{4.9}$$

From this equation, the critical scale factor $a_{\rm cr}$ can be obtained in terms of M as

$$a_{\rm cr}/a_0 \simeq 1.0 \times 10^{-4} \left(\frac{\Omega_{\rm gdm} h^2}{0.11}\right)^{1/3} \left(\frac{1.0{\rm eV}}{M}\right)^{4/3} \ . \tag{4.10}$$

Thus, it is found that for $M \gtrsim 1$ eV the critical scale factor $a_{\rm cr}$ is smaller than the scale factor at the matter-radiation equality, $a_{\rm eq}$. In the previous section, we have obtained the lower bound for M from the background evolution as $M > 1$ eV. Hence, if this bound is satisfied, we can use the low energy effective theory, at least, after the matter-radiation equality. Hereinafter, we consider the case for $M > 1$ eV.

This GDM description by X is equivalent to fluid-like description by Eqs. (3.6)-(3.9), when X is near the condensate position, i.e., $|X - M^4| \ll M^4$. However, the sound speed will approach to $c_s^2 = \frac{1}{3}$ in this description and will never exceed the speed of light even if the energy density evolves to infinity as $|X - M^4|/M^4 \gg 1$, as is shown in Fig. 1. The equation of state parameter, $w = P_{\rm gdm}/\rho_{\rm gdm}$, also approaches to 1/3. Therefore we can solve the system in a numerically stable way. It should be noted, however, that this description should be considered only as a regulator of the fluid description considered in the previous section, and it will be invalid when X is far away from the condensate position. Still one needs to interpret the results where $a < a_{\rm cr}$ with caution.

In fact, we have to extend our analysis before $a_{\rm cr}$ because the density perturbations should be solved numerically from the early epoch when the corresponding Fourier modes are outside the cosmic horizon. Therefore, in the analysis in this section we extrapolate the behavior of GDM beyond $a_{\rm cr}$ with a reasonable asuumption that the sound speed of GDM had been satulated as $c_s^2 = 1/3$ for $a < a_{\rm cr}$. This treatment corresponds to an assumption that the density perturbation of GDM can not grow in the radiation dominated era even when the low energy effective theory is broken.

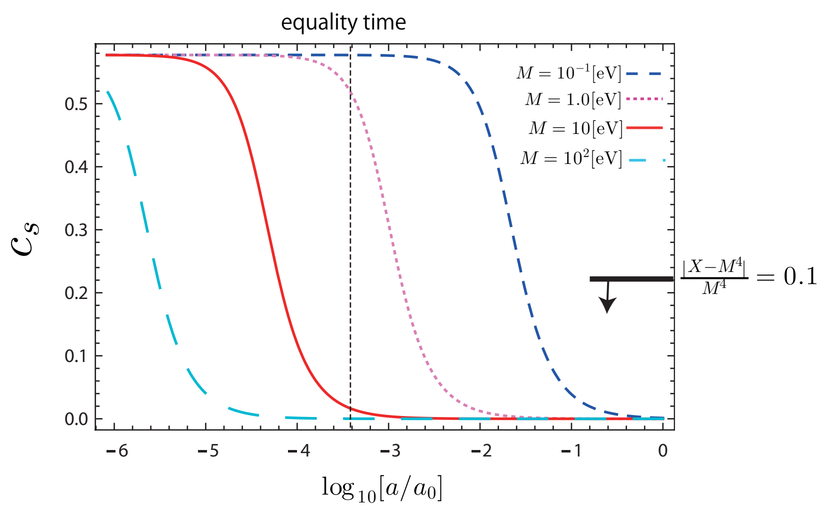

Figure 1: The evolution of the sound speed of GDM for various values of M. The blue short-dashed line is for cosmological constant plus ghost dark matter (ΛGDM) with $M = 10^{-1}$ eV, the magenta dotted line is for the ΛGDM with $M = 1.0$ eV, the red solid line is for the ΛGDM with $M = 10$ eV and the cyan long-dashed line is for the ΛGDM with $M = 10^2$ eV. In all plots we set $\Omega_\Lambda = 0.7$ and $\Omega_{\rm gdm} = 0.3$ (or $\Omega_{\rm cdm} = 0.3$ for the ΛCDM). The vertical black dashed line represents the matter-radiation equality. The downward arrow shows the regime of the validity of the low energy effective theory ($|X - M^4|/M^4 < 0.1$).

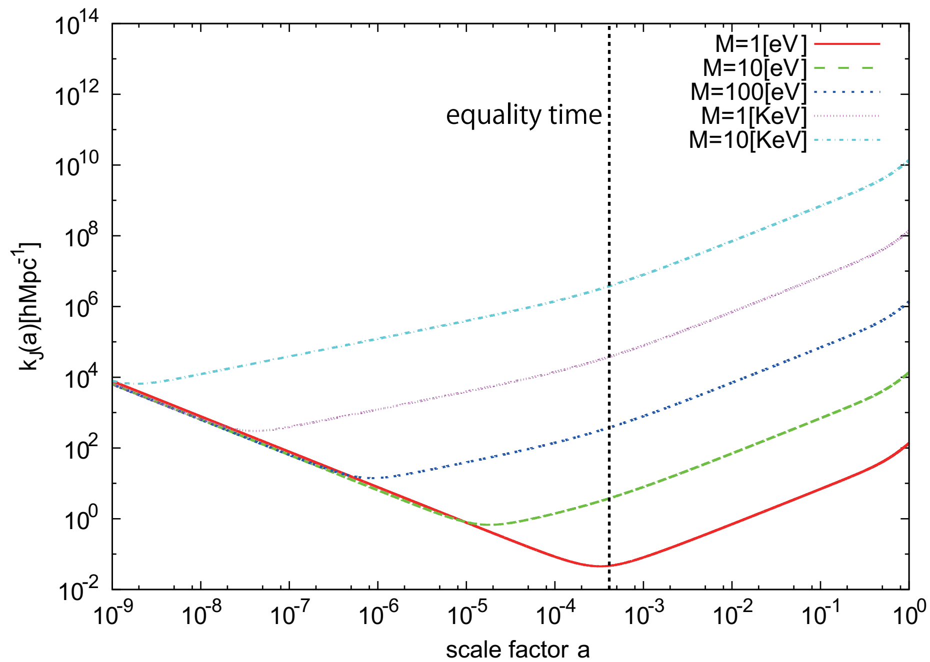

Figure 2: The comoving Jeans wavenumber for various values of M as indicated in the figure. From up to bottom, the lines are for $M = 10, 1\text{keV}, 100, 10, 1$ eV. The vertical dashed line shows the matter-radiation equality time. The turnover of the Jeans wavenumber occurs due to the change of the sound speed of GDM, which roughly shows the boundary of the regime of validity of the low energy effective theory.

The corresponding Jeans wavenumber is plotted in Fig. 2. From the figure, we can see a turnover of the Jeans wavenumber k_J for each M. This turnover of the Jeans wavenumber occurs at the transition of the sound speed of the GDM between $c_s = 1/\sqrt{3}$ and $c_s \propto a^{-3/2}$. The Jeans wavenumber is given by

$$k_{J,\mathrm{turn}} = \frac{3}{\sqrt{2}} a_{\mathrm{turn}} H(a_{\mathrm{turn}}) \;, \tag{4.11}$$

where we denote the scale factor at the turnover epoch by a_{turn}. We can easily find that $a_{\mathrm{turn}} \simeq a_{\mathrm{cr}}$, where a_{cr} is given by Eq. (4.10). For $M \gtrsim 1$ eV, of course, $a_{\mathrm{turn}} \simeq a_{\mathrm{cr}}$ is smaller than a_{eq}.

4.2.2 Perturbation evolution

Let us consider the evolution of the matter perturbations. In synchronous gauge, the growth of density contrast of GDM, δ_{gdm}, is described by the differential equations [22]

$$\delta''_{\mathrm{gdm}} + \left(1 + 3c_s^2 - 6w_{\mathrm{gdm}}\right) \mathcal{H}\delta'_{\mathrm{gdm}}$$
$$+ \Bigg\{ \frac{3}{2}\mathcal{H}^2 (c_s^2 - w_{\mathrm{gdm}})(1 - 6w_{\mathrm{gdm}} - 3\bar{w})$$

$$+c_s^2k^2 + 3\mathcal{H}(c_s^{2\prime} - w'_{\rm gdm})\Big\}\delta_{\rm gdm}$$

$$= -(1+w_{\rm gdm})\frac{h''}{2} - (1+w_{\rm gdm})(1-3w_{\rm gdm})\mathcal{H}\frac{h'}{2} - \frac{w'_{\rm gdm}h'}{2}, \qquad (4.12)$$

$$h'' + \mathcal{H}h' = -3\mathcal{H}^2(1+\bar{c_s}^2)\bar{\delta}, \qquad (4.13)$$

where a prime denotes a derivative with respect to conformal time, $\mathcal{H} = a'/a$, h is the metric perturbation, $\bar{\delta}$, $\bar{c_s}$, $\bar{w}$ are total component's density fluctuation, sound speed and equation of state, respectively, defined as

$$\bar{\delta} = \frac{\sum \rho_i\delta_i}{\sum \rho_i}, \quad \bar{w} = \frac{\sum \rho_i}{\sum p_i}, \quad \bar{c_s^2} = \frac{\sum \dot{\rho}_i c_{s,i}^2}{\sum \dot{\rho}_i}\ . \qquad (4.14)$$

The evolution of perturbations in the sound horizon depends on the time dependence of the sound speed. In the present model the sound speed is $c_s^2 = 1/3$ in the very early universe, and $c_s^2 \propto a^{-3/2}$ after some critical epoch. Therefore, the perturbation evolution in the sound horizon is divided into two cases. They are; case (a): $w_{\rm gdm} = c_s^2 = 1/3$, and case (b): $w_{\rm gdm}, c_s \ll 1$.

Note that the case (a) may not be a correct description of GDM scenario in the sense that the low energy effective theory is broken at large redshifts where $a < a_{\rm cr} (\lesssim a_{\rm eq})$. However, it would be natural to expect that the density perturbation in GDM can not grow very much if the sound speed of GDM had been saturated as $c_s \sim O(1)$. This situation would be effectively taken into account by setting $c_s^2 = \frac{1}{3}$ because in this case the density perturbation can not grow at subhorizon scales. Therefore we shall assume case (a) in the following analysis to solve the perturbations in GDM for the energy scales in which the effective theory can not be applied.

- **case (a):** $w_{\rm gdm} = c_s^2 = 1/3$

In this case, $c'_s, w'_{\rm gdm} = 0$. So, differential equations(Eq. (4.12), Eq. (4.13)) are reduced to

$$\delta''_{\rm gdm} + c_s^2k^2\delta_{\rm gdm} = -\frac{2}{3}h'', \qquad (4.15)$$

$$h'' + \mathcal{H}h' = -4\mathcal{H}^2\bar{\delta}. \qquad (4.16)$$

Roughly speaking, $h'' \propto \mathcal{H}^2\bar{\delta}$ from the second equation, and using the fact that $\mathcal{H} \ll c_s k$ in sound horizon, the first equation becomes

$$\delta''_{\rm gdm} + c_s^2k^2\delta_{\rm gdm} = 0. \qquad (4.17)$$

Therefore, the density perturbation of GDM in this case cannot grow and only oscillate.

• **case (b) :** $w_{\rm gdm}, c_s \ll 1$

In this case, $c_s' = -\frac{3}{2}\mathcal{H}c_s$ and $w_{\rm gdm}' = -3\mathcal{H}w_{\rm gdm}$. So, Eq. (4.12) becomes

$$\delta_{\rm gdm}'' + \mathcal{H}\delta_{\rm gdm}' + \left(c_s^2 k^2 - \frac{3}{2}\mathcal{H}^2\right)\delta_{\rm gdm} = 0. \tag{4.18}$$

Since $\mathcal{H}'(\eta) \ll c_s'(\eta)k$, we use WKB approximation

$$\delta_{\rm gdm} = A(\eta)\exp\left(i\int_{\eta_0}^{\eta} c_s k d\eta\right), \tag{4.19}$$

to solve this equation. In this approximation, $A'(\eta)$ is roughly in proportion to $\mathcal{H}$ and $c_s k \gg \mathcal{H}$. So the term proportional to $\mathcal{H}^2$ can be neglected. Then, the equation becomes

$$\left(2c_s A' + c_s' A + c_s \mathcal{H} A\right) ik \exp\left(i\int_{\eta_0}^{\eta} c_s k d\eta\right) = 0. \tag{4.20}$$

Then we obtain $A \propto a^{1/4}$ since $c_s' = -\frac{3}{2}\mathcal{H}c_s$. Thus the growth rate of GDM for case (b) is

$$\delta \propto a^{1/4}, \tag{4.21}$$

in the sound horizon. Therefore, the density perturbation of GDM for case (b) gradually *grows* in the sound horizon. The above consideration can be generalized in the case where $c_{s,} \propto a^{-n} (n \neq 0)$. In this case, $c_s' = -n\mathcal{H}c_s$, so the evolution of perturbation in sound horizon is

$$\delta \propto a^{(n-1)/2}, \tag{4.22}$$

which is derived from Eq. (4.20).

The evolution of GDM density perturbations in ΛGDM cosmology can be divided into four cases by comparing the wavelength of the modes and the Jeans length. Let us define $a_{\rm J}(k)$ as a scale factor when a mode becomes larger than the Jeans scale, namely, when the mode exits the sound horizon. We define the four cases from the smallest scale to the largest one, depending on $a_{\rm J}(k)$ and the matter-radiation equality time denoted by $a_{\rm eq}$ as follows;

• **case I** ($k < k_{J,\rm turn}$)**:**

the modes of interest never cross the sound horizon during the cosmic history,

• **case II** ($k_{J,\rm eq} > k > k_{J,\rm turn}$)**:**

the modes of interest exit the sound horizon before the matter-radiation equality,

• **case III** ($a_0 > a_{\rm J}(k)$; $k > k_{J,\rm eq}$)**:**

the modes of interest exit the sound horizon after the matter-radiation equality,

• **case IV** ($a_{\rm J}(k) > a_0$; $k > k_{J,{\rm eq}}$):

the modes do not exit the sound horizon until today.

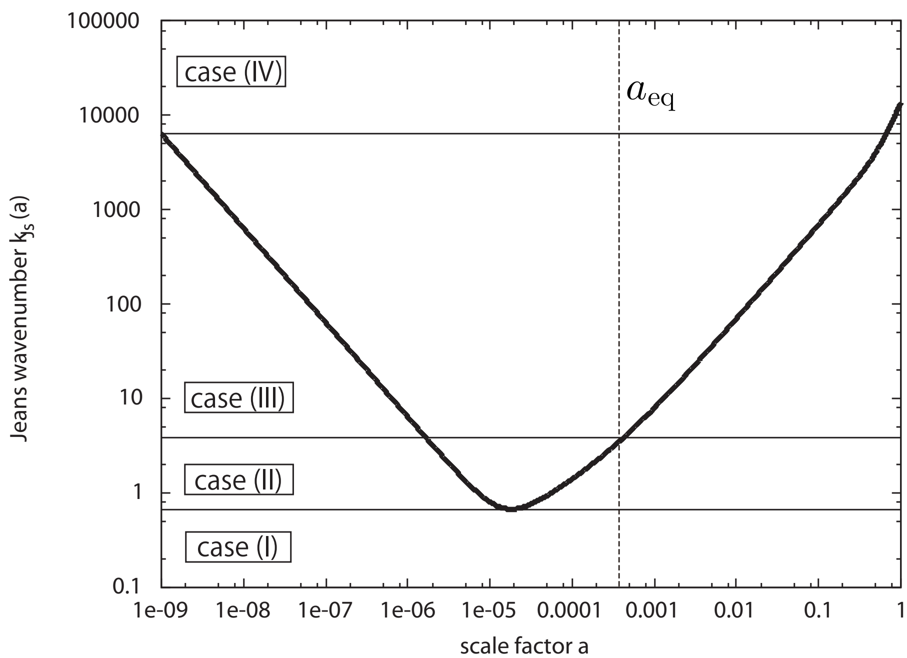

Figure 3: Four cases (case I - IV) divided for the Jeans wavenumber for the ΛGDM with $M = 10$eV due to the difference of the evolution of the perturbation. Thick solid line shows the evolution of the Jeans wavenumber, vertical thin dashed line shows the matter-radiation equality.

For example, Fig.3 shows typical wavenumbers of four cases for the ΛGDM with $M = 10$ eV. If the mode belongs to the case I, the evolution of the mode is similar to that in the ordinary CDM one because the mode is always outside the sound horizon. However, if the modes belong to the cases II - IV, the modes enter the sound horizon at some epochs, where the perturbation evolution of GDM becomes different from that of CDM. Then, let us consider how the modes entering the sound horizon of GDM can be suppressed and estimate the k-dependence of the matter power spectrum. In the $M \geq 1$eV models, the small scale perturbations cross the sound horizon in the radiation dominated era as shown in Fig.2. However, the time of exiting the sound horizon dependends on the wavenumber of the mode k and also the cut-off scale M as mentioned above.

From these considerations, one can estimate the power spectrum of GDM model. In the following, for simplicity, we neglect the phase of oscillations in the sound horizon. In fact, it is found to be important quantitatively and we will include this effect in the numerical calculation which will be presented later in order to calculate evolution of perturbations and the matter power spectrum more precisely. We approximate $\delta_{\rm gdm} \propto a$ and $\propto \log a$ outside the sound horizon in the matter and radiation dominated era ($a_{\rm cr} < a < a_{\rm eq}$), respectively. Because of the fact that $\delta_{\rm gdm} \propto a^{1/4}$ inside the sound horizon when $a \gtrsim a_{\rm cr}$, we find

$$\delta_{\rm gdm}(a_0) \sim \begin{cases} \delta_{\rm cdm}(a_0) & \text{for} \quad k < k_{J,\rm turn} \\ (a_{\rm J}/a_{\rm cr})^{1/4} \log\left(a_{\rm eq}/a_J\right)(a_0/a_{\rm eq})\,\delta_{\rm gdm}(a_{\rm hc}) & \text{for} \quad k_{J,\rm eq} > k > k_{J,\rm turn} \\ (a_{\rm J}/a_{\rm cr})^{1/4}\,(a_0/a_J)\,\delta_{\rm gdm}(a_{\rm hc}) & \text{for} \quad a_0 > a_{\rm J}(k);\ k > k_{J,\rm eq} \\ (a_0/a_{\rm cr})^{1/4}\,\delta_{\rm gdm}(a_{\rm hc}) & \text{for} \quad a_{\rm J}(k) > a_0;\ k > k_{J,\rm eq} \end{cases} \tag{4.23}$$

where we have introduced $a_{\rm hc}$ as the scale factor when the modes of interest enter in the Hubble horizon. In these equations, only $a_{\rm J}$ has k dependence, namely, $a_{\rm J} \propto k^2$ in the radiation era and $a_{\rm J} \propto k$ in the matter era. Meanwhile, in the standard cold dark matter cosmology, we find $\delta_{\rm cdm}(a_0) \sim \log(a_{\rm eq}/a_{hc})\,(a_0/a_{\rm eq})\,\delta_{\rm gdm}(a_{\rm hc})$. Therefore, the k dependence of the GDM power spectrum at present can be found to be

$$\frac{P_{\rm gdm}(k)}{P_{\rm cdm}(k)} \propto \begin{cases} 1 & \text{for} \quad k < k_{J,\rm turn} \\ (\log k)^{-2} & \text{for} \quad k_{J,\rm eq} > k > k_{J,\rm turn} \\ k^{-2} & \text{for} \quad a_0 > a_{\rm J}(k);\ k > k_{J,\rm eq} \\ (\log k)^{-2} & \text{for} \quad a_{\rm J}(k) > a_0;\ k > k_{J,\rm eq} \end{cases} . \tag{4.24}$$

where we have used $x^{-1/4}\log(x) \approx O(1)$ for $x > 1$.

The analysis so far is based on the rough estimate and only appropriate for qualititative understanding. In order to evaluate the perturbation amplitude quantitatively and take into account the effects neglected in the above analysis, we calculate the evolution numerically using the modified CAMB code [23].

In Fig.4, we depict the evolutions of density perturbations of GDM with different wavenumbers ($k = 0.1, 1, 10, 100$ Mpc^{-1}) for two different model parameters ($M =$ 20 eV and 100 eV). We also depict those of the baryon density for comparison. At largest scales (top panels) the evolution of GDM density perturbations is almost identical to that of standard CDM, because the mode is always outside the sound horizon. On the other hand, the evolutions of the modes at small scales are depicted in the third and fourth panels. In these panels the amplitude of GDM exhibits oscillations when the perturbations are inside the sound horizon. The growth of density perturbation of GDM inside the sound horizon for $w_{\rm gdm}, c_s^2 \ll 1$ is clearly seen, which has a dependence of $a^{1/4}$ as derived in our analytic estimate. We need, however, to see the second-top left panel ($k = 1$ Mpc^{-1} and $M = 20$ eV) with care. The panel shows the marginal case where the mode exits the sound horizon immediately after entering the horizon. In this case, we observe that the amplitude of $\delta_{\rm gdm}$ does not experience any logarithmic growth which is expected for the CDM case. We found that this brings a large difference in amplitudes at the present universe about an order of magnitude, which is seen by comparing with the second-top right panel.

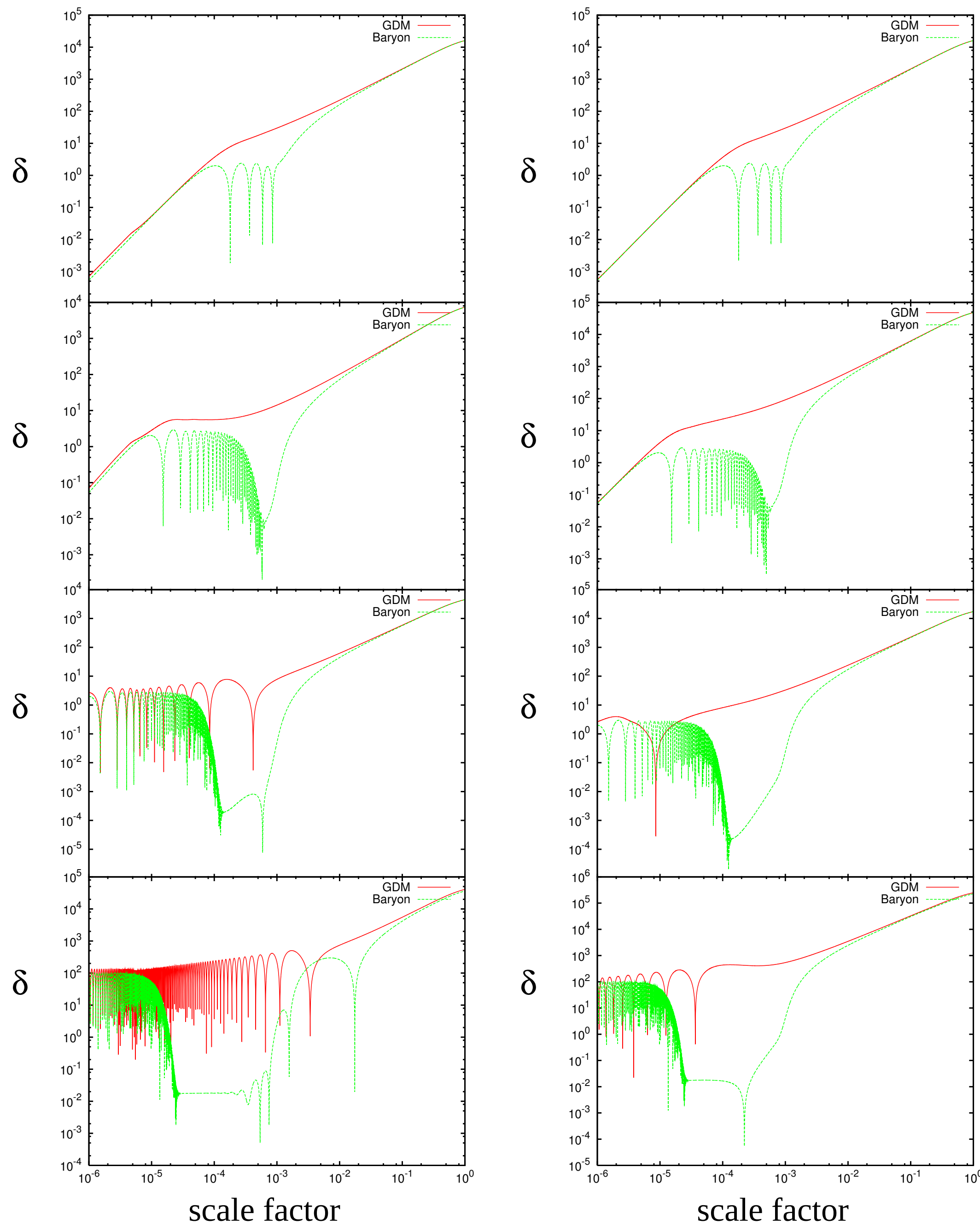

Figure 4: The evolution of density perturbations of GDM and baryon fluid in Ghost condensation model. Left panels are for $M = 20$ eV model, right panels are for $M = 100$ eV model. The panels from top to bottom are for $k = 0.1, 1, 10, 100$ Mpc^{-1}.

The net effect in the matter power spectrum is the *deficit* of power for $k \gtrsim k_{J,\rm turn}$, which is shown in Fig.5. From this figure, we can find that the matter power spectrum alters not at $k_{J,\rm eq}$ as discussed in Sec.4.1, but at $k_{J,\rm turn}$. That is because we have extended the analysis beyond the critical epoch $a_{\rm cr}$ in our numerical calculations, by extrapolating the behavior of GDM beyond $a_{\rm cr}$ with a reasonable assumption that the sound speed of GDM had been satulated as $c_s^2 = 1/3$. In this case the density perturbation of GDM only oscillates inside the sound horizon and does not experience any logarithmic growth during the radiation dominated era which is expected in the ΛCDM model. The power deficit at $k \gtrsim k_{J,\rm eq}$ is more reasonable because it is a result derived mainly from the perturbation evolution at the subhorizon scales at the regime where the effective low-energy theory is applicable, i.e., $(X - M^4)/M^4 < 1$ or $a > a_{\rm cr}$, under a simple assumption that the density perturbation of GDM can not grow deep in the radiation dominated era, even when the low energy effective theory is broken.

The evolution of the baryon density perturbation is affected only through gravity. In the standard cosmology, it is known that after the fluctuation is erased by diffusion damping the baryon fluctuation should start to grow in time again with terminal velocity falling into the gravitational potential of clustered dark matter [24]. In the GDM model, however, the baryon fluid can not have terminal velocity because the dark matter density fluctuation oscillates in the sound horizon and hence the gravitational potential. As the GDM ceases oscillating, the amplitude of the density fluctuation of baryon fluid starts growing with oscillation, which is clearly seen in the left bottom panel. In any case, the baryon density fluctuation quickly catches up the dark matter density one after decoupling.

The matter power spectrum of our universe can be probed through various cosmological observations, such as clustering of galaxies [25], cosmic shear [27, 26, 28], Lyman-α forest [29, 30, 31], and so on. At present, almost all observational data support the CDM paradigm, roughly for the wavenumber $k/h \lesssim 1\ \mathrm{Mpc}^{-1}$. In the models considered in this paper, this observational fact gives us a constraint on the model parameter M. The constraint is derived through the suppression of the matter power spectrum, in the same way to obtain the constraints on the hot and/or warm dark matter models (or in other words, masses of neutrinos and/or warm dark matter particles) [32]. By looking at the matter power spectrum obtained in the present analysis (Fig.5), we conclude that the model parameter should be

$$M \gtrsim 10\ \mathrm{eV}\ , \tag{4.25}$$

which is a stronger constraint than that obtained only from the background evolution considered in Sec. 3. This result is consistent with the rough analytic estimate given by Eq. (4.2).

Finally we should note that the way of suppression in the GDM models is different from that in hot or warm dark matter models. In the GDM models the suppression is the power law while for hot and warm models the power is exponentially suppressed. This fact might be used to distinguish between GDM and other dark matter models.

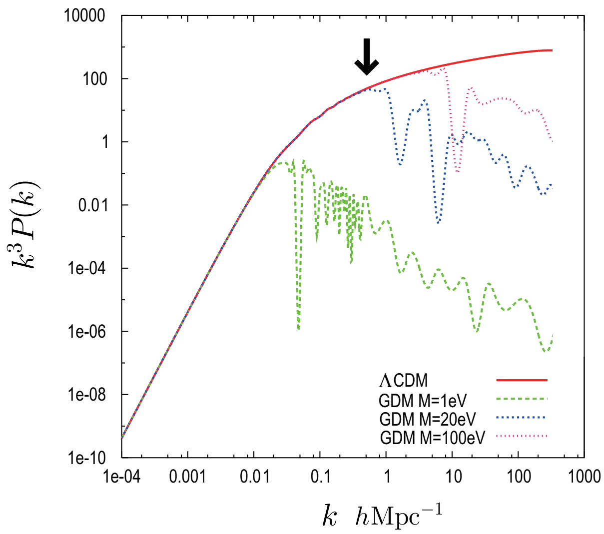

Figure 5: The matter power spectrum for GDM model. Red solid line is a matter power spectrum of ΛCDM model. The GDM power spectra are plotted for three models : $M =$ 1 eV (green dashed line), $M = 20$ eV (blue short-dashed line), $M = 100$ eV (pink dotted line). For modes $k > k_{J,\mathrm{turn}}$ (shown by a down arrow for $M = 20$ eV), the power spectrum of GDM model is suppressed compared to that of ΛCDM model.

5. Summary and discussion

We have investigated the possibility that ghost condensation may serve as an alternative to dark matter, which had not been investigated in detail. In the present paper we have considered the ΛGDM universe, i.e. a late-time universe dominated by a cosmological constant and ghost dark matter. We have investigated the Friedmann-Robertson-Walker (FRW) background evolution and the large-scale structure of the ΛGDM universe, and have found a lower bound on the scale of spontaneous Lorentz breaking as $M \gtrsim 10$ eV. This bound is compatible with the phenomenological upper bound $M \lesssim 100$ GeV known in the literature.

As we have reviewed in Sec. 2, ghost condensation is the simplest Higgs mechanism for gravity in the sense that the number of Nambu-Goldstone boson is only one. The structure of the low-energy effective field theory is completely determined by the symmetry breaking pattern and this makes it possible for us to give robust predictions of the theory as far as the system is in the regime of validity of the effective theory. In Sec. 2 we have also reviewed the infrared modification of linearized gravity as well as some non-linear dynamics and the phenomenological lower bound mentioned above.

In Sec. 3 we have provided a simple description of ghost dark matter and investigated the FRW background evolution. In subsection 3.1 we have shown that, under a certain condition, the background evolution and the behavior of large-scale perturbations of ghost dark matter can be described by a fluid with the equation of state $P_{\rm gdm} \propto \rho_{\rm gdm}^2/M^4$. This description has been used throughout this paper, except for numerical studies where the field picture turns out to be more convenient for some technical reasons. In subsection 3.2, as a possible cosmological scenario relating the late time evolution of ghost dark matter to the early universe, we have considered a generation mechanism of ghost dark matter at the end of ghost inflation [21]. In subsection 3.3 we have shown that the background FRW evolution in the ΛGDM universe is indistinguishable from that in the standard ΛCDM universe if $M \gtrsim 1$ eV.

In Sec.4 we have investigated the large-scale structure of the ΛGDM universe. Since the GDM has the effective sound speed unlike the standard CDM, small scale perturbations are suppressed. The suppression of the matter power spectrum occurs in a way different from that in models with hot dark matter or warm dark matter particles. We have given an analytic treatment to predict the matter power spectrum observed today and have also calculated the power spectrum numerically. The analytic treatment can be extended for general dark matter models in which dark matter has a finite sound speed whose time dependence is the power law of scale factor. By comparing the GDM power spectrum with the observed one, one can get a lower bound on the scale M. The result we obtained is $M \gtrsim 10$ eV.

The constraint obtained in this paper can be improved further by observations of the matter power spectrum at smaller scales. For example, 21cm-line observations and/or the precise determination of the reionization epoch will provide us plenty of information about the matter power spectrum at smaller scales and hence stronger limits on the ghost condensation scale M. We leave these interesting subjects for future investigation.

As suggested in [11], ghost condensation may provide an alternative explanation for the acceleration of the present universe if $M \sim 10^{-3}$ eV. If the cosmological constant in the symmetric phase (with $\dot{\phi} = 0$) is zero then the effective cosmological constant in the broken phase, i.e. the ghost condensate, (with $\dot{\phi} = M^2$) is positive [4] and of order $O(M^4/M_{\rm Pl}^2)$ unless fine-tuned. Unfortunately, the condition $M \sim 10^{-3}$ eV is not compatible with the lower bound on M found in the present paper under the assumption that ghost dark matter is responsible for all dark matter in the universe. Therefore, it is not easy for the ghost condensate to be a simultaneous alternative to dark energy and dark matter unless fine-tuned.

On the other hand, as shown in subsection 3.2, ghost inflation is compatible with ghost dark matter. Ghost dark matter is naturally produced at the end of ghost inflation. Moreover, the lower bound (3.16) from ghost inflation can be satisfied simultaneously with not only the phenomenological upper bound (2.29) but also the bound from ghost dark matter. Detailed investigation of the combination of ghost inflation and ghost dark matter is certainly worthwhile as a future work.

[4]Note that in the standard Higgs mechanism, the effective cosmological constant in the broken phase would be negative if the cosmological constant in the symmetric phase is zero.

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