Gravitation and inertia; a rearrangement of vacuum in gravity

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We address the gravitation and inertia in the framework of *general gauge* principle (GGP) which accounts for *gravitation gauge group* G_R generated by hidden local internal symmetry implemented on the flat space. Following the method of phenomenological Lagrangians, we connect the group G_R to nonlinear realization of the Lie group of *distortion* G_D of local internal properties of six-dimensional flat space M_6, which is assumed as a toy model underlying four-dimensional Minkowski space. We study geometrical structure of the space of parameters and derive the Maurer-Cartan's structure equations. We treat distortion fields as Goldstone fields, to which the metric and connection are related, and infer the group invariants and calculate conserved currents. The agreement between proposed gravitational theory and available observational verifications is satisfactory. Unlike the GR, this theory is free of fictitious forces, which prompts us to address separately the inertia from a novel view point. We construct relativistic field theory of inertia, which treats inertia as distortion of local internal properties of flat space M_2 conducted under the *distortion inertial fields*. We derive the *relativistic law of inertia* (RLI) and calculate inertial force acting on the photon in gravitating system. In spite of totally different and independent physical sources of gravitation and inertia, the RLI furnishes justification for introduction of the Principle of Equivalence. Particular attention is given to realization of the group G_R by the hidden local internal symmetry of abelian group $U^{loc} = U(1)_Y \times diag[SU(2)]$ implemented on the space M_6. This group has two generators, third component T^3 of isospin and hypercharge Y implying $Q^d = T^3 + Y/2$, where Q^d is the *distortion charge* operator assigning the number -1 to particles, but +1 to anti-particles. This entails two neutral gauge bosons that coupled to T^3 and Y. We address the rearrangement of vacuum state in gravity resulting from these ideas. The neutral complex Higgs scalar breaks the vacuum symmetry leaving the gravitation subgroup intact. The resulting massive distortion field component may cause an additional change of properties of spacetime continuum at huge energies above the threshold value.

Keywords: Modified theories of gravity, Spontaneous breaking of symmetries, Field theory of inertia, Principle of Equivalence, A rearrangement of vacuum

I. INTRODUCTION

More than four centuries passed since the famous far-reaching discovery of Galileo (in $1602-1604$) that all bodies fall at the same rate [1], which led to an early empirical version of suggestion that gravitation and inertia may somehow result from a single mechanism. Besides describing these early gravitational experiments, Newton in *Principia Mathematica* [2] has proposed a comprehensive approach to studying the relation between the gravitational and inertial masses of a body. Ever since there is an ongoing quest to understand the reason for the equality of gravitational and inertial forces, which remains an intractable mystery. From its historical development this can be regarded as furnishing immediate support for the Principle of Equivalence asserted by Einstein for General Relativity (GR), which preserves the idea of relativity of all kinds of motion. Currently, the Earth-Moon-Sun system provides the best solar system arena for testing the Principle of Equivalence, for a review see e.g. [3, 4]. Any theory of gravitation might explain both the attraction of masses and inertia in consistent terms. However, a nature of the relationship of gravity and inertia continues to elude us and, beyond the Principle of Equivalence, there has been little progress in discovering their true relation. Moreover, it seemed that the inertia cannot be ultimately identified with the gravity as it is proposed in GR, because there are important reasons to question the validity of this description. Actually, there are several empirical effects that seem incomprehensible in this framework. The experiments by [5–7] tested the important question of anisotropy of inertia stemming from the idea that the matter in our galaxy is not distributed isotropically with respect to the earth, and hence if the inertia is due to gravitational interactions then the inertial mass of a body will depend on the direction of its acceleration with respect to the direction towards the center of our galaxy. However, these experiments do not found such anisotropy of mass. For example, the most sensitive test is obtained in [7] from a nuclear magnetic resonance experiment, where the increase in sensitivity over that which one could obtain from Mössbaur effect is due to the far narrower line width obtainable for a transition with a Li^7 nucleus of spin $I = 3/2$ in its ground state as compared with a nucleus in an excited state. If the mass anisotropy effect is present, there will be three different intervals which will lead to a triplet nuclear resonance line, if the structure is resolved, or to a single broadened line if the structure is unresolved. The magnetic field was of about 4700 gauss. The south direction in the horizontal plane points within 22 degrees towards the center of our galaxy, and 12 hour later this same direction along the earth's horizontal plane points 104 degrees away from the galactic center. If the nuclear structure of Li^7 is treated as a single $P_{3/2}$ proton in a central nuclear potential, the variation Δm of mass with direction, if it exists, was found to satisfy $\frac{\Delta m}{m} \leq 10^{-20}$. This proves that there is no anisotropy of mass which is due to the effects of mass in our galaxy. If the curvature of Riemannian space is associated with gravitational interaction then it would indicate an universal feature equally suitable for action on all the matter fields at once. Then another objection is that this is rather applicable only for gravity but not for inertia since the inertia depends solely on the state of motion of individual test particle or coordinate frame of interest. So, the curvature arisen due to acceleration of coordinate frame of interest relates to this coordinate system itself and does not acts at once on all the other systems or matter fields. Such interesting aspects which deserve further investigations, unfortunately, have attracted little attention in subsequent developments. This state of affairs has not much changed up to present, as well as the RLI still remain unknown. The present paper aims to fill this gap. Furnishing justification for introduction of the Principle of Equivalence, in addition to those of available experimental verifications [3, 4], we must also assign a high importance to the prove on the theoretical basis.

Another purpose of present article is to explore the rearrangement of vacuum state in gravity at huge energies, which will be of vital interest for the physics of superdense matter in very compact astrophysical sources [8–11] and references therein. Note that the Riemannian space interacting quantum field theory cannot be a satisfactory ground for addressing this question. The geometrical interpretation of gravitation arisen from the dual character of the metrical tensor in its metrical and gravitational aspects, is a noteworthy result of GR. Although this interpretation has advantage in solving the problems of cosmology, nevertheless such a distinction of gravitational field among the fields yields the difficulties in the unified theories of all interactions of elementary particles, and in quantization of gravitation. Therefore, the GR as a geometrized theory of gravitation clashes from the very outset with basic principles of field theory. This rather stems from the fact that Riemannian geometry, in general, does not admit a group of isometries, i.e., Poincaré transformations no longer act as isometries and, for example, it is impossible to define energy-momentum as Noether local currents related to exact symmetries. This posed severe problems in Riemannian space interacting quantum field theory. The major unsolved problem is the non-uniqueness of the physical vacuum and associated Fock space. Actually, a peculiar shortcoming of the interacting quantum field theory in curved spacetime is the following two key questions to be addressed yet: a) an absence of the definitive concept of space-like separated points, particularly, in canonical approach, and the 'light-cone' structure at each spacetime point; b) the separation of positive- and negative-frequencies for completeness of the Hilbert-space description. Due to it, a definition of positive frequency modes cannot, in general, be unambiguously fixed in the past and future which leads to $|in> \neq |out>$, because the state $|in>$ is unstable against decay into many particle $|out>$ states due to interaction processes allowed by lack of Poincaré invariance. Non-trivial Bogolubov transformation between past and future positive frequency modes implies that particles are created from the vacuum and this is one of the reasons for $|in> \neq |out>$. Note that a remarkable surge of activity of investigations towards an extension of GR has arisen recently. They are expressible geometrically in the language of fundamental structure known as a fiber bundle. This provides an unified picture of gravity modified models based on several Lie groups, see e.g. [12–29]. All these approaches have their own advantages, but in the same time they are subject to many uncertainties. Currently no single theory has been uniquely accepted as the convincing gauge theory of gravitation, which will be able successfully to address the aforementioned problems.

A. Rational

To innovate the solution to the problems involved, in this paper we develop on the *general gauge* principle (GGP), an early version of which is given in [11, 30, 31]. The GGP accounts for *gravitation gauge group* G_R generated by hidden local internal symmetries implemented on the flat space M_6. Involving the auxiliary flat space M_6, with the whole set of well-defined Killing's vectors, just has a single aim as a guiding tool in dealing with an intricate 'jungle' of curved geometry. In this paper much more will be done (Sect.2) to make clear and rigorous these early results and formulations. The following part of the present paper will be the original contribution, whereas we relate the group G_R to the Lie group G_D of *distortion* of local internal properties of flat space M_6. It can be achieved by nonlinear realization of the group G_D in the framework of method of phenomenological Lagrangians. This approach was originally introduced by Coleman, Wess and Zumino [32–34] in the context of internal symmetry groups. It was later extended to the case of spacetime symmetries by Isham, Salam, and Strathdee [35, 36] considering the nonlinear action of $GL(4, \mathbb{R})$ mod the Lorentz subgroup, see [15] and references therein for a comprehensive review. We study geometrical structure of the space of parameters in terms of Cartan's calculus of exterior forms and derive the Maurer-Cartan's structure equations. We derive key relation which uniquely determines, for given distortion field, the six angles of distortion rotations around each axes of the M_6. We treat distortion fields as Goldstone fields to which the metric and connection are related. We infer group invariants and calculate conserved currents. The metric is no more a fundamental dynamical field. The fundamental field is distortion gauge field and, thus, both the actions and the equations of motion depend on the concept of gauge potential. The metric and connection may be derived from this gauge field. To test the proposed gravitation theory, we derive the line element in particular case of static and spherically symmetric gravitational field. Traditionally the solar system is a laboratory that offers many opportunities to improve tests of relativistic gravity. The usual Eddington-Robertson-Schiff parameters β and γ used to describe these tests are perhaps in some sense the most important parameters of the parameterized post-Newtonian (PPN) formalism [37–40]. We rather show that the agreement is satisfactory between the proposed gravitation theory and available observational verifications [4, 40–55]. We complete the proposed theoretical basis of distortion of spacetime by exploring, further, two major problems of inertia and rearrangement of vacuum state in gravity.

We construct the relativistic field theory of inertia which similarly to gravitation theory treats the inertia effects as a distortion of local internal properties of flat spacetime continuum. We motivate this approach as follows. Unlike the GR, proposed gauge field theory of gravitation is free of fictitious forces, because the infinite-parameter group of general covariance is no longer in use. Instead, the preferred systems and group of transformations of the, so-called, *real-curvilinear coordinates* relate solely to real gravitational fields. In spite of totally different and independent physical sources of gravitation and inertia, still we might expect that the inertial force is of the same nature as gravitational force. Namely we ascribe the effects associated with gravity and inertia to spacetime geometry itself, and that both phenomena arise due to the distortion of local internal properties of flat space. To trace this line, we involve besides the distortion gauge fields being responsible only for gravitation, also the *distortion inertial fields* which account for the inertia separately. Seeking a replacement for the unobservable Newtonian *absolute spacetime*, which is necessary to assign a meaning to Newtonian *absolute acceleration*, instead we explore the geometry of two-dimensional flat space M_2. Similar reasoning leads us, further, to the conclusion that an alteration of uniform motion of test particle under the unbalanced force is the immediate cause of the real distortion of the local internal properties of the space M_2 conducted under the distortion inertial field. This necessarily, in the first place, with equal justice could be interpreted as a definite criterion for the universal *absolute* acceleration of test particle or coordinate frame of interest, and in the second place, will give us the fundamental RLI. This we might expect to hold on the basis of an intuition founded on a past experience limited to low velocities, and which were implicit in the ideas of Galileo and Newton as to the nature of inertia. The major premise is that the centrifugal endeavor of particles to recede from the axis of rotation is directly proportional to the quantity of the *absolute* circular acceleration, which, for example, concave water surface in Newton's famous rotating bucket experiments. In this framework, the *relative* acceleration (in Newton's terminology) (both magnitude and direction), in contrary, cannot be the cause of the distortion of the space M_2 and, thus, it does not produce inertia effect. Therefore, the real *inertia* effects can be an empirical indicator of *absolute* acceleration. We calculate the inertial force acting on the photon in gravitating system of particles that are bound together by their mutual gravitational attraction. A particular attention is given to the theoretical justification for introduction of the Principle of Equivalence.

Finally, the developments on the GGP are applied to address the rearrangement of vacuum state in gravity. The objections concerning non-uniqueness of the physical vacuum can be circumvented immediately due to one of the underlying principles that in the flat space interacting quantum field theory the vacuum is well-determined and unique $|in>=|out>$ (up to a phase factor). In realization of G_R we implement the simplest hidden gauge symmetry of abelian group $U^{loc}=U(1)_Y \times diag[SU(2)]$ on the M_6 which entails two neutral gauge bosons. Spontaneous symmetry breaking is achieved in standard manner by introducing the neutral complex Higgs scalar. Non-vanishing vacuum expectation value (VEV) leaves one Goldstone boson which is gauged away from the scalar sector. But it essentially reappears in the gauge sector providing the longitudinally polarized spin state of one of gauge bosons that acquires mass through its coupling to Higgs scalar. The massless component of distortion field is responsible for gravitational interactions. In the resulting theory, simultaneously with the strong gravity, the massive distortion field component may cause a substantial change of properties of spacetime continuum at huge energies above the threshold value.

This paper is organized as follows: In Sect.2 a number of useful mathematical concepts of GGP are reviewed for the reader's convenience. We will refrain from providing lengthy details of the formalism of GGP and unitary map. For these the reader is referred to Appendix. In Sect.3 we relate the group G_R to the Lie group G_D by constructing its nonlinear realization. In Sect.4 we construct the relativistic field theory of inertia and give theoretical justification for introduction of the Principle of Equivalence. In Sect.5 we address the rearrangement of vacuum state in gravity. Conclusions are presented in Sect.6. The specific topics dealt with in the Appendix are further details on the GGP. We will be brief and often suppress the indices without notice. Unless otherwise stated we take natural units, $h=c=1$. The quantities denoted by wiggles throughout this paper refer to distorted (curved) space, but the quantities referring to flat space are left without wiggles.

II. THE GGP PRELIMINARIES

For the benefit of the reader, a brief outline of the framework of GGP are given in this Section and in Appendix to make the rest of the paper understandable. We have used a combined geometrical structure known as a fiber bundle, which provides a unified picture of theory based on the local internal gauge symmetries. The gravity, as a gauge theory, could be achieved by introducing a generalized gauge transformation law (Eq. (1)) which enables the gauging of external spacetime groups.

Given the principal fiber bundle $\widetilde{\mathbb{P}}(R_4,\, G_R;\, \widetilde{\pi})$ with the structure group G_R, the local coordinates $\widetilde{p} \in \widetilde{\mathbb{P}}$ are $\widetilde{p} = (\widetilde{x}, U_R)$, where $\widetilde{x} \in R_4$ and $U_R \in G_R$, the total bundle space $\widetilde{\mathbb{P}}$ is smooth manifold, the surjection $\widetilde{\pi}$ is a smooth map $\widetilde{\pi} : \widetilde{\mathbb{P}} \to R_4$. The base space is assumed to be curved four dimensional Riemannian space R_4 in order to describe the effects of gravitation. A set of open coverings $\{\widetilde{\mathcal{U}}_i\}$ of R_4 with $\widetilde{x} \in \{\widetilde{\mathcal{U}}_i\} \subset R_4$ satisfy $\bigcup_\alpha \widetilde{\mathcal{U}}_\alpha = R_4$. The fibration is given as $\bigcup_{\widetilde{x}} \widetilde{\pi}^{-1}(\widetilde{x}) = \widetilde{\mathbb{P}}$. The local gauge will be the diffeomorphism map $\widetilde{\chi}_i : \widetilde{\mathcal{U}}_i \times_{R_4} G_R \to \widetilde{\pi}^{-1}(\widetilde{\mathcal{U}}_i) \in \widetilde{\mathbb{P}}$, since $\widetilde{\chi}_i^{-1}$ maps $\widetilde{\pi}^{-1}(\widetilde{\mathcal{U}}_i)$ onto the direct (Cartesian) product $\widetilde{\mathcal{U}}_i \times_{R_4} G_R$. Here $\times_{R_4}$ represents the fiber product of elements defined over space R_4 such that $\widetilde{\pi}(\widetilde{\chi}_i\,(\widetilde{x},\, U_R)) = \widetilde{x}$ and $\widetilde{\chi}_i(\widetilde{x},\, U_R) = \widetilde{\chi}_i(\widetilde{x},\, (id)_{G_R})U_R = \widetilde{\chi}_i(\widetilde{x})U_R\ \forall \widetilde{x} \in \{\widetilde{\mathcal{U}}_i\}$, $(id)_{G_R}$ is the identity element of group G_R. Let the collection of matter fields of arbitrary spins $\widetilde{\Phi}(\widetilde{x})$ (the various suffixes are left implicit) take values in standard fiber over $\widetilde{x} :\ \widetilde{\pi}^{-1}(\widetilde{\mathcal{U}}_i) = \widetilde{\mathcal{U}}_i \times \widetilde{\mathbb{F}}_{\widetilde{x}}$. The fiber $\widetilde{\pi}^{-1}$ at $\widetilde{x} \in R_4$ is diffeomorphic to $\widetilde{\mathbb{F}}$, where $\widetilde{\mathbb{F}}$ is the fiber space, such that $\widetilde{\pi}^{-1}\,(\widetilde{x}) \equiv \widetilde{\mathbb{F}}_{\widetilde{x}} \approx \widetilde{\mathbb{F}}$. The action of structure group G_R on $\widetilde{\mathbb{P}}$ defines an isomorphism of the Lie algebra $\widetilde{\mathfrak{g}}$ of G_R onto the Lie algebra of vertical vector fields on $\widetilde{\mathbb{P}}$ tangent to the fiber at each $\widetilde{p} \in \widetilde{\mathbb{P}}$ called fundamental. Whereas, the tangent and cotangent bundles, respectively, are $\widetilde{T}(\widetilde{\mathbb{P}})$ and $\widetilde{T}^*(\widetilde{\mathbb{P}})$, $\widetilde{T}_p(\widetilde{\mathbb{P}})$ is the space of tangents at $\widetilde{p} \in \widetilde{\mathbb{P}}$, i.e. $\widetilde{T}_p(\widetilde{\mathbb{P}}) \in \widetilde{T}(\widetilde{\mathbb{P}})$. The metric is the section of conjugate vector bundle $S^2\widetilde{T}^*(\widetilde{\mathbb{P}})$ (symmetric part of tensor degree): $\hat{g} : \widetilde{T}(\widetilde{\mathbb{P}}) \times \widetilde{T}(\widetilde{\mathbb{P}}) \to C^\infty(R_4)$, where a section is a smooth map $S : R_4 \to \widetilde{\mathbb{P}}$, such that $S(\widetilde{x}) \in \widetilde{\pi}^{-1}(\widetilde{x}) = \widetilde{\mathbb{F}}_{\widetilde{x}} \forall \widetilde{x} \in R_4$, and satisfies $\pi \circ S = (id)_{R_4}$, where $(\circ)$ represents the group composition operation, where $(id)_{R_4}$ is the identity element of R_4. It assigns to each point $\widetilde{x} \in R_4$ a point in the fiber over $\widetilde{x}$. The general coordinate transformations $\delta\widetilde{x} = f(\widetilde{x})$, where $f(\widetilde{x})$ is an arbitrary function of coordinates $\widetilde{x}$, yield the infinite-parameter group of general covariance in R_4 if only the functions $f(\widetilde{x})$ can be expanded in power series of $\widetilde{x}$. The expansion coefficients are considered as the group-parameters, and that the group-algebra includes an infinite number of generators.

Remark: An invariance of the Lagrangian $L_{\widetilde{\Phi}}$ of matter fields $\widetilde{\Phi}(\widetilde{x})$ under the infinite-parameter group of general covariance in R_4 implies an invariance of $L_{\widetilde{\Phi}}$ under the *gravitation gauge group* G_R and vice versa if, and only if, the generalized local gauge transformations of the fields $\widetilde{\Phi}(\widetilde{x})$ and their covariant derivative $\nabla_\mu\widetilde{\Phi}(\widetilde{x})$ are introduced by finite local $U_R(\in G_R)$ gauge transformations as

$$
\begin{aligned}
&\widetilde{\Phi}'(\widetilde{x}) = U_R\,(\widetilde{x})\,\widetilde{\Phi}(\widetilde{x}),\\
&\left[g^\mu(\widetilde{x})\,\nabla_\mu\widetilde{\Phi}(\widetilde{x})\right]' = U_R\,(\widetilde{x})\,\left[g^\mu(\widetilde{x})\,\nabla_\mu\widetilde{\Phi}(\widetilde{x})\right],
\end{aligned}
\tag{1}
$$

where ∇_μ denotes the covariant derivative agreed with the metric, $g^\mu(\widetilde{x}) \to \widetilde{e}^\mu(\widetilde{x})$ for the fields of spin ($j = 0, 1$), and $g^\mu(\widetilde{x}) = V^\mu_\alpha(\widetilde{x})\,\gamma^\alpha$ for the spinor field ($j = \frac{1}{2}$), where $V^\mu_\alpha(\widetilde{x}) =< \widetilde{e}^\mu,\, \widetilde{e}_\alpha >$ are the components of affine tetrad vectors $\widetilde{e}^\alpha$ in used coordinate net $\widetilde{x}^\mu$ [56], γ^α are the Dirac's matrices. The unitary matrix $U_R\,(\widetilde{x})$ will be determined below.

Next, suppose the massless gauge field $a(x)\,(\equiv a_\mu(x))$ takes values in Lie algebra $\mathfrak{g}$ of abelian group U^{loc}, which is a local form of expression of connection in principle fiber bundle $\mathbb{P}(M_4,\, U^{loc};\, \pi)$ with the structure group U^{loc} and the surjection π. The base space is the flat Minkowski space M_4, so, a set of open coverings $\{\mathcal{U}_i\}$ of M_4 with $x \in \{\mathcal{U}_i\} \subset M_4$ satisfy $\bigcup_\alpha \mathcal{U}_\alpha = M_4$. The metric is the section of conjugate vector bundle $\hat{\eta} : T(\mathbb{P}) \times T(\mathbb{P}) \to C^\infty(M_4)$, whereas the symmetric components (η_{lk}) of metrical tensor can be given in basis (e_l). The matter fields $\Phi(x)$ of arbitrary spin are the sections of vector bundles associated with abelian group U^{loc}. They take values in standard fiber which is the Hilbert vector space where a linear representation $U(x)$ of group U^{loc} is given. This space can be regarded as Lie algebra of group U^{loc} upon which Lie algebra acts according to law of adjoint representation: $a \leftrightarrow ad\,a\ \Phi \to [a\,,\Phi]$. We adopt the following conventions: Greek indices stand for variables in R_4, Latin indices refer to M_4, and that $\psi^\mu_l \equiv \partial_l\,\widetilde{x}^\mu$ where $\partial_l = \partial/\partial\, x^l$. Aforesaid is the mathematical tools of conventional gauge dynamics. Now, to involve a drastic revision of a role of gauge fields in physical concept of curved geometry, below we generalize this scheme by exploring a new special type of *distortion* gauge fields assumed acting on external spacetime groups. While, a local internal gauge symmetry $U^{loc}(1)$ remains hidden symmetry as far as it is screened by the gravitation gauge group G_R.

Theorem 1: For any generalized gauge field dynamics of Eqs. (1) defined on R_4 the underlying (surjective) conventional gauge field dynamics can always be constructed on M_4.

Proof: The following three steps are the very foundation of our construction procedure which went into the proof of this theorem.

First step: We assume that the basis vector (e) undergoes *distortion* transformations under the *distortion* gauge field (a):

$$
\widetilde{e}_\mu(a) = D^l_\mu(a)\,e_l. \tag{2}
$$

The transformation matrix $D(a)$ will be determined in Sect.3.

Second step: We construct the diffeomorphism $\widetilde{x}^\mu(x, a) : M_4 \to R_4$ by seeking the new holonomic coordinates $\widetilde{x}^\mu(x, a)$ as the solutions of the first-order partial differential equations

$$\widetilde{e}_\mu(a)\,\psi^\mu_l = \Omega^m_l(F)e_m, \tag{3}$$

where $\Omega^m_l(F) = \delta^m_l + \omega^m_l(F)$, the (F) denotes antisymmetrical tensor of gauge field $F_{nk} = \partial_n\, a_k - \partial_k\, a_n$, and hence the tensor $\Omega^m_l(F)$ has a null variational derivative $(\delta\Omega^m_l(F)/\delta a_n) = 0$ at local variations of connection $a_n \to a_n + \delta a_n$.

Third step: We consider a smooth unitary map of all the matter fields and their covariant derivatives:

$$\begin{aligned} &R(a) : \Phi \to \widetilde{\Phi}, \\ &S(a)\,R(a) : \left(\gamma^k D_k \Phi\right) \to \left(g^\nu(x)\nabla_\nu \widetilde{\Phi}\right), \end{aligned} \tag{4}$$

where $R(a)$ is the unitary map matrix, $S(F)$ is the gauge invariant scalar function (see Eq. (6) and App.2), $D_k = \partial_k - i\text{æ}\, a_k$, æ is the gauge coupling constant which relates to Newton gravitational constant as in Eq. (37).

The conditions of integrability $\partial_k\, \psi^\mu_l = \partial_l\, \psi^\mu_k$ and non-degeneracy $(\|\psi\| \neq 0)$ necessarily hold [57, 58], therefore, the following constrain is imposed upon the tensor $\Omega^m_l(F)$: $\partial_k(D^\mu_m \Omega^m_l) = \partial_l(D^\mu_m \Omega^m_k)$, the solution of which can be written in general form $\Omega^m_l(F) = D^m_\nu(a)\partial_l \Theta^\nu(a, F)$, where $\Theta^\mu(a, F)$ are the arbitrary functions such that $\partial_k\partial_l\Theta^\mu(a, F) = \partial_l\partial_k\Theta^\mu(a, F)$. Hence, the equation (3) yields the bilinear form $d\,\widetilde{s}^2$ on R_4 :

$$\begin{aligned} &d\,\widetilde{s}^2 = g_{\mu\nu}\, d\,\widetilde{x}^\mu d\,\widetilde{x}^\nu = ds^2_\chi \equiv \\ &\Omega^m_l(F)\Omega^m_k(F) dx^l dx^k = inv(\Lambda,\, U^{loc}), \end{aligned} \tag{5}$$

where ds^2_χ is the Lorentz (Λ) and gauge (U^{loc}) invariant line element given on M_4. Denoting $\chi_l = \omega^m_l(F)e_m$, and $\chi^\mu_l = \psi^\mu_l - D^\mu_l$, we may derive the following gauge invariant scalar functions:

$$\begin{aligned} &\chi(F) =< e^l,\, \chi_l(F) >= \omega^l_l(F) = tr\,\omega(F), \\ &S(F) = \tfrac{1}{4}\psi^l_\mu(a, F) D^\mu_l(a) = 1 + \tfrac{1}{4} tr\,\omega(F). \end{aligned} \tag{6}$$

In what follows, we take the form

$$\omega^m_l(F) = \delta^m_l \omega(x)(F), \tag{7}$$

where we do not initially specify the scalar function $\omega(F)$ apart that $\omega(0) = 0$. Instead, at some intermediate stage in the analysis we adopt an expansion form (Subsect.3.3). The curvature of the space R_4 is zero if $(\partial\psi^l_\mu/d\widetilde{x}^\nu) = \Gamma^\lambda_{\mu\nu}\,\psi^l_\lambda$ [59], where $\Gamma^\lambda_{\mu\nu}$ denote the Christoffel symbols agreed with the metric $g_{\mu\nu}(a) = D^l_\mu(a) D^l_\nu(a)$. In illustration of the point at issue, the Eqs. (4) explicitly may read

$$\begin{aligned} &\widetilde{\Phi}^{\mu\cdots\delta}(\widetilde{x}) = \psi^\mu_l \cdots \psi^\delta_m\, R(a)\,\Phi^{l\cdots m}(x) \equiv \\ &(R_\psi)^{\mu\cdots\delta}_{l\cdots m}\,\Phi^{l\cdots m}(x), \end{aligned} \tag{8}$$

and that

$$\begin{aligned} &g^\nu(x)\nabla_\nu \widetilde{\Phi}^{\mu\cdots\delta}(\widetilde{x}) = \\ &S(F)\,\psi^\mu_l \cdots \psi^\delta_m\, R(a)\,\gamma^k D_k\, \Phi^{l\cdots m}(x). \end{aligned} \tag{9}$$

Using the gauge transformations in M_4, it is a straightforward to determine from Eq. (8) the matrix $U_R\,(\widetilde{x})$ in terms of matrices U and R. Actually,

$$\widetilde{\Phi}'(\widetilde{x}) = U_R\,(\widetilde{x})\,\widetilde{\Phi}(\widetilde{x}) = U_R\, R_\psi(a)\,\Phi = R'_\psi\,\Phi' = R'_\psi\, U\,\Phi.$$

Similarly, this can be determined from Eq. (9) too:

$$\begin{aligned} &\left(g^\nu(x)\nabla_\nu\widetilde{\Phi}(\widetilde{x})\right)' = U_R\,(\widetilde{x})\,\left(g^\nu(x)\nabla_\nu\widetilde{\Phi}(\widetilde{x})\right) = \\ &U_R\, S(F)\, R_\psi(a)\,\left(\gamma^k D_k \Phi\right) = S(F')\, R'_\psi\,\left(\gamma^k D_k\, \Phi\right)' = \\ &S(F')\, R'_\psi\, U\,\left(\gamma^k D_k\, \Phi\right). \end{aligned}$$

Hence $U_R = R'_\psi\, U\, R^{-1}_\psi$, where $R'_\psi \equiv R_\psi(a')$ and the (a') denotes U^{loc}-transformed gauge field. Based on the Theorem 1 we may extend conventional gauge principle to involve gravity in the GGP scheme by requiring that: *The physical system of the fields $\widetilde{\Phi}(\widetilde{x})$ defined on R_4 must always be invariant under the finite local gauge transformations U_R of the Lie group of gravitation G_R.*

$$\begin{tikzcd} \widetilde{\Phi}'(\widetilde{x}) = U_R\,\widetilde{\Phi}(\widetilde{x}) & & \widetilde{\Phi}(\widetilde{x}) \arrow[ll, "U_R = R'_\psi\, U\, R^{-1}_\psi"'] \\ \Phi'(x) = U\,\Phi(x) \arrow[u, "R'_\psi(\widetilde{x},\, x)"] & & \Phi(x) \arrow[ll, "U"'] \arrow[u, "R_\psi(\widetilde{x},\, x)"'] \end{tikzcd}$$

The scheme of GGP.

Although, in the reminder of this article we have explored the simplest abelian symmetry U^{loc} as hidden symmetry, however, one may envisage that a straightforward extension should be to achieve the full machinery of the GGP scheme for non-abelian symmetries. We conclude, on the observations above that out of all the arbitrary coordinates in R_4 the *real-curvilinear* coordinates $\widetilde{x}(x, a)$ can be distinguished which are derived from Eq. (3) at all Lorentz (Λ) and gauge (U^{loc}) transformations of variables (x) and (a). Hence, unlike GR, the wider infinite-parameter group of general covariance in R_4 is no longer in use. Therefore, some Lorentz or gauge transformation necessarily underlies the arbitrary transformation $\widetilde{x} \to \widetilde{x}'$ of real-curvilinear coordinates which relate solely to real gravitational fields. This prompts us to treat the inertia separately (Sect.4). In case of zero curvature, the Eq. (3) can be satisfied globally in M_4 by setting $\psi^\mu_l = D^\mu_l = V^\mu_l = (\partial x^\mu/\partial X^l)$, $\|D\| \neq 0$, $\chi_l = 0$, where X^l are the inertial coordinates. In this, one has conventional gauge theory given on the M_4 in both curvilinear and inertial coordinates. At this point, we have discussed all the mathematical tools that complete the formalism of GGP by making clear and rigorous the early results and formulations [11, 30, 31]. In what follows we shall present the original contribution.

III. NONLINEAR REALIZATION OF DISTORTION GROUP G_D

The nonlinear realization technique [32, 33] provides a way to determine the transformation properties of fields defined on the quotient space G/H. Constructing nonlinear realization of the Lie group of distortion G_D, first, within the scheme of the GGP we necessarily introduce the language of a conceptual six-dimensional geometry of M_6, which is assumed as a toy model underlying the M_4. This replacement appears to be indispensable in discussion of the distortion of local internal properties of spacetime continuum, and it mostly manifests its virtues in constructing the relativistic field theory of inertia (Sect.4). So, let M_6 be the smooth differentiable six-dimensional flat space with the decomposition law as follows:

$$\begin{array}{l} M_6 = R^3_+ \oplus R^3_- = R^3 \oplus T^3, \\ sgn(R^3) = (+++), \quad sgn(T^3) = (---). \end{array} \tag{10}$$

The $e_{(\lambda\alpha)} = O_\lambda \times \sigma_\alpha \quad (\lambda = \pm,\, \alpha = 1,2,3)$ are linear independent unit basis vectors at the point (p) of interest of given three-dimensional space R^3_λ. The unit vectors O_λ and σ_α imply

$$< O_\lambda,\, O_\tau >= {}^*\delta_{\lambda\tau}, \quad < \sigma_\alpha,\, \sigma_\beta >= \delta_{\alpha\beta}, \tag{11}$$

where $\delta_{\alpha\beta}$ is the Kronecker symbol, and ${}^*\delta_{\lambda\tau} = 1 - \delta_{\lambda\tau}$. Consequently, three spatial $e_\alpha = \xi \times \sigma_\alpha$ and three temporal $e_{0\alpha} = \xi_0 \times \sigma_\alpha$ components are the basis vectors, respectively, in spaces R^3 and T^3, where $O_\pm = (1/\sqrt{2})(\xi_0 \pm \xi), \quad \xi_0^2 = -\xi^2 = 1, \quad < \xi_0,\, \xi >= 0$. Within this scheme, we are presumably allowed to perceive directly the three-dimensional ordinary space R^3, but not the three - dimensional time space T^3 being orthogonal to the former. In using this language it is important to guard a reduction to the space M_4 which can be achieved in the following way.

1) In case of free flat space M_6, the subspace T^3 is isotropic. And so far it contributes in line element just only by the square of the moduli $t = |\mathbf{x}^0|,\, \mathbf{x}^0 \in T^3$, then, the reduction $M_6 \to M_4 = R^3 \oplus T^1$ can be readily achieved if for conventional *time* we use $t = |\mathbf{x}^0|$.

2) In case of curved space, the reduction $R_6 \to R_4$ can be achieved if we use the projection $(\widetilde{e}_0)$ of the temporal component $(\widetilde{e}_{0\alpha})$ of basis six-vector $\widetilde{e}\ (\widetilde{e}_\alpha,\, \widetilde{e}_{0\alpha})$ on the given *universal* direction $(\widetilde{e}_{0\alpha} \to \widetilde{e}_0)$. By this we choose the *time* coordinate. Actually, the Lagrangian of physical fields defined on R_6 is the function of scalars such as $A_{(\lambda\alpha)}B^{(\lambda\alpha)} = A_\alpha B^\alpha + A_{0\alpha}B^{0\alpha}$, then upon the reduction of temporal components of six-vectors $A_{0\alpha}B^{0\alpha} = A^{0\alpha} < \widetilde{e}_{0\alpha}, \widetilde{e}_{0\beta} > B^{0\beta} = A^0 < \widetilde{e}_0, \widetilde{e}_0 > B^0 = A_0 B^0$ we may fulfill a reduction to R_4.

A. Distortion of local internal properties of the M_6

First, we consider distortion transformations of the ingredient unit vectors O_τ under the distortion gauge field (a):

$$\begin{array}{l} \widetilde{O}_{(+\alpha)}(a) = \mathcal{Q}^\tau_{(+\alpha)}(a)\, O_\tau = O_+ + æ\, a_{(+\alpha)} O_-, \\ \widetilde{O}_{(-\alpha)}(a) = \mathcal{Q}^\tau_{(-\alpha)}(a)\, O_\tau = O_- + æ\, a_{(-\alpha)} O_+, \end{array} \tag{12}$$

where $\mathcal{Q} \left(= \mathcal{Q}^\tau_{(\lambda\alpha)}(a) \right)$ is the element of the group Q. This violates the first relation in Eq. (11) because of $\widetilde{O}^2_{(\lambda\alpha)}(a) = 2æ\, a_{(\lambda\alpha)} \neq 0$ for given λ and α. Next, we assume that this induces the distortion transformations of ingredient unit vectors σ_β, which, in turn, undergo the rotations: $\widetilde{\sigma}_{(\lambda\alpha)}(\theta) = \mathcal{R}^\beta_{(\lambda\alpha)}(\theta)\, \sigma_\beta$, where $\mathcal{R}(\theta) \in SO(3)$ is the element of the group of rotations of planes involving two arbitrary axes around orthogonal third axis in the given ingredient space R^3_λ. Then, resulting basis vectors $\widetilde{\sigma}_{(\lambda\alpha)}(\theta)$ of each three-dimensional ingredient space R^3_λ retain the orthogonality condition between themselves, but violate it between the basis vectors of different ingredient spaces. That is, $< \widetilde{\sigma}_{(\lambda\alpha)},\, \widetilde{\sigma}_{(\tau\beta)} >_{\alpha\neq\beta} \neq 0, \quad \text{at} \quad \lambda \neq \tau$. In fact, distortion transformations of basis vectors (O) and (σ) are not independent, and rather governed by the spontaneous breaking of distortion symmetry (see Eq. (24)). To avoid a further proliferation of indices, hereafter we will use upper case Latin (A) in indexing $(\lambda\alpha)$, etc. The infinitesimal transformations then read

$$\begin{array}{l} \delta\, \mathcal{Q}^\tau_A(a) = æ\, \delta\, a_A X^\tau_\lambda \,\in Q, \\ \delta\, \mathcal{R}(\theta) = -\frac{i}{2} M_{\alpha\beta} \delta\, \omega^{\alpha\beta} \,\in SO(3), \end{array} \tag{13}$$

provided by generators $X^\tau_\lambda = {}^*\delta^\tau_\lambda$ and $I_i = \frac{\sigma_i}{2}$, where σ_i are the Pauli's matrices, $M_{\alpha\beta} = \varepsilon_{\alpha\beta\gamma} I_\gamma$ and $\delta\, \omega^{\alpha\beta} = \varepsilon_{\alpha\beta\gamma} \delta\, \theta_\gamma$. Transformation matrix $D(a,\, \theta) = \mathcal{Q}(a) \times \mathcal{R}(\theta)$ is the element of distortion group $G_D = Q \times SO(3)$:

$$\begin{array}{l} D_{(d\, a^A,\, d\, \theta^A)} = I + d\, D_{(a^A,\, \theta^A)}, \\ d\, D_{(a^A,\, \theta^A)} = i \left[d\, a^A X_A + d\, \theta^A I_A \right], \end{array} \tag{14}$$

where $I_A \equiv I_\alpha$ at given λ. We may join to each point $(a,\, \theta)$ of group space the Descartes' reper which is equal, in group sense, to the reper joint to point of origin $(0,\, 0)$. This is introduced to ensure that the vector $(a,\, \theta;\, a + d\, a,\, \theta + d\, \theta)$ has the same analytical expression of the vector $(0,\, 0;\, d\, a',\, d\, \theta)'$: (Eq. (14))

$$D_{(a,\, \theta)}\, d\, D^{-1}_{(a,\, \theta)} = i[\omega^A\, X_A + \vartheta^A\, I_A], \tag{15}$$

where we denote $d\, a'^A = \omega^A(a,\, \theta;\, d\, a,\, d\, \theta)$ and $d\, \theta'^A = \vartheta^A(a,\, \theta;\, d\, a,\, d\, \theta)$. Then,

$$\begin{array}{l} d\, e_A = e_A\, d\, F_A = O_A(d) \times \sigma_A + O_A \times \sigma_A(d) = \\ i[\omega^A(d)\, X_A + \vartheta^A(d)\, I_A]\, e_A, \end{array} \tag{16}$$

where $e_A \equiv (\exp F_A)$. The functions $\omega^A(d)$ and $\vartheta^A(d)$ will be determined in the next subsection.

B. A spontaneous breaking of distortion symmetry

Following the method of phenomenological Lagrangians [13, 15, 32–34, 60] and references therein, our goal is to treat the distortion group G_D and its stationary subgroup $H = SO(3)$, respectively, as dynamical group and its algebraic subgroup. But the generators X_A (Eq. (13)) of group Q do not complete the group H to the dynamical group G_D, therefore, they cannot be interpreted as the generators of quotien space G_D/H, and that the distortion fields a_A cannot be identified directly with the Goldstone fields arisen in spontaneous breaking of distortion symmetry G_D. These objections, however, can be circumvented if we define the pair of the original basis vectors O_λ in terms of orthogonal unit vectors ξ_0 and ξ. The distortion transformations Eq. (12) of basis vectors $O_\pm$ then immediately become rotations of the group $SO(3)$ of modified basis vectors

$$\overline{\widetilde{O}}_A = \frac{1}{\sqrt{2}}\left(\widetilde{\xi}_{0A}(\overline{\theta}) + \epsilon_A\,\widetilde{\xi}_A(\overline{\theta})\right) = \cos\overline{\theta}_A \widetilde{O}_A, \tag{17}$$

where $\tan\overline{\theta}_A \equiv -æ\, a_A,\ \epsilon_{(+\alpha)} = -\epsilon_{(-\alpha)} = 1$, and that

$$\begin{pmatrix} \widetilde{\xi}_{0A}(\overline{\theta}) \\ \widetilde{\xi}_A(\overline{\theta}) \end{pmatrix} = \begin{pmatrix} \cos\overline{\theta}_A & \sin\overline{\theta}_A \\ -\sin\overline{\theta}_A & \cos\overline{\theta}_A \end{pmatrix} \begin{pmatrix} \xi_0 \\ \xi \end{pmatrix}. \tag{18}$$

Here, a rotation on $\overline{\theta}_{(+\alpha)}$ is clockwise, while on $\overline{\theta}_{(-\alpha)}$ is counterclockwise. Consequently, the distortion group $G_D = Q \times SO(3)$ can be mapped in one-to-one manner onto the group $G_D = SO(3) \times SO(3)$ which, in turn, is isomorphic to chiral group $SU(2) \times SU(2)$. In this case the method of phenomenological Lagrangians is well known. For a convenience, throughout this subsection we leave the Greek indices implicit unless otherwise stated: $A = (\lambda i) \to i = 1, 2, 3,$. But it goes without saying that all the results obtained refer to the given R^3_λ space. Three I_i among six generators of the group correspond to isotopic transformations, and three K_i- to special chiral transformations mixing the states of different parities. They imply the conventional commutation relations

$$\begin{aligned} &[I_i,\, I_j] = i\varepsilon_{ijk} I_k, \quad [I_i,\, K_j] = i\varepsilon_{ijl} K_l, \\ &[K_i,\, K_j] = i\varepsilon_{ijk} I_k, \end{aligned} \tag{19}$$

of invariant subgroup $H = SO(3)$, with the generators I_i, and of quotien space G_D/H, with the generators K_i, where ε_{ijk} denotes the antisymmetric unit tensor. Three modified parameters $\overline{a}^i(a)$ of the quotien space G_D/H of adjacent classes, with respect to which the Lagrangian of physical fields is not invariant, can be identified with three Goldstone fields. They are introduced to make provisions for the Eq. (16), which incorporated into Eq. (17) yields

$$\begin{aligned} &d\,\overline{e}_A(\overline{a}) = \overline{e}_A(a)\, d\,\overline{F}_A(\overline{\theta}, \theta) = \overline{O}_A(d) \times \sigma_A + \\ &\overline{O}_A \times \sigma_A(d) = i[\omega^i(\overline{a}, d\,\overline{a})\, K_i + \vartheta^l(\overline{a}, d\,\overline{a})\, I_l]\, \overline{e}_A(\overline{a}). \end{aligned} \tag{20}$$

This is written in terms of generators of the group $SU(2) \times SU(2)$, where $\overline{e}_A(\overline{a}) = \overline{O}_A(\overline{a}) \times \sigma_A(\overline{a})$. We are at once led to seek the function $\overline{\theta}(\theta)$ if $d\,\overline{F}_A(\overline{\theta}, \theta)$ constitutes a total differential, i.e., $\omega^i(\overline{a}, d\,\overline{a})$ and $\vartheta^l(\overline{a}, d\,\overline{a})$ are Cartan's forms. This implies

$$\frac{\partial^2\, \overline{F}_A}{\partial\,\overline{\theta}\,\partial\,\theta} = \frac{\partial^2\, \overline{F}_A}{\partial\,\theta\,\partial\,\overline{\theta}}, \tag{21}$$

where $\overline{\theta} \equiv \overline{\theta}_A$ and $\theta \equiv \theta_A$. Using the infinitesimal transformations (see Eq. (13))

$$d\,\sigma_{(\lambda l)} = \sigma_{(\lambda l)}(d) = \tfrac{1}{2}\varepsilon_{lkj}\sigma_k d\,\theta_{(\lambda j)}, \tag{22}$$

it is straightforward to calculate the partial derivative

$$\frac{\partial\, F_A}{\partial\,\theta} = \frac{\partial\,\sigma_A}{\sigma_A\,\partial\,\theta} \equiv \frac{\sigma_A(\partial)}{\sigma_A\,\partial\,\theta} = \frac{1}{\sin\,\theta_A}. \tag{23}$$

The similar relation holds for the vectors $d\,\widetilde{\xi}_{0A}(\overline{\theta})$ and $d\,\widetilde{\xi}_A(\overline{\theta})$ (Eq. (18)), and that $(\partial\, F_A/\partial\,\overline{\theta}) = (1/\sin\overline{\theta}_A)$. Upon the reduction, the holonomy condition Eq. (21) becomes $(\partial/\partial\,\theta)\left(1/\sin\overline{\theta}_A\right) = (\partial/\partial\,\overline{\theta})\left(1/\sin\theta_A\right)$, with nontrivial solution $\theta_A = \overline{\theta}_A$. Hence we arrive at

$$\tan\,\theta_A = -æ\, a_A. \tag{24}$$

Given distortion field a_A, the key relation (24) uniquely determines six angles θ_A of rotations around each of six (A) axes. We are now in position to derive the Maurer-Cartan's structure equations. According to Poincaré's theorem, the exterior derivative $(')$ of total differential form is zero, and that the Eq. (20) gives

$$\begin{aligned} &(\overline{e}_A(a)\, d\,\overline{F}_A)' = \overline{e}_A(a)\, \left([d\,\overline{F}_A, \delta\,\overline{F}_A] + (d\,\overline{F}_A)'\right) = \\ &i\left([\omega^i(\overline{a}, d\,\overline{a})\, K_i + \vartheta^l(\overline{a}, d\,\overline{a})\, I_l]\, \overline{e}_A(a)\right)' = 0, \end{aligned} \tag{25}$$

where $[d\,\overline{F}_A, \delta\,\overline{F}_A] = (d\,\overline{F}_A)' = 0$. In calculating of exterior differential and exterior product of the forms, in Eq. (25) the differentials of functions $(K_i\,\overline{e}_A)$ and $(I_l\,\overline{e}_A)$ figured in bilinear differential

$$\begin{aligned} &\delta\, d\,\overline{e}_A = i[\delta\,\omega^i(d)\, X_i + \delta\,\vartheta^l(d)\, I_l]\, \overline{e}_A + \\ &i[\omega^i(d)\, \delta(K_i\,\overline{e}_A) + \vartheta^l(d)\, \delta(I_l\,\overline{e}_A)], \end{aligned} \tag{26}$$

are defined according to Eq. (20). Equating to zero the coefficients at the same linearly independent generators of exterior differential in Eq. (25), this yields the following system of equations:

$$\begin{aligned} &(\omega^i)' = [\omega^k,\, \vartheta^\beta]\, \varepsilon_{ik\beta}, \\ &(\vartheta^\gamma)' = [\vartheta^\alpha,\, \vartheta^\beta]\, \varepsilon_{\gamma\alpha\beta}/2 + \varepsilon_{\gamma ki}[\omega^k,\, \omega^i]/2. \end{aligned} \tag{27}$$

Defining new forms $\omega^i_k = \vartheta^\beta \varepsilon_{ik\beta}$, and using the relations $\varepsilon_{k\alpha\beta}\,\varepsilon_{ijk} = (\varepsilon_{i\beta k}\,\varepsilon_{k\alpha j} - \varepsilon_{i\alpha k}\,\varepsilon_{k\beta j})$, which stem from the Yacobi's identity

$$\{[K_i[I_\alpha,\, I_\beta]]\} + \quad \text{permutations} \equiv 0, \tag{28}$$

the Eqs. (27) yield the Maurer-Cartan's structure equations

$$\begin{aligned} &(\omega^i)' = [\omega^k,\, \omega^i_k], \\ &(\omega^i_k)' = -R^l_{jki}[\omega^k,\, \omega^i]/2 + [\omega^k_j,\, \omega^l_k], \end{aligned} \tag{29}$$

where $R^l_{jki} = -\varepsilon_{lj\gamma}\varepsilon_{\gamma ki}$ is the curvature of the group space. These equations, as usual, describe the motion of orthogonal reper joint to given point of group space. The forms ω^i and ω^i_k are interpreted as transformations of translation and rotation of the orthogonal reper, i.e., the rotation $(\vartheta^l I_l)$ belongs to stationary group H, but the translation reads $(\omega^i K_i)$. The invariant constraint Eq. (24) has as an immediate consequence that there always exists the rotation transformation of stationary subgroup H annulling the change in equation of Cartan's forms arisen from the translation transformation of the quotien space $\overline{Q} = G_D/H$. The forms ω^i and ϑ^l can be used to construct the group invariants, namely the phenomenological Lagrangians. The Lagrangian of Goldstone fields $(\overline{a})$ can be identified with the square of interval of the geodesic line, with minimal number of derivatives, between the infinitely closed points $\overline{a}^i$ and $\overline{a}^i + d\,\overline{a}^i$:

$$L_{\overline{a}}(\eta) = \tfrac{1}{2}\,\omega^i(\overline{a},\,\partial_A\,\overline{a})\,\omega^i(\overline{a},\,\partial_A\,\overline{a}), \tag{30}$$

where $\varepsilon_{\alpha il}\varepsilon_{l\alpha j} = -\delta_{ij}$ is the metrical tensor of the group space, $\partial_A = (\partial/\partial\eta^A)$, η is the local coordinates in open neighborhood of $p \in M_6$. In normal coordinates it becomes

$$\begin{aligned}&L_{\overline{a}}(\eta) = (\partial_A\,\overline{a})^2/2+\\&(\delta_{ik} - \overline{a}^i\overline{a}^k/\overline{a}^2)\left(\sin^2\,\sqrt{\overline{a}^2}/\sqrt{\overline{a}^2} - 1\right)\partial_A\,\overline{a}^i\,\partial_A\,\overline{a}^k/2.\end{aligned} \tag{31}$$

Since the massless gauge field (a) associates with the gauge group U^{loc}, the Lagrangian Eq. (31) should be equated to undegenerated Killing form defined on the Lie algebra of the group U^{loc} in adjoint representation

$$L_{\overline{a}}(\eta) = L_a(\eta) = -\tfrac{1}{4} < F_{AB}(a),\, F^{AB}(a) >_K, \tag{32}$$

where $F_{AB}(a)$ is the antisymmetrical tensor of gauge field (a). The Goldstone fields $(\overline{a})$ can be determined from the Eq. (32) as the functions of gauge field (a). The covariant derivatives of matter fields Φ interacting with the Goldstone fields $(\overline{a})$ can be determined by means of the form ϑ^α as

$$L = L_0\left(\Phi,\,\partial_A\Phi + \vartheta^\alpha(\overline{a},\,\partial_A\,\overline{a})T_\alpha\,\Phi\right), \tag{33}$$

where $L_0\,(\Phi,\,\partial_A\Phi)$ is the Lagrangian of interacting matter fields classified by the linear representations T_α of the subgroup H. The Lagrangians (32) and (33) are invariant with respect to distortion translations and rotations. This must be completed by the transformations of the fields Φ

$$\Phi' = (\exp\left[i\theta'^\alpha(\overline{a},\,g)T_\alpha\right])\,\Phi, \tag{34}$$

where $\theta'^\alpha(\overline{a},\,g)$ is given by Eq. (A31).

C. Static line element with spherical-symmetry

The field equations can be derived from an invariant action $S = S_a + \widetilde{S}_{\widetilde{\Phi}}$, which is similar to Eq. (A1). The action of distortion gauge field S_a given on the flat space M_6 is invariant under the Lorentz (Λ) and gauge (U^{loc}) groups, while the action of matter fields $\widetilde{S}_{\widetilde{\Phi}}$ given on the curved space R_6 is invariant under the gauge group of gravitation G_R. Field equations may immediately follow in terms of Euler-Lagrange variations carried out in the spaces M_6 and R_6, respectively. The field equation for dimensionless potential $x_A \equiv æ\,a_A$ can be obtained from the Lagrangian (32) as

$$\begin{aligned}&\partial^B\partial_B\,x_A - (1-\zeta_0^{-1})\partial_A\partial^B\,x_B =\\&-\tfrac{1}{2}\,æ^2\,\sqrt{g(x)}\,\tfrac{\partial g^{BC}(x)}{\partial x_A}\widetilde{T}_{BC},\end{aligned} \tag{35}$$

where $\partial_B = \partial/\partial\eta^B$, η^B are the coordinates in the given space R_B^3, $\widetilde{T}_{BC}$ denotes the energy-momentum tensor, ζ_0 is the gauge fixing parameter. To render our discussion here more transparent, below we clarify a relation between gravitational and coupling constants. To assist in obtaining actual solutions from field equations, we may consider the weak-field limit and shall envisage that the right hand side of Eq. (35) should be in the form

$$-\tfrac{1}{2}\,(4\pi G_N)\,\sqrt{g(x)}\,\tfrac{\partial g^{BC}(x)}{\partial x_A}\widetilde{T}_{BC}. \tag{36}$$

Hence, we may assign to the Newton's gravitational constant G_N the value

$$G_N = æ^2/4\pi. \tag{37}$$

Remark: Although the distortion gauge field (a_A) is vector field, nevertheless the key relation Eq. (37) shows that only the gravitational attraction presents in proposed theory of gravitation.

To obtain a feeling for this point we may consider physical systems which are static as well as spherically symmetrical. Upon the reduction $R_6 \to R_4$, we have the group of motions $SO(3)$ with two-dimensional space-like orbits S^2 where the standard coordinates are $\widetilde{\theta}$ and $\widetilde{\varphi}$. The stationary subgroup of $SO(3)$ acts isotropically upon the tangent space at the point of sphere S^2 of radius $\widetilde{r}$. So, the bundle $\widetilde{\pi} : R_4 \to \widetilde{R}^2$ has the fiber $S^2 = \widetilde{\pi}^{-1}(\widetilde{x})$, $\quad \widetilde{x} \in R_4$ with a trivial connection on it, where $\widetilde{R}^2$ is the quotient-space $R_4/SO(3)$. The coordinates $\widetilde{x}^\mu(\widetilde{t},\widetilde{r},\widetilde{\theta},\widetilde{\varphi})$ implying the diffeomorphism $\widetilde{x}^\mu(x,a) : M_4 \to R_4$ exist in the whole region $\widetilde{\pi}^{-1}(\widetilde{\mathcal{U}}_i) \in R_4$, where $\widetilde{x}^{0r} \equiv \widetilde{t}$, $\quad \widetilde{x}^{0\theta} = \widetilde{x}^{0\varphi} = 0$. In outside of configuration of given mass, the field equation Eq. (35) can be written in Feynman gauge as $\nabla^2 a_0 = 0$, which has the solution $x_0 = -r_g/2r$, where $x_0 \equiv æ\,a_0(r)$, r_g is the gravitational radius. The components of transformation matrix are $D_{\widetilde{0}}^0 = 1 + x_0$, $\quad D_{\widetilde{r}}^r = 1 - x_0$, $\quad D_{\widetilde{\theta}}^\theta = D_{\widetilde{\varphi}}^\varphi = 1$. From Eq. (3) and Eq. (7), we obtain

$$\tfrac{\partial\widetilde{x}^\mu}{\partial x^l} \equiv \psi_l^\mu = D_l^\mu(1 + \omega(F)). \tag{38}$$

It can be easily verified that either $\partial_r\psi_0^0 \neq \Gamma_{01}^0\psi_0^0$, or $\partial_r\psi_1^1 \neq \Gamma_{11}^1\psi_1^1$, etc., and the curvature of the space R_4 is not zero. The line element then reads

$$\begin{aligned}&d\,\widetilde{s}^2 = (1 - \tfrac{r_g}{2r})^2\,d\widetilde{t}^2 - (1 + \tfrac{r_g}{2r})^2\,d\widetilde{r}^2-\\&\widetilde{r}^2(\sin^2\widetilde{\theta}\,d\,\widetilde{\varphi}^2 + d\,\widetilde{\theta}^2),\end{aligned} \tag{39}$$

provided, the Eq. (38) and Eq. (39) give the relation

$$\frac{dg_{00}}{dr} = \frac{dg_{00}}{d\widetilde{r}}\frac{1+\omega(F)}{1+\frac{r_g}{2r}} = \frac{r_g}{r^2}(1-\frac{r_g}{2r}), \tag{40}$$

for determining the function $\widetilde{r}(r)$. We must now turn to the actual correspondence between the expression (39) for the line element surrounding an attracting central body and the observational facts of astronomy. The investigating methods are so well known that it will be sufficient for our purposes merely to indicate the classical tests of GR conducted in solar-system dealing only with the shape of the trajectories of photons and planets. All these tests are carried out in empty space and in gravitational fields that are to a good approximation static and spherically symmetric. Therefore, in sufficient approximation, at great distances from the central body

$$F = (4r_g^2 æ^2 F_{mn}F_{mn})^{1/4} = r_g/r << 1, \tag{41}$$

we may take expansion of function $\omega(F) = \lambda_1 F + \lambda_2 F^2 + \cdots$, and that from Eq. (40) we obtain

$$\widetilde{r} = r(1 + \alpha_1 F + \alpha_2 F^2 + \cdots), \tag{42}$$

provided $\alpha_1 = 1/2 - \lambda_1$, $\alpha_2 = \lambda_2 + 1/4 + 4(\lambda_1 - 1/4)^2$, etc. Then, in terms up to the second order in $\widetilde{F}\quad(= r_g/\widetilde{r})$, which is an approximation of interest for available observational verifications, the temporal component of metrical tensor reduces to $g_{00} \simeq 1 - \widetilde{F} + (\lambda_1 - 1/4)\widetilde{F}^2$. With these provisions, Eq. (39) is reduced to standard Schwarzschild line element with the metrical tensor components as follows:

$$\begin{aligned} &g_{00} \simeq 1 - \frac{r_g}{\widetilde{r}} + (\lambda_1 - \tfrac{1}{4})\frac{r_g^2}{\widetilde{r}^2}, \\ &g_{11} \simeq -(1 + \frac{r_g}{\widetilde{r}} + \cdots), \\ &g_{22} = -\widetilde{r}^2, \quad g_{33} = -\widetilde{r}^2 \sin^2\widetilde{\theta}. \end{aligned} \tag{43}$$

The free adjustable parameter λ_1 in Eq. (43) can be written in terms of Eddington-Robertson expansion parameters as $\lambda_1 = 1/4 + 2(\beta - \gamma)$. While γ controls also other relativistic effects, in particular those related to gravitomagnetism, it mainly affects electromagnetic propagation. The differential displacement of the stellar images near the Sun historically was the first experimental effect to be investigated and is now of great importance in accurate astrometry. The bending of a light ray also increases the light-time between two points, an important effect usually named after its discoverer I. I. Shapiro [49]. Several experiments to measure this delay have been successfully carried out, using *wide-band* microwave signals passing near the Sun and transponded back, either passively by planets, or actively, by space probes, see [40, 46]. The very long baseline interferometry (VLBI) has achieved accuracies of better than 0.1 mas (milliarcseconds of arc), and regular geodetic VLBI measurements have frequently been used to determine the space curvature parameter γ [43, 45, 47, 48, 50], resulting in the accuracy of better than $\sim$ 0.045% in the tests of gravity via astrometric VLBI observations. Detailed analysis of VLBI data have yielded a consistent stream of improvements $\gamma = 1.0000.003$ [47, 48], $\gamma = 0.99960.0017$ [45], $\gamma = 0.999940.00031$ [43] and $\gamma = 0.999830.00045$ [50] resulting in the accuracy of better than $\sim$ 0.045%. The major advances in several disciplines notably in microwave spacecraft tracking, high precision astrometric observations, and lunar laser ranging (LLR) suggest new experiments. LLR, a continuing legacy of the Apollo program, provided improved constraint on the combination of parameters $4\beta - \gamma - 3$ [52–55]. The analysis of LLR data [53] constrained this combination as $4\beta - \gamma - 3 = (4.0 \pm 4.3) \times 10^{-4}$, leading to an accuracy of $\sim$ 0.011% in verification of general relativity via precision measurements of the lunar orbit. A significant improvement was reported in 2003 from Doppler tracking of the Cassini spacecraft while it was on its way to Saturn [41], with a result $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$. This was made possible by the ability to do Doppler measurements using both X-band (7175 MHz) and Ka-band (34316 MHz) radar, thereby significantly reducing the dispersive effects of the solar corona. In addition, the 2002 superior conjunction of Cassini was particularly favorable: With the spacecraft at 8.43 astronomical units from the Sun, the distance of closest approach of the radar signals to the Sun was only $1.6R_\odot$. This experiment has reached the current best accuracy of $\sim$ 0.002% [42]. Keeping in mind aforesaid, the best fit for satisfactory agreement between the proposed theory of gravitation and observation can be reached at $\lambda_1 - 1/4 = (2.95 \pm 3.24) \times 10^{-5}$.

IV. THE RELATIVISTIC FIELD THEORY OF INERTIA

As we mentioned in Sect.2, in the proposed theory of gravitation, the preferred systems and group of transformations of the *real-curvilinear* coordinates relate only to the real gravitational fields. This prompts us to introduce separately the *distortion inertial fields*, which have other physical source than that of gravitation, and construct the relativistic field theory of inertia. The latter, similarly to gravitation theory, treats the *inertia* as a distortion of local internal properties of flat M_2 space. The geometry of Sect.3 is a language which is almost indispensable for the treatment of this problem.

A. The case of unbalanced net force other than gravitational

First, we shall discuss the inertia effects in particular case when the relativistic test particle accelerated in the flat space under unbalanced net force other than gravitational. Let us concentrate our attention on the first observer in two-dimensional Minkowski flat space $M_2 = R^1_{(+)} \oplus R^1_{(-)} = R^1 \oplus T^1$, being regarded as in a state of rest or uniform motion. Suppose this unaccelerated observer for the position of free test particle in M_2 uses the inertial coordinate frame $S_{(2)}$ corresponding to spatial $q \in R^1$ and temporal $t \in T^1$ variables $q^a(q^1, q^0) \equiv (q, t)$ $(a = 1, 0)$, and to the formula for interval $d\hat{\eta}^2 = ds_q^2 = dt^2 - dq^2$, while the ingredient spaces $R^1_{(\pm)}$ are spanned by the coordinates $\eta^{(\pm1)}$, respectively. Translating this into the language of geometry of the Sect.3, upon reduction we may write

$$\begin{array}{l} d\hat{\eta} = (O_+ d\eta^{(+1)} + O_- d\eta^{(-1)}) \times \sigma_1 = \\ d\hat{q} \equiv e_0 dt + e_q dq, \end{array} \tag{44}$$

where $e_0 = \xi_0 \times \sigma_1$ and $e_q = \xi \times \sigma_1$ are, respectively, the temporal and spatial basis vectors along the axes of $S_{(2)}$, and that $q = (1/\sqrt{2})(\eta^{(+1)} - \eta^{(-1)})$, $t = (1/\sqrt{2})(\eta^{(+1)} + \eta^{(-1)})$, and $v^{(\pm1)} = (d\eta^{(\pm1)}/dt) = (1/\sqrt{2})(1 \pm v_q)$, $v_q = (dq/dt) = const$. The law of inertia states that a free particle in motion of uniform speed ($v_q = const$) in a straight line in free space R^1 tends to stay in this motion and a particle at rest tends to stay at rest unless acted upon by an unbalanced force. Below, we introduce the most important for the treatment of inertia new concepts of *relative* state and universal, so-called, *absolute* state of ingredient space $R^1_{(\pm)}$. The key measure for these states will be the magnitude of the velocity components $(v^{(+1)}, v^{(-1)})$ of particle of interest.

Definition. The space $R^1_{(\pm)}$ is in the *absolute* (abs) state if $v^{(\pm1)} = 0$; the space $R^1_{(\pm)}$ is in the *relative* (rel) state if $v^{(\pm1)} \neq 0$.

According to it, the space M_2 can be realized respectively in the following three states: the *semi-absolute* states (rel, abs) or (abs, rel), and *total relative* state (rel, rel). It is remarkable that the *total-absolute* state (abs, abs) of M_2, which is equivalent to the unobservable Newtonian *absolute* two-dimensional spacetime, cannot be realized because of $v^{+1} + v^{-1} = \sqrt{2}c$ (we re-instate the factor (c)), where $v^{\pm1} \geq 0$. The existence of *absolute* state of $R^1_{(+)}$ is the immediate cause of the light traveling in free space R^1 along q-axis with the resulting maximal velocity $v_q = c$, respectively, in $(+)$-direction in case of $(v^{(+1)}, 0) \Leftrightarrow$ (rel, abs) and in $(-)$- direction in case of $(0, v^{(-1)}) \Leftrightarrow$ (abs, rel). Also note that the *absolute* state of $R^1_{(+)}$ manifests its *absolute* character in the important for special relativity fact that the resulting velocity of light in free space R^1 is the same for all inertial frames, that is, if $v^{(\pm1)} = 0$ then $v^{(\pm1)} = v^{(\pm1)\prime} = v^{(\pm1)\prime\prime} = ... = 0$. The velocity $v^{(\pm1)} \neq 0$ is the measure of difference from the *absolute* state. We might expect that this has a substantial effect in alteration of particle motion under the unbalanced force. Similar reasoning prompts us, further, to introduce the *distortion inertial field* potential which depends on the rate of change of this measure and allows the physical interpretation of the RLI as follows:

RLI: *The rate of change of constant velocity* $(v^{(\pm1)})$ *(both magnitude and direction) of massive* (m) *test particle under the unbalanced net force* (f) *is the immediate cause of distortion of local internal properties of flat space* $M_2 \to R_2$ *conducted under the distortion inertial field potential*

$$æa^{(\pm1)}_{(in)}(q, t) = \pm\varrho(q, t, m, f)v^{(\pm1)}. \tag{45}$$

The function $\varrho(q, t, m, f)$, which dependents on the rate of change of the $v^{(\pm1)}$, will be determined below. Following general prescription of Eq. (12), the distortion transformations of basis vectors O_λ may be recast as

$$\begin{array}{l} \widetilde{O}_{(+1)} = \mathcal{Q}^\tau_{(+1)}(a)\, O_\tau = O_+ + æa^{(+1)}_{(in)} O_-, \\ \widetilde{O}_{(-1)} = \mathcal{Q}^\tau_{(-1)}(a)\, O_\tau = O_- + æa^{(-1)}_{(in)} O_+. \end{array} \tag{46}$$

Now let a second observer, who makes measurements using a frame of reference $\widetilde{S}_{(2)}$ which is held stationary in distorted space R_2, uses for the test particle the real-curvilinear coordinates $\widetilde{q}^a(\widetilde{q}, \widetilde{t})$, where $\widetilde{q} = (1/\sqrt{2})(\widetilde{\eta}^{(+1)} - \widetilde{\eta}^{(-1)})$, $\widetilde{t} = (1/\sqrt{2})(\widetilde{\eta}^{(+1)} + \widetilde{\eta}^{(-1)})$. The choice of (46) has completely fixed the original form of the interval we are to use $d\widetilde{s}_q^2 \equiv d\hat{\widetilde{\eta}}^2$, provided

$$\begin{array}{l} d\hat{\widetilde{\eta}} = (\widetilde{O}_{(+1)} d\widetilde{\eta}^{(+1)} + \widetilde{O}_{(-1)} d\widetilde{\eta}^{(-1)}) \times \sigma_1 = \\ d\hat{\widetilde{q}} = \widetilde{e}_0 d\widetilde{t} + \widetilde{e}_q d\widetilde{q}, \end{array} \tag{47}$$

where $\widetilde{e}_0 = \widetilde{\xi}_{01} \times \sigma_1$ and $\widetilde{e}_q = \widetilde{\xi}_1 \times \sigma_1$ are, respectively, the temporal and spatial basis vectors, and that $\widetilde{\xi}_{01} = (1/\sqrt{2})(\widetilde{O}_{(+1)} + \widetilde{O}_{(-1)})$, $\widetilde{\xi}_1 = (1/\sqrt{2})(\widetilde{O}_{(+1)} - \widetilde{O}_{(-1)})$. The Eq. (3) now gives

$$\begin{array}{l} d\widetilde{\eta}^A = \frac{\partial\widetilde{\eta}^A}{\partial\eta^C} d\eta^C = \\ \left[< \widetilde{O}^A, O_C > + < \widetilde{O}^A, \chi_C(e, F) > \right] d\eta^C, \end{array} \tag{48}$$

where the capital Latin indices A, C, etc. run over (±1). In terms of corresponding matrices $(\cdots)$ it can be rewritten as

$$(d\widetilde{\eta}^A) = \left[(< \widetilde{O}^A, O_C >) + (< \widetilde{O}^A, \chi_C >) \right] (d\eta^C), \tag{49}$$

where $(d\widetilde{\eta}^A) = \begin{pmatrix} d\widetilde{\eta}^{(+1)} \\ d\widetilde{\eta}^{(-1)} \end{pmatrix}$ and so on. Denoting $\mathcal{Q}_\eta = (< \widetilde{O}_A, O^C >)$, we obtain

$$\begin{array}{l} (< \widetilde{O}^A, O_C >) = \mathcal{Q}_\eta^{-1} = \\ \gamma_a \begin{pmatrix} 1 & -æa^{(+1)}_{(in)} \\ -æa^{(-1)}_{(in)} & 1 \end{pmatrix}, \end{array} \tag{50}$$

where $\gamma_a = (1 - æ^2 a^{(+1)}_{(in)} a^{(-1)}_{(in)})^{-1} = (1 + \varrho^2/2\gamma_q^2)^{-1}$, $\gamma_q = (1 - v_q^2)^{-1/2}$. The transformation equation for coordinates becomes

$$d\widetilde{\eta}^{(\pm1)} = \gamma_a (d\eta^{(\pm1)} \mp \varrho v^{(\pm1)} d\eta^{(\mp1)}), \tag{51}$$

while $\chi_{(\pm1)} = e_{(\pm1)}\omega(0) = 0$. Actually, the temporal $æa^0_{(in)} = \varrho v_q$ and spatial $æa^1_{(in)} = \varrho$ components of inertial field yield $F_{10} = \partial_q(\varrho v_q) - \partial_0 \varrho = 0$.

Then, $d\hat{\tilde{q}} = d\hat{q}$ or $d\tilde{s}_q^2 = ds_q^2 = dt^2/\gamma_q^2$, that is, $g_{ab} = (\partial q^c/\partial\tilde{q}^a)(\partial q^d/\partial\tilde{q}^b)\eta_{cd}$, $(a,b,c,d = 0,1)$, where the components of the metrical tensor η_{cd} assume the values $(-1,1,0)$. Introducing new coordinates $d\tilde{\eta}'^A = \gamma_a^{-1}d\tilde{\eta}^A$, the metrical tensor transformed to $g'_{AB} = (\partial\tilde{\eta}^C/\partial\tilde{\eta}'^A)(\partial\tilde{\eta}^D/\partial\tilde{\eta}'^B)g_{CD} = \gamma_a^2 g_{AB}$, where $g_{AB} = D_A^C D_B^C$. To retain former notational conventions, from now on we will omit the prime at the quantities $\tilde{\eta}'^{(A)}$, $\tilde{q}'$, $g'_{AB}, ...$ The Eq. (51) then becomes

$$d\tilde{\eta}^{(\pm1)} = d\eta^{(\pm1)} \mp \varrho v^{(\pm1)}d\eta^{(\mp1)} = (v^{(\pm1)} \mp \tfrac{\varrho}{2\gamma_q^2})dt. \tag{52}$$

The generally covariant expression for interval is $d\tilde{s}_q^2 = (d\hat{\tilde{q}})^2 = g_{ab}d\tilde{q}^a d\tilde{q}^b$, provided

$$\begin{aligned} g_{00} &= (1+\tfrac{\varrho^2}{2\gamma_q^2})^{-2}[(1+\tfrac{\varrho v_q}{\sqrt{2}})^2 + \tfrac{\varrho^2}{2}], \\ g_{11} &= -(1+\tfrac{\varrho^2}{2\gamma_q^2})^{-2}[(1-\tfrac{\varrho v_q}{\sqrt{2}})^2 - \tfrac{\varrho^2}{2}], \\ g_{10} &= g_{01} = -\sqrt{2}\varrho\,(1+\tfrac{\varrho^2}{2\gamma_q^2})^{-2}. \end{aligned} \tag{53}$$

It is easily verified that the resulting curvature is not zero because of inequalities $\partial_q\psi_0^0 \neq \Gamma_{01}^0\psi_0^0$, or $\partial_q\psi_1^1 \neq \Gamma_{11}^1\psi_1^1$, etc, where $\psi_l^\mu = D_l^\mu$. The difference of the vectors $d\hat{\tilde{\eta}}$ (44) and $d\hat{\tilde{\eta}}$ (47) could be interpreted by the second observer as being due to the distortion of basis vectors $O_\pm$ of flat space. However, this difference with equal justice could be interpreted by him as a definite criterion for the *absolute* character of his own state of acceleration, rather than to any absolute quality of distortion of local internal properties of flat space. To prove this assertion, we may derive from the Eq. (52) the general transformation equations for spatial and temporal intervals to accelerated axes as

$$\begin{aligned} d\tilde{q} &= dq(1+\tfrac{\varrho v_q}{\sqrt{2}}) - \tfrac{\varrho}{\sqrt{2}}dt, \\ d\tilde{t} &= dt(1-\tfrac{\varrho v_q}{\sqrt{2}}) + \tfrac{\varrho}{\sqrt{2}}dq. \end{aligned} \tag{54}$$

The foregoing transformation equations give a reasonable change at low velocities $\varrho/\sqrt{2} << 1 \quad (v_q \sim 0)$: $d\tilde{q} \simeq dq - (\varrho/\sqrt{2})dt, \quad d\tilde{t} \simeq dt$, which become conventional transformation equations to accelerated $(a \neq 0)$ axes at $\varrho/\sqrt{2} = \int_0^t a dt'$. This immediately indicates that we may introduce (in Newton's terminology) the *absolute* acceleration as

$$\mathbf{a}_{abs} \equiv e_q \frac{d\varrho}{\sqrt{2}ds_q}. \tag{55}$$

We may also introduce the, so-called, *inertial* acceleration

$$\mathbf{a}_{in} \equiv e_q a^1 = e_q \frac{d^2\tilde{q}}{ds_q^2} = e_q \frac{1}{\sqrt{2}}(\frac{d^2\tilde{\eta}^{(+1)}}{ds_q^2} - \frac{d^2\tilde{\eta}^{(-1)}}{ds_q^2}). \tag{56}$$

Combining (52), (55) and (56), we obtain the key relation

$$\gamma_q \mathbf{a}_{in} = -\mathbf{a}_{abs}. \tag{57}$$

Suppose the position of test particle in the space M_4, in general, is specified by the coordinates $x^l(s)$ $(l = 1,2,3,0)$ with respect to the axes of inertial system $S_{(4)}$. The specific problem that now arises is to obtain equations connecting the *absolute* acceleration (Eq. (55)) given in the inertial system $S_{(2)}$ to the unbalanced relativistic force ([59]): $f^l(\mathbf{f}, f^0) = m(d^2x^l/ds^2) = \Lambda_k^l(\mathbf{v})F^k$, exerted on the test particle. Here $F^k(\mathbf{F}, 0)$ is the force in the proper reference frame of test particle, $\Lambda_k^l(\mathbf{v})$ is the Lorentz transformation matrix $(i,j = 1,2,3)$: $\Lambda_j^i = \delta_{ij} - v_i v_j(\gamma-1)/\mathbf{v}^2$, and $\Lambda_i^0 = \gamma v_i$, where $\gamma = (1-\mathbf{v}^2)^{-1/2}$. Then the two systems can be chosen so that the axis e_q of $S_{(2)}$ lies along the acting force $\mathbf{f} = e_f|\mathbf{f}|$ while the time coordinates in the two systems are taken the same $q^0 = x^0 = t$. This choice $(e_q = e_f)$ of unit vectors, which can always be made, implies $v_q = (e_f \cdot \mathbf{v})$, and that the rate of change of the measure of difference from the *absolute* state of massive (m) test particle under the unbalanced net force $f^l(\mathbf{f}, f^0)$ other than gravitational can be determined as

$$\frac{1}{\sqrt{2}}\frac{d\varrho}{ds_q} = \frac{1}{m}|f^l| = \frac{1}{m\gamma}|\mathbf{f}|. \tag{58}$$

The key relation (57) provides quantitative means for the RLI as

$$\begin{aligned} \mathbf{f}_{(in)} &= m\mathbf{a}_{in} = e_q(-m\Gamma_{ab}^1(\varrho)\frac{d\tilde{q}^a}{d\tilde{s}_q}\frac{d\tilde{q}^b}{d\tilde{s}_q}) = \\ &-m\mathbf{a}_{abs}/\gamma_q = -[\mathbf{F} + (\gamma-1)\mathbf{v}(\mathbf{v}\cdot\mathbf{F})/v^2]/\gamma_q\gamma, \end{aligned} \tag{59}$$

where $\mathbf{f}_{(in)}$ is the *inertial* force, also we have taken into account that the trajectory of the particle in curved space R_2 is given by the equation for the geodesic. At low velocities $v_q \simeq |\mathbf{v}| \simeq 0$, the Eq. (59) reduces to conventional law of inertia

$$\mathbf{f}_{(in)} = -\mathbf{F}. \tag{60}$$

At high velocities $|\mathbf{v}| \sim 1$, we have $e_f \simeq e_v$, where $\mathbf{v} = e_v|\mathbf{v}|$, and that $v_q \simeq |\mathbf{v}| \sim 1$. The Eq. (59) then gives

$$\mathbf{f}_{(in)} \simeq -e_v(e_v \cdot \mathbf{F})/\gamma, \tag{61}$$

which vanishes in the limit of the photon $(m = 0)$. This can be easily understood. Certainly, the acceleration of photon in free space along its traveling direction is impossible, as well as its deflection by some force is also discarded because otherwise its velocity becomes greater than (c). Thus, it takes force to disturb inertia state, i.e. to make *absolute* $(\mathbf{a}_{abs})$ acceleration. The *absolute* acceleration is due to the existence of the *absolute* state of ingredient space $R^1_{(\pm)}$ and, evidently, it is admitted as *the immediate cause of the real distortion of the local internal properties of flat space* M_2. The *relative* $(d\varrho/ds_q = 0)$ acceleration (in Newton's terminology) (both magnitude and direction), in contrary, cannot be the cause of the distortion of the space and, thus, it does not produce inertia effect.

B. Involving gravitation; the Principle of Equivalence

In the development of the relativistic field theory of inertia in the more general case when gravitational action is involved, we are at once led to seek equations in the form of generally covariant tensor expression using any set of general coordinates which we may desire to introduce. Let the distortion gauge field (a_l) underlies gravitation. Then, the generally covariant expression for interval in four-dimensional Riemannian space R_4 is $d\widetilde{s}^2 = g_{\mu\nu}(a)d\widetilde{x}^\mu d\widetilde{x}^\nu$, and $\Gamma^\lambda_{\mu\nu}(a)$ denotes affine connection agreed with the metric $g_{\mu\nu}(a)$. Although inertial forces do not exactly cancel for freely falling systems in an inhomogeneous or time-dependent gravitational field, in accordance with the Principle of Equivalence, we may still maintain for a sufficiently small region that the effects of gravitation could be removed by the use of local freely falling coordinate frame $S_4^{(l)}$, having the natural acceleration due to gravity for that region. This implies that the local spacetime structure can be identified with the Minkowski spacetime possessing Lorentz symmetry, and that the physical laws in the frame $S_4^{(l)}$ will be the same as in any inertial coordinate frame $S_{(4)}$. This is similar to assumption of approximate replacement of a curved surface by its tangent plane at a given point, made use of in geometrical considerations. Therefore, we can always choose *natural coordinates* $X^\alpha(X,Y,Z,T) = (\mathbf{X}, T)$ with respect to the axes of the system $S_4^{(l)}$ in immediate neighbourhood of any spacetime point $(\widetilde{x}_p) \in R_4$ in question, over a differential region taken small enough so that we can neglect the spatial and temporal variations of gravity for the range involved. In this coordinates, the special relativity formula for interval will be $dS^2 = \eta_{lk}dX^l dX^k = d\widetilde{s}^2$, where the components of the metrical tensor η_{lk} assume the special relativity values $(-1,1,0)$, and the first differential coefficients of the η_{lk} with respect to these coordinates will be zero at the point (p). In general, however, the second differential coefficients will not be zero except for the special case of spacetime that actually is flat. The values of metrical tensor $g_{\mu\nu}(a)$ and affine connection $\Gamma^\lambda_{\mu\nu}(a)$ at the point $(\widetilde{x}_p)$ is necessarily sufficient information for determination of the natural coordinates $X^\alpha(\widetilde{x}^\mu)$ in the small region of neighbourhood of selected point [59]. The whole scheme outlined in the previous subsection will then hold in the frame $S_4^{(l)}$, provided as a preliminary step we first examine the possibility of re-expressing the special relativity formula for the free $(\mathbf{f}_{(l)} = 0)$ test particle $dU^\alpha/dS = d^2X^\alpha/dS^2 = 0$, $(\alpha = 1,2,3,0)$ in generally covariant form as $DU^\alpha/D\widetilde{s} = D^2X^\alpha/D\widetilde{s}^2 = 0$, where $\mathbf{f}_{(l)}$ is the special relativity value of unbalanced relativistic force other than gravitational in the frame $S_4^{(l)}$ and, according to the general prescription $D/D\widetilde{s}$ is the covariant derivative along the curve $\widetilde{x}^\mu(\widetilde{s}) \in R_4$. The relativistic gravitational force $f^\mu_g(\widetilde{x})$ exerted on the test particle of the mass (m) is

$$f^\mu_g(\widetilde{x}) = m\frac{d^2\widetilde{x}^\mu}{d\widetilde{s}^2} = -m\Gamma^\mu_{\nu\lambda}(a)\frac{d\widetilde{x}^\nu}{d\widetilde{s}}\frac{d\widetilde{x}^\lambda}{d\widetilde{s}}. \quad (62)$$

The frame $S_4^{(l)}$ will be valid if only the gravitational force given in this coordinate frame

$$f^\alpha_{g(l)} = \frac{\partial X^\alpha}{\partial \widetilde{x}^\mu} f^\mu_g, \quad (63)$$

could be removed by the inertial force which, in turn, is due to the distortion inertial field potential Eq. (45). Whereas, the two systems S_2 and $S_4^{(l)}$ can be chosen so that the axis e_q of $S_{(2)}$ now lies along $(e_q = e_f)$ the acting net force $\mathbf{f} = \mathbf{f}_{(l)} + \mathbf{f}_{g(l)}(a)$, while the time coordinates in the two systems are taken the same $q^0 = t = X^0 = T$. Instead of Eq. (58), we now have

$$\frac{1}{\sqrt{2}}\frac{d\varrho}{ds_q} = \frac{1}{m}|f^\alpha_{(l)} + f^\alpha_{g(l)}(a)|. \quad (64)$$

Hence, in general, the RLI can be obtained as

$$\begin{aligned} \mathbf{f}_{(in)} = m\mathbf{a}_{in} = e_q(-m\Gamma^1_{ab}(\varrho)\frac{d\widetilde{q}^a}{d\widetilde{s}_q}\frac{d\widetilde{q}^b}{d\widetilde{s}_q}) = \\ -\frac{m\mathbf{a}_{abs}}{\gamma_q} = -\frac{e_f}{\gamma_q}|f^\alpha_{(l)} - m\frac{\partial X^\alpha}{\partial \widetilde{x}^\sigma}\Gamma^\sigma_{\mu\nu}(a)\frac{d\widetilde{x}^\mu}{dS}\frac{d\widetilde{x}^\nu}{dS}|. \end{aligned} \quad (65)$$

In spite of totally different and independent sources of gravitation and inertia, at $\mathbf{f}_{(l)} = 0$ when the mass (m) is canceled out in Eq.(65), the RLI indeed furnishes justification for introduction of the Principle of Equivalence. A remarkable feature is that the inertial force is of the same nature as gravitational force, i.e., both are due to the distortion of local internal properties of the flat space. The non-vanishing inertial force acting on the photon of energy $h\nu$, and that of effective mass $\left(h\nu/c^2\right)$ after inserting a factor (c^2) which so far was suppressed, can be obtained from the Eq. (65), at $\mathbf{f}_{(l)} = 0$, as

$$\begin{aligned} \mathbf{f}_{(in)} = -\left(\frac{h\nu}{c^2}\right)e_f|\frac{\partial X^\alpha}{\partial \widetilde{x}^\sigma}\Gamma^\sigma_{\mu\nu}(a)\frac{d\widetilde{x}^\mu}{dT}\frac{d\widetilde{x}^\nu}{dT}| = \\ -\left(\frac{h\nu}{c^2}\right)e_f|(\frac{d^2\widetilde{t}}{dT^2})\frac{dX^\alpha}{d\widetilde{t}} + (\frac{d\widetilde{t}}{dT})^2\frac{\partial X^\alpha}{\partial \widetilde{x}^i}\frac{du_i}{d\widetilde{t}}|, \end{aligned} \quad (66)$$

provided $e_f = (\mathbf{X}/|\mathbf{X}|)$, $v_q = (e_f \cdot \mathbf{u}) = |\mathbf{u}|$, $(\gamma_q = \gamma)$ where $(\mathbf{u})$ is the velocity of photon and $(d\mathbf{u}/d\widetilde{t})$ is the acceleration, and that, $g_{\mu\nu}(a)(d\widetilde{x}^\mu/dT)(d\widetilde{x}^\nu/dT) = 0$. To obtain some feeling for this, we may turn now to the (PPN) approximation [37–40], which can be regarded as a deformation of a background asymptotically flat Minkowski metric. In this context we calculate the inertial force for the photon in gravitating system of particles that are bound together by their mutual gravitational attraction to order $\bar{v}^2 \sim G_N\bar{M}/\bar{r}$ of small parameter, where $\bar{v}$, $\bar{M}$ and $\bar{r}$ are typical , respectively, the average values of their velocities, masses and separations. In doing this, we may expand the metrical tensor to the following order: $g_{00} = 1 + \overset{2}{g}_{00} + \overset{4}{g}_{00} + ...$, $g_{ij} = -\delta_{ij} + \overset{2}{g}_{ij} + \overset{4}{g}_{ij} + ...$, $g_{i0} = \overset{3}{g}_{i0} + \overset{5}{g}_{i0} + ...$, where $\overset{N}{g}_{\mu\nu}$ denotes the term of order $\bar{v}^N$. Taking into account the standard expansions of affine connection $\Gamma^\sigma_{\mu\nu} = \overset{2}{\Gamma}{}^\sigma_{\mu\nu} + \overset{4}{\Gamma}{}^\sigma_{\mu\nu} + ...$ for the components Γ^i_{00}, Γ^i_{jk}, Γ^0_{0i}, and that $\Gamma^\sigma_{\mu\nu} = \overset{3}{\Gamma}{}^\sigma_{\mu\nu} + \overset{5}{\Gamma}{}^\sigma_{\mu\nu} + ...$ for the components Γ^i_{0j}, Γ^0_{00}, Γ^0_{ij}, where $\overset{2}{\Gamma}{}^i_{00} = \overset{2}{\Gamma}{}^0_{0i} = -(1/2)(\partial \overset{2}{g}_{00}$

$/\partial\widetilde{x}^i)$ etc., hence to the required accuracy we obtain

$$\begin{array}{l}\overset{2}{\mathbf{f}}_{(in)}= -\left(\frac{h\nu}{c^2}\right) e_f|\,(\frac{\partial \overset{1}{X^\alpha}}{\partial\widetilde{x}^\sigma})(\frac{d^2\overset{2}{\widetilde{x}^\sigma}}{dT^2})\,| = -\left(\frac{h\nu}{c^2}\right)(\frac{d\overset{2}{\mathbf{u}}}{d\widetilde{t}})= \\ -\left(\frac{h\nu}{c^2}\right)[-2\nabla\phi+\mathbf{4u(u\cdot\nabla\phi)+O(\bar{v}^3)}],\end{array} \quad (67)$$

where ϕ is the Newton's potential, such that $\overset{2}{g}_{00}= 2\phi$, $\overset{2}{g}_{ij}= 2\delta_{ij}\phi$, and $|\mathbf{u}| = 1+2\phi+O(\bar{v}^3)$.

V. THE REARRANGEMENT OF VACUUM STATE

Collecting together the results just established in previous sections we finally arrive at discussion of the rearrangement of vacuum in gravitation. To trace this line, in realization of the gravitation gauge group G_R we implement the abelian local group

$$U^{loc} = U(1)_Y \times \overline{U}(1) \equiv U(1)_Y \times diag[SU(2)], \quad (68)$$

with the group elements of $exp\,[i\frac{Y}{2}\,\theta_Y(\eta)]$ of $U(1)_Y$ and $exp\,[iT^3\,\theta_3(\eta)]$ of $\overline{U}(1)$. The group Eq. (68) leads to the renormalizable theory on M_6 because gauge invariance gives conservation of charge, also ensures the cancelation of quantum corrections that would otherwise result in infinitely large amplitudes. This has two generators, the third component T^3 of isospin $\mathbf{T}$ related to the Pauli spin matrix $\frac{\tau}{2}$, and hypercharge Y implying

$$Q^d = T^3 + \frac{Y}{2},$$

where Q^d is the *distortion charge* operator assigning the number -1 to particles, but +1 to anti-particles. The group Eq. (68) entails two neutral gauge bosons W^3_A of $\overline{U}(1)$ or that coupled to T^3, and B_A of $U(1)_Y$, or that coupled to hypercharge Y. Gauge invariant Lagrangian of fermion field is given in standard form $\mathcal{L} = \overline{\psi}(\eta)i\gamma^A\,D_A\psi(\eta)$, provided by covariant derivative $D_A\,\psi(\eta) = \left(\partial_A - ig\,T^3\,W^3_A - ig'\,(Y/2)\,B_A\right)\,\psi(\eta)$, and g, g' being the $\overline{U}(1)$, $U(1)_Y$ coupling strength, respectively. Spontaneous symmetry breaking can be achieved by introducing the neutral complex scalar Higgs field

$$\phi = \begin{pmatrix} 0 \\ \phi^0 \end{pmatrix}, \quad Y(\phi) = 1, \quad \phi^0 = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

with the standard potential energy density function $V(\phi) = -\mu^2\phi^+\,\phi + \lambda\,(\phi^+\,\phi)^2$, where $\mu^2 > 0$, $\lambda > 0$. This is ingredient of the gauge invariant Lagrangian of Higgs field $\mathcal{L}_H = (D_A\,\phi)^+\left(D^A\,\phi\right) - V(\phi)$, where $D_A\,\phi(\eta) = (\partial_A - ig\,T^3\,W^3_A - ig'\,(Y/2)\,B_A)\,\phi(\eta)$. Minimization of the vacuum energy fixes non-vanishing VEV:

$$<\phi>_0 \equiv <0|\phi|0> = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = \left(\frac{\mu^2}{\lambda}\right)^{1/2},$$

leaving one Goldstone boson. The VEV of spontaneously breaks the theory, leaving the $U(1)_d$ subgroup intact. The unitary gauge

$$\phi(\eta) = U^{-1}(\xi_3)\begin{pmatrix} 0 \\ \frac{v+\zeta(\eta)}{\sqrt{2}} \end{pmatrix}, \; U(\xi_3) = \exp\left[\frac{i\xi_3\cdot\tau^3}{v}\right],$$

is parameterized by two real shifted fields ξ_3 and ζ, such that $<0|\xi_3|0> = <0|\zeta|0> = 0$. The gauge transformation

$$\phi' = U(\xi_3)\,\phi = \frac{v+\zeta}{\sqrt{2}}\,\chi, \quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

leads to $V(\phi') = \mu^2\zeta^2 + \lambda v\zeta^3 + (\lambda/4)\,\zeta^4$, which gives the mass of Higgs boson $M_H = \sqrt{2}\,\mu$. An examination of the v^2-terms in the kinetic piece of the Lagrangian $\mathcal{L}_H = (D_A\,\phi')^+\left(D^A\,\phi'\right) - V(\phi')$ reveals the mass terms for the physical gauge bosons:

$$\begin{array}{l}\frac{v^2}{2}\,\left|\left(i\frac{g}{2}\,\tau^3\,W'^3_A + ig'\,\frac{Y}{2}\,B'_A\right)\,\chi\right|^2 = \\ \frac{1}{2}\,(Z_A,\,A_A)\begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^A \\ A^A \end{pmatrix}.\end{array} \quad (69)$$

The mass matrix can be diagonalized by the standard orthogonal transformations:

$$\begin{array}{l} Z_A = \cos\theta_W\,W'^3_A + \sin\theta_W\,B'_A, \\ A_A = \sin\theta_W\,W'^3_A + \cos\theta_W\,B'_A, \\ M_Z = \frac{v}{2}\sqrt{g^2+g'^2}, \quad M_A = 0, \end{array} \quad (70)$$

where $\tan\theta_W = g'/g$. Namely, the neutral gauge field W'^3_A mixes with the abelian gauge field B'_A to form the physical states Z_A and A_A, with the masses M_Z and M_A, respectively. For neutral current we get

$$\mathcal{L}_{int} = æ\left(\mathcal{J}^{(0)}_A\,A^A + \mathcal{J}^{(M)}_A\,Z^A\right) \equiv æ\,\left(\mathcal{J}_A\,a^A\right), \quad (71)$$

where $æ = g\,\sin\theta$, and

$$\begin{array}{l} \mathcal{J}^{(0)}_A = \overline{\psi}(\eta)i\gamma_A\,Q^d\,\psi(\eta), \\ \mathcal{J}^{(M)}_A = \overline{\psi}(\eta)i\gamma_A\,Q^{(in)}\,\psi(\eta), \\ \mathcal{J}_A = \overline{\psi}(\eta)i\gamma_A\,\psi(\eta), \quad Q^{(in)} = \frac{T^3 - \sin^2\theta_W,\,Q^{gr}}{\sin\theta_W\,\cos\theta_W},. \end{array} \quad (72)$$

These relations show that an additional substantial change of properties of the spacetime continuum besides the curvature may be arisen at huge energies. A more detailed analysis and calculations on this will be presented on the later date.

VI. CONCLUDING REMARKS

Overall, we would sum up our investigation as follows. Following the powerful method of phenomenological Lagrangians, in the framework of GGP we connect the gravitation gauge group G_R to nonlinear realization of the Lie group G_D of distortion of the flat space M_6. The fundamental fields are distortion gauge fields, and that the metric and connection are related to these gauge fields. The agreement between this theory and observation is satisfactory. On the suggested theoretical basis, we construct the relativistic field theory of inertia which treats inertia as a distortion of local internal properties of flat space M_2. We derive the RLI which furnishes justification for introduction of Principle of Equivalence. Finally, we address the rearrangement of vacuum state in gravity. Whereas, the missing ingredient is the Higgs boson. The principle assumption went into the building of this approach that the Higgs boson is coupled only with distortion field, but not with the matter fields. The matter fields have interacted with the Higgs boson only via the metrical tensor. The four parameters g, g', μ, λ are inserted by hand, which consequently determine two coupling constants $æ_A$, $æ_Z$, and two masses M_H, M_Z. However, two relations can be imposed upon these parameters. First, the Compton wave-length λ_Z of massive component Z_a is finite, which will be of vital interest for the physics of superdense matter in very compact astrophysical sources if, for example, we set $\lambda_Z = 2\hbar/cv\sqrt{g^2+g'^2} \leq 0.4\,fm$, where $\simeq 0.4\,fm$ is the distance between the particles at nuclear density (we re-instate $\hbar$ and c). Second, we have $2\sqrt{\pi\,G_N} = g\,g'/\sqrt{g^2+g'^2}$.

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Appendix A: Further topics on the GGP

1. Field equations

Field equations can be derived from an invariant action

$$S = S_a + S_{\widetilde{\Phi}} = \int \sqrt{-\eta}\, L_a\, d^4x + \int \sqrt{-g}\, L_{\widetilde{\Phi}}\, d^4\widetilde{x}, \quad \text{(A1)}$$

where L_a is the Lagrangian of distortion field (a), $L_{\widetilde{\Phi}}$ is the Lagrangian of matter fields, whereas the dependence on distortion gauge field comes only through the components of metrical tensor. The L_a is invariant under (Λ)-coordinate and U^{loc}-gauge groups. The Lagrangian $L_{\widetilde{\Phi}}$, in turn, is invariant under gauge group of gravitation G_R. In terms of Euler-Lagrange variations in M_4 and R_4, we readily obtain

$$\begin{array}{l} \frac{\delta\left(\sqrt{-\eta}\, L_a\right)}{\delta\, a_l} = -\frac{\partial\, g^{\mu\nu}}{\partial\, a_l}\frac{\delta(\sqrt{-g}\, L_{\widetilde{\Phi}})}{\delta\, g^{\mu\nu}} = -\frac{\sqrt{-g}}{2}\,\frac{\partial\, g^{\mu\nu}}{\partial\, a_l}\widetilde{T}_{\mu\nu}, \\ \frac{\delta \widetilde{L}_{\widetilde{\Phi}}}{\delta\, \widetilde{\Phi}} = 0, \quad \frac{\delta\, L_{\widetilde{\Phi}}}{\delta\overline{\widetilde{\Phi}}} = 0, \end{array} \quad \text{(A2)}$$

where $\widetilde{T}_{\mu\nu}$ is the energy-momentum tensor of matter fields $\widetilde{\Phi}(\widetilde{x})$.

2. The unitary map matrix

In this subsection we will determine the unitary map matrix $R(a)$ and gauge invariant scalar function $S(F)$ for the fields of spin 0, 1, and 1/2. Our strategy is as follows: we, first, insert Eq. (8) into Eq. (9) to obtain an identity, and then equate the coefficients in front of $\partial\,\Phi$ and Φ to zero. In this way we may obtain the required relations to determine $R(a)$ and $S(F)$.

1) A straightforward calculation for the fields of spin $j = 0, 1$ gives the unitary matrix

$$R(\widetilde{x}, x) = R(x)\, R_g(\widetilde{x}) = \exp\left(-i\Theta(x) - \Theta_g(\widetilde{x})\right), \quad \text{(A3)}$$

provided

$$\begin{array}{l} \Theta(x) = æ \int_0^x a_l(x)\, d\,x^l, \\ \Theta_g(\widetilde{x}) = \int_0^{\widetilde{x}} \left[R^+\,\Gamma_\mu\, R + \psi^{-1}\widetilde{\partial}_\mu\,\psi\right]\, d\,\widetilde{x}^\mu, \end{array} \quad \text{(A4)}$$

where $\psi \equiv (\psi_l^\mu)$, $\Theta_g = 0$ for scalar field, and $\Theta_g + \Theta_g^+ = 0$ for vector field because of $\Gamma_\mu + \Gamma_\mu^+ = 0$, Γ_μ is the connection. The scalar function $S(F)$ is given in the Eq. (6):

$$S(F) = \tfrac{1}{4}\, R^+(\psi_\mu^l\, D_l^\mu)\, R = \tfrac{1}{4}\,\psi_\mu^l\, D_l^\mu. \quad \text{(A5)}$$

2) Unitary map of spinor field $\Psi(x)$ $\quad (j = 1/2)$ can be written as

$$\begin{array}{l} \widetilde{\Psi}(\widetilde{x}) = R(a)\,\Psi(x), \\ g^\mu(\widetilde{x})\nabla_\mu\,\widetilde{\Psi}(\widetilde{x}) = S(F)\, R(a)\,\gamma^l D_l\,\Psi(x), \end{array} \quad \text{(A6)}$$

where

$$\begin{array}{l} \nabla_\mu = \widetilde{\partial}_\mu + \Gamma_\mu, \\ \Gamma_\mu(\widetilde{x}) = (1/2)\,\Sigma^{\alpha\beta}\, V_\alpha^\nu(\widetilde{x})\widetilde{\partial}_\mu\, V_{\beta\nu}(\widetilde{x}), \\ \widetilde{R} = \gamma^0 R\gamma^0, \quad \Sigma^{\alpha\beta} = \frac{1}{4}[\gamma^\alpha, \gamma^\beta], \\ \Gamma_\mu(\widetilde{x}) = \frac{1}{4}\Delta_{\mu,\alpha\beta}(\widetilde{x})\gamma^\alpha\gamma^\beta, \end{array} \quad \text{(A7)}$$

$\Delta_{\mu,\alpha\beta}(\widetilde{x})$ denote the Ricci rotation coefficients. The unitary map matrix $R(a)$ is in the form of Eq. (A3), provided we make single change

$$\Theta_g(\widetilde{x}) = \tfrac{1}{2}\int_0^{\widetilde{x}} R^+\left\{g^\mu\Gamma_\mu R,\, g_\nu\, d\,\widetilde{x}^\nu\right\}. \quad \text{(A8)}$$

The calculations now give

$$\begin{array}{l} S(F) = \frac{1}{8K}\psi_\mu^l\left\{\widetilde{R}^+ g^\mu R,\, \gamma_l\right\} = inv, \\ K = \widetilde{R}^+\, R = \widetilde{R}_g^+\, R_g, \end{array} \quad \text{(A9)}$$

and hence

$$\begin{array}{l} K = \exp\left(-\frac{1}{2}\int_0^{\widetilde{x}}\left(\left\{R^+\widetilde{\Gamma}_\mu^+ g^\mu,\, g_\nu d\widetilde{x}^\nu\right\}R + \right.\right. \\ \left.\left. R^+\left\{g^\mu\Gamma_\mu R,\, g_\nu d\widetilde{x}^\nu\right\}\right)\right), \end{array} \quad \text{(A10)}$$

where $\widetilde{\Gamma}_\mu^+ = \gamma^0\Gamma_\mu^+\gamma^0$. Taking into account a commutation relation $[R, g_\nu] = 0$, and substituting [61]:

$$\widetilde{\Gamma}_\mu^+ g^\nu + g^\nu\Gamma_\mu = -\nabla_\mu g^\nu = 0, \quad \text{(A11)}$$

we obtain $K = 1$. Note that

$$\widetilde{U}_R^{+} U_R = \widetilde{R} U^{+} \widetilde{R'}^{+} R' U R^{+} = \widetilde{R} U^{+} \widetilde{R}^{+} R U R^{+}, \quad \text{(A12)}$$

and $\widetilde{R'}^{+} R' = \widetilde{R}^{+} R = 1$, therefore

$$\widetilde{U}_R^{+} U_R = \gamma^0 U_R^{+} \gamma^0 U_R = 1. \quad \text{(A13)}$$

3. The GGP for any spin

The results above can be readily extended to any spin j. In doing this we employ the Bargman-Wigner's wave functions for higher-spin in flat space [62]. The formalism of GGP will then hold, wheras the field of arbitrary spin j can be treated as a system of $2j$ fermions of half-integer spin. The wave function of spin-$j = n/2$ particle with momentum $\vec{p}$ defined on the M_4 can be obtained by Lorentz transformation from the symmetric Dirac spinor of rank n corresponding to the particle in the rest $U_{\alpha_1 \ldots \alpha_n}(0)$ implying $(\gamma_4 - 1)_\beta^{\beta'} U_{\beta' \beta_2 \ldots \beta_n}(0)$ for each index

$$U_{\beta_1 \ldots \beta_n}(\vec{p}) = S_{\beta_1}^{\beta_1'}(\alpha(\vec{p})) \ldots S_{\beta_n}^{\beta_n'}(\alpha(\vec{p})) U_{\beta_1' \ldots \beta_n'}(0), \quad \text{(A14)}$$

where

$$\overline{U}'(\vec{p})\, U'(\vec{p}) = \overline{U}(\Lambda^{-1}\vec{p})\, U(\Lambda^{-1}\vec{p}). \quad \text{(A15)}$$

A spin part is written $\Sigma_{\mu\nu} = (1/2) \sum_{r=1}^{n} \Sigma_{\mu\nu}^{(r)}$, where a matrix $\Sigma_{\mu\nu}^{(r)}$ acts only on the r-th index

$$\left(\Sigma_{\mu\nu}^{(r)}\right)_{\alpha_1 \ldots \alpha_n}^{\beta_1 \ldots \beta_n} = \delta_{\alpha_1}^{\beta_1} \cdots \delta_{\alpha_{r-1}}^{\beta_{r-1}} \left(\Sigma_{\mu\nu}^{(r)}\right)_{\alpha_r}^{\beta_r} \delta_{\alpha_{r+1}}^{\beta_{r+1}} \cdots \delta_{\alpha_n}^{\beta_n}, \quad \text{(A16)}$$

that $\Sigma_{\mu\nu}^{(r)} = (1/4)[\gamma_\mu^r, \gamma_\nu^r]$, and

$$\left(\gamma_\mu^r\right)_{\alpha_1 \ldots \alpha_n}^{\beta_1 \ldots \beta_n} = \delta_{\alpha_1}^{\beta_1} \cdots \delta_{\alpha_{r-1}}^{\beta_{r-1}} \left(\gamma_\mu^r\right)_{\alpha_r}^{\beta_r} \delta_{\alpha_{r+1}}^{\beta_{r+1}} \cdots \delta_{\alpha_n}^{\beta_n}. \quad \text{(A17)}$$

The spin-j field $\Phi_{\beta_1 \ldots \beta_n}(x)$ (Eq. (A14)) takes values in standard fiber over $x : \pi^{-1}(\mathcal{U}_i) = \mathcal{U}_i \times \mathbb{F}_x$. In the framework of GGP, the mapped spin-j field $\widetilde{\Phi}_{\beta_1 \ldots \beta_n}^{(r)}(\widetilde{x})$, where $\widetilde{\Phi}^{(r)} = R^{(r)} \Phi$, takes values in the fiber over $\widetilde{x} : \widetilde{\pi}^{-1}(\widetilde{\mathcal{U}}_i) = \widetilde{\mathcal{U}}_i \times \widetilde{\mathbb{F}}_{\widetilde{x}}$. The unitary map matrix $R^{(r)}$ is given in the form of Eq. (A3) and Eq. (A8), but now refers to the r-th index. The Lagrangian of this field will be invariant under the local gauge transformations

$$\begin{gathered}\widetilde{\Phi}'(\widetilde{x}) = U_R^{(r)}\, \widetilde{\Phi}(\widetilde{x}), \\ \left(g_{(r)}^{\mu}(\widetilde{x})\, \nabla_\mu^{(r)} \widetilde{\Phi}(\widetilde{x})\right)' = U_R^{(r)} \left(g_{(r)}^{\mu}(\widetilde{x})\, \nabla_\mu^{(r)} \widetilde{\Phi}(\widetilde{x})\right),\end{gathered} \quad \text{(A18)}$$

where $g_{(r)}^{\mu}(\widetilde{x}) = V_\alpha^\mu(\widetilde{x}) \gamma_{(r)}^{\alpha}$, $\nabla_\mu^{(r)}$ denotes the covariant derivative on R_4 defined only for the r-th index by the conventional substitution [56]:

$$\begin{gathered}\nabla_\mu^{(r)} \widetilde{U}_{\beta_1 \ldots \beta_n} \to \\ \Lambda_\alpha^{\alpha'}(\widetilde{x}) S_{\beta_1}^{\beta_1'}(\alpha(\Lambda)) \ldots S_{\beta_n}^{\beta_n'}(\alpha(\Lambda)) \nabla_\mu^{(r)} \widetilde{U}_{\beta_1 \ldots \beta_n}.\end{gathered} \quad \text{(A19)}$$

Here, as usual, we denote $\nabla_\alpha^{(r)} = V_\alpha^\mu\, (\widetilde{\partial}_\mu + \Gamma_\mu^{(r)})$, and that

$$\Gamma_\mu^{(r)} = \tfrac{1}{2} \Sigma_{(r)}^{\alpha\beta}\, V_\alpha^\nu(\widetilde{x})\, \partial_\mu V_{\beta\nu}(\widetilde{x}) = \tfrac{1}{4} \Delta_{\mu,\alpha\beta}^{(r)}\, \gamma_{(r)}^{\alpha}\, \gamma_{(r)}^{\beta}, \quad \text{(A20)}$$

$\Delta_{\mu,\alpha\beta}^{(r)}(\widetilde{x})$ are the Ricci rotation coefficients. The Eqs. (A18) hold if

$$g_{(r)}^{\mu}\, \nabla_\mu^{(r)} \widetilde{\Phi}^{(r)} = R^{(r)}\, S^{(r)}\, (\gamma^l\, D_l\, \Phi), \quad \text{(A21)}$$

where $D_l = \partial_l - ig\, a_l(x)$, the $R^{(r)}$, $S^{(r)}(F)$ are, respectively, unitary map matrix and gauge invariant scalar function of Eq. (A6) but referred to r-th index. According to the results of subsection A.2, we get

$$\widetilde{U_R^{(r)}}^{+}\, U_R^{(r)} = \gamma^0\, U_R^{(r)+}\, \gamma^0\, U_R^{(r)} = 1. \quad \text{(A22)}$$

The Lagrangian of the spin-j field may be recast as

$$\begin{aligned} &L(x) = J_\psi \widetilde{L}(\widetilde{x}) = \\ &J_\psi \left\{ \tfrac{i}{2} \left[\overline{\widetilde{\Phi}}^{(r)}(\widetilde{x})\, g_{(r)}^{\mu}(\widetilde{x}) \nabla_\mu^{(r)} \widetilde{\Phi}^{(r)}(\widetilde{x}) - \right.\right. \\ &\left. (\nabla_\mu^{(r)} \overline{\widetilde{\Phi}}^{(r)}(\widetilde{x}))\, g_{(r)}^{\mu}(\widetilde{x}) \widetilde{\Phi}^{(r)}(\widetilde{x}) \right] - \\ &\left. m \overline{\widetilde{\Phi}}^{(r)}(\widetilde{x}) \widetilde{\Phi}^{(r)}(\widetilde{x}) \right\} = \\ &J_\psi \left\{ S^{(r)}(a) \tfrac{i}{2} \left[\overline{\Phi}\, \gamma_{(r)}^{l} D_l\, \Phi - (D_l \overline{\Phi})\, \gamma_{(r)}^{l} \Phi \right] - m \overline{\Phi}\, \Phi \right\}, \end{aligned} \quad \text{(A23)}$$

where $J_\psi \equiv \|\psi\| \sqrt{-g}$. Generalized Bargman-Wigner's equation for the spin-$j = \frac{n}{2}$ particle in curved space stems from the Lagrangian eq. (A23):

$$\begin{gathered}\left(g'^{\mu}\, \nabla'_\mu \widetilde{\Phi}^{(r)} - m\right)_\beta^{\beta'} \widetilde{\Phi}_{\beta' \beta_2 \ldots \beta_n}(\widetilde{\vec{p}}) = \\ \left[R'\, (S'\, D - m)\right]_\beta^{\beta'} \Phi_{\beta' \beta_2 \ldots \beta_n}(\vec{p}) = 0,\end{gathered} \quad \text{(A24)}$$

where $R', S', \ldots$ refer to index β'.

4. The conserved currents

The general transformation of the group G_D is written as

$$G_D = \overline{Q}(\overline{a}) H(\theta), \quad \text{(A25)}$$

where the transformation $\overline{Q}(\overline{a})$ belongs to the left adjacent class G_D/H. Acting from the left by the group element g we define the transformations of parameters $\overline{a}$ and θ:

$$G_D(g) \overline{Q}(\overline{a}) H(\theta) = \overline{Q}(\overline{a}'(\overline{a}, g)) H(\theta'(\theta, \overline{a}, g)). \quad \text{(A26)}$$

The Cartan's forms allow an exponentiation of the group element $\overline{Q}(a) = \exp(i\overline{a}^i K_i)$, which determines the modified distortion fields $\overline{a}(a)$

$$d\, \overline{e}_A(\overline{a}) = \left[\exp(-i\overline{a}^i K_i)\, d\, \exp(i\overline{a}^i K_i)\right] \overline{e}_A(\overline{a}). \quad \text{(A27)}$$

Exponential parametrization of the finite transformations of the group is equivalent to the choice of normal coordinates in quotien space G_D/H. The solution of the structure equations (29) reads

$$\begin{aligned}&\omega^i(\overline{a},\, d\,\overline{a}) = (\sin\sqrt{\tau}/\sqrt{\tau})^i_k\; d\,\overline{a}^k,\\ &\vartheta^\alpha(\overline{a},\, d\,\overline{a}) = \left[(1-\cos\sqrt{\tau})\,/\tau\right]^i_k\; d\,\overline{a}^k\,\varepsilon_{\alpha il}\,\overline{a}^l,\\ &\tau^i_k = -\varepsilon_{ij\alpha}\varepsilon_{\alpha kl}\,\overline{a}^j\,\overline{a}^k.\end{aligned} \tag{A28}$$

By virtue of the relations

$$\begin{aligned}&\left(\tau^{(n)}\right)^i_j = (\overline{a}^2)^{n-1}\left(\tau^{(1)}\right)^i_j,\\ &\tau^i_j = \varepsilon_{ilk}\varepsilon_{kjn}\overline{a}^l\overline{a}^n = \overline{a}^2\delta_{ij} - \overline{a}^i\overline{a}^j,\end{aligned} \tag{A29}$$

the Eq. (A28) may be recast as

$$\begin{aligned}&\omega^i_A = \omega^i(\overline{a},\, \partial_A\,\overline{a}) = \partial_A\,\overline{a}^i +\\ &(\delta_{ik} - \overline{a}^i\overline{a}^k/\overline{a}^2)\left(\sin\sqrt{\overline{a}^2}/\sqrt{\overline{a}^2} - 1\right)\partial_A\,\overline{a}^k,\\ &\vartheta^i_A = \vartheta^i(\overline{a},\, \partial_A\,\overline{a}) = \left[\left(1-\cos\sqrt{\overline{a}^2}\right)/\overline{a}^2\right]\partial_A\,\overline{a}^l\,\varepsilon_{ilj}\,\overline{a}^j.\end{aligned} \tag{A30}$$

In exponential parametrization the Eq. (A26) gives

$$\begin{aligned}&G_D(g)\exp(i\overline{a}^iK_i)\exp(i\theta^\alpha I_\alpha) =\\ &\exp\left[i\overline{a}'^i(\overline{a},\, g)K_i\right]\exp\left[i\theta'^\alpha(\overline{a},\, g)I_\alpha\right].\end{aligned} \tag{A31}$$

According to the Eq. (A31), the transformation of parameters $\overline{a}$ and θ at the infinitesimal translation $G_D(g) = (1 + i\varepsilon^i\,K_i) + O(\varepsilon^2)$ can be written as

$$\begin{aligned}&(1 + i\varepsilon^i\,K_i)\exp(iK_i\overline{a}^i) =\\ &\exp\left[iK_i(\overline{a}^i + \delta\,\overline{a}^i(\overline{a},\, \varepsilon))\right]\exp\left[i\delta\,\theta^\alpha(\overline{a},\, \varepsilon))I_\alpha\right].\end{aligned} \tag{A32}$$

Employing the Feynman's method of ordering by means of auxiliary parameter (t) [63] which implies $\int_x^y A(t)dt = A(y-x)$, in standard manner we expand

$$\begin{aligned}&\exp\left[iK_i(\overline{a}^i + \delta\,\overline{a}^i)\right] = \exp(iK_i\overline{a}^i) + i\exp(iK_i\overline{a}^i)\times\\ &\times\int_0^1 dt\,\exp(-iK_i\overline{a}^i\,t)\,(\delta\,\overline{a}^i\,K_i)\,\exp(iK_i\overline{a}^i\,t) + \cdots.\end{aligned} \tag{A33}$$

Then the Eq. (A32) gives

$$i\varepsilon^i\,K_i(1) = i\int_0^1 dt\delta\,\overline{a}^iK_i(t) + i\delta\,\theta^\alpha(\overline{a},\,\theta)I_\alpha, \tag{A34}$$

where

$$K_j(t) = \exp(-iK\overline{a}^it)K_j\exp(iK_i\overline{a}^it). \tag{A35}$$

In analogy, defining the $I_\alpha(t)$, and taking derivatives of the $K_j(t)$ and $I_\alpha(t)$ with respect to parameter (t), we obtain

$$\begin{aligned}&\tfrac{\partial}{\partial t}K_j(t) = i\varepsilon_{\alpha ji}\overline{a}^i\,I_\alpha(t); \quad K_j(0) = K_j,\\ &\tfrac{\partial}{\partial t}I_\alpha(t) = i\varepsilon_{l\alpha k}\overline{a}^k\,K_l(t); \quad I_\alpha(0) = 0,\end{aligned} \tag{A36}$$

the solution of which can be written in the form of Eq. (A28)

$$\begin{aligned}&K_j(t) = (\cos\sqrt{\tau}\,t)^i_j\;K_j +\\ &i\,(\sin\sqrt{\tau}\,t/\sqrt{\tau})^l_j\;\varepsilon_{\alpha lk}\,\overline{a}^k\,I_\alpha.\end{aligned} \tag{A37}$$

Inserting this solution into the Eq. (A34) and equating the coefficients at the same generators K_jI_α, we finally obtain

$$\begin{aligned}&\delta\,\overline{a}^i(\overline{a},\,\theta)) = -(\sqrt{\tau}\,\coth\sqrt{\tau})^i_k\varepsilon^k + O(\varepsilon^2),\\ &\delta\,\theta^\alpha(\overline{a},\,\theta)) = \{[\sin\sqrt{\tau} - \coth\sqrt{\tau}(1-\\ &\cos\sqrt{\tau})]/\sqrt{\tau}\}^l_i\,\varepsilon^i\,\varepsilon_{\alpha lk}\overline{a}^k.\end{aligned} \tag{A38}$$

The transformation of parameters in case of rotation $G_D(g') = (1 + i\xi^\alpha\,I_\alpha) + O(\xi^2)$ can be obtained in the same manner as

$$\delta\,\overline{a}^i(\xi)) = -\varepsilon_{i\alpha k}\,\xi^\alpha\,\overline{a}^k + O(\xi^2), \quad \delta\,\theta^\alpha(\overline{a},\,\xi)) = \xi^\alpha. \tag{A39}$$

Let the fields undergo the infinitesimal transformations

$$\Phi^j(\eta) \to \Phi^j(\eta) + \varepsilon(\eta)^k\,\textstyle\prod^j_k(\Phi), \tag{A40}$$

where $\prod^j_k(\Phi)$ is the nonlinear function of the fields. The current related to these transformations

$$\begin{aligned}&J^A_k = -\delta\,L(\Phi,\,\partial\,\Phi)/\delta\,(\partial_A\,\varepsilon^k) =\\ &-\textstyle\prod^j_k(\Phi)\,\delta\,L(\Phi,\,\partial\,\Phi)/\delta\,(\partial_A\,\Phi^j),\end{aligned} \tag{A41}$$

implies

$$\partial_A\,J^A_k = -\delta\,L(\Phi,\,\partial\,\Phi)/\delta\,\varepsilon^k.$$

If the Lagrangian is invariant with respect to constant transformations of Eq. (A40), then the corresponding currents are conserved. According to equations (A28), (A38) and (A39) the conserved currents can be calculated in normal coordinates as

$$\begin{aligned}&J^k(\overline{a},\,\partial\,\overline{a}) \equiv \omega^k(2\overline{a},\,\partial\,\overline{a}) = (\sin 2\sqrt{\tau}/2\sqrt{\tau})^k_i\partial\,\overline{a}^i,\\ &J^\alpha(\overline{a},\,\partial\,\overline{a}) \equiv \vartheta^\alpha(2\overline{a},\,\partial\,\overline{a}) =\\ &-\left[(1-\cos 2\sqrt{\tau})\,/2\tau\right]^i_k\;\partial\,\overline{a}^k\,\varepsilon_{\alpha il}\,\overline{a}^l,\end{aligned} \tag{A42}$$

where we left the indices (A) implicit.

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