The cosmological perturbation theory in loop cosmology with holonomy corrections

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Abstract

In this paper we investigate the scalar mode of first-order metric perturbations over spatially flat FRW spacetime when the holonomy correction is taken into account in the semi-classical framework of loop quantum cosmology. By means of the Hamiltonian derivation, the cosmological perturbation equations is obtained in longitudinal gauge. It turns out that in the presence of metric perturbation the holonomy effects influence both background and perturbations, and contribute a non-trivial sector S_h in the cosmological perturbation equations.

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I. INTRODUCTION

In loop quantum cosmology two main quantum gravity effects lead to remarkable modifications to the standard description of the early universe(for a detailed review, see Ref. [1]). One is due to the holonomy correction and the other is due to the inverse volume correction. Such modifications can successfully avoid the Big Bang singularity [2–5], and replace it by the Big Bounce even at the semi-classical level [6, 7]. In addition, it is very interesting to notice that quantum gravity effects may lead to the occurrence of the super-inflationary phase [8]. As shown in Ref. [9], such a super-inflationary phase can also resolve the horizon problem with only a few number of e-foldings. Therefore, it is possible to construct a phase of inflation or an alternative to inflation in the framework of loop quantum cosmology.

As we all known, the inflationary phase is crucial for understanding the structure formation and anisotropies of the CMB. In order to address these issues in the framework of loop quantum cosmology, we must consider the cosmological perturbation theory with modifications due to quantum gravity effects. In the earlier work by Bojowald *et al.*[10, 11], by means of the Hamiltonian derivation they have obtained the cosmological perturbation equation with inverse volume corrections for scalar modes in longitudinal gauge. They show that super-horizon curvature perturbations are not preserved. Recently, they have also derived the gauge-invariant quantities and the corresponding gauge-invariant cosmological perturbation equations with inverse volume corrections for scalar modes [12, 13]. In addition, the vector modes and tensor modes with corrections from loop quantum gravity have been investigated [14, 15].

At the same time, some pioneer work have already been devoted to understanding the primordial power spectrum in the perturbation theory of LQC [9, 16–21]. First of all, in Ref. [9, 18], it is shown that a scale invariant spectrum can be obtained. More importantly these attempts imply that the quantum gravity effects may leave an imprints on the power spectrum which can be potentially detected in the future experiments such as the Planck satellite. However,above considerations are restricted to the scalar field perturbations with fixed background. To provide a complete and more precise understanding on the perturbation theory in loop cosmology, it is essential to take the metric perturbation into account. Along this direction it is worthwhile to point out that another potential observables, primordial gravitational waves have already been investigated intensively in LQC [22].

Although, in Ref. [10–13] the cosmological perturbation equations with inverse volume corrections have been derived in longitudinal gauge and gauge-invariant manner respectively, the metric perturbations with holonomy corrections is still absent. In the present paper, by means of the Hamiltonian derivation, we will derive the cosmological perturbation equations with holonomy corrections in longitudinal gauge.

The outline of our paper is the following. For comparison, we firstly present a brief review on the perturbation equations in standard classical cosmology in section II. After introducing the basic variables in loop cosmology in section III, we will demonstrate a detailed derivation on the cosmological perturbation equation with holonomy corrections in section IV. The discussion is given in section VI.

II. THE CLASSICAL COSMOLOGICAL PERTURBATION EQUATIONS

Before proceeding to the effective loop quantum cosmology with holonomy corrections, we first briefly review the classical perturbation equations in standard cosmology. A detailed derivation can be found in Ref. [23]. Let us now consider a spatially flat background metric of FRW type

$$ds^2 = a^2(\eta)(-d\eta^2 + \delta_{ab} dx^a dx^b) \ . \tag{1}$$

where η is the conformal time. The spatial part of the metric describes isotropic and homogeneous 3-surfaces. Then one can perturb the background metric

$$ds^2 = a^2(\eta)\left[-(1+2\Phi)d\eta^2 + (1-2\Psi)\delta_{ab} dx^a dx^b\right] \ . \tag{2}$$

Here we only consider the scalar modes in longitudinal gauge, which is thus diagonal. Through this paper, we will consider the scalar field φ as the matter source. Expanding the Einstein's equation linearly, one can obtain the cosmological perturbation equation

$$\nabla^2\Phi - 3\mathbb{H}\dot{\Phi} - (\dot{\mathbb{H}} + 2\mathbb{H}^2)\Phi = 4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi} + \bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi) \ , \tag{3}$$

$$\ddot{\Phi} + 3\mathbb{H}\dot{\Phi} + (\dot{\mathbb{H}} + 2\mathbb{H}^2)\Phi = 4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi} - \bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi) \ , \tag{4}$$

$$\partial_a(\dot{\Phi} + \mathbb{H}\Phi) = 4\pi G\dot{\bar{\varphi}}\delta\varphi_{,a} \ , \tag{5}$$

where a dot denotes a derivative with respect to the conformal time η. $\mathbb{H}$ is the Hubble expansion rate in the conformal time, and for later convenience, we have identified a^2 with $\bar{p}$ which is introduced in (10). Note that in the case of vanishing anisotropic stresses, two scalar functions Φ and Ψ coincide, $\Phi = \Psi$. Therefore, in above equations we have set $\Phi = \Psi$, which simplifies the equations considerably[1]. Moreover, among these equations above only two of them are independent. Combining these equations, one can obtain the following second order differential equation for Φ

$$\ddot{\Phi} - \nabla^2\Phi + (6\mathbb{H} + 2\bar{p}\frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}})\dot{\Phi} + (2\dot{\mathbb{H}} + 4\mathbb{H}^2 + \frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}}\mathbb{H})\Phi = 0 \ . \tag{6}$$

In addition, the background and the perturbed Klein-Gordon equation can respectively expressed as

$$\ddot{\bar{\varphi}} + 2\mathbb{H}\dot{\bar{\varphi}} + \bar{p}V_{,\varphi}(\bar{\varphi}) = 0 \ , \tag{7}$$

$$\delta\ddot{\varphi} + 2\mathbb{H}\delta\varphi - \bigtriangledown^2\delta\varphi + \bar{p}V_{,\bar{\varphi}\bar{\varphi}}(\bar{\varphi}) + 2\bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\Phi - 4\dot{\bar{\varphi}}\dot{\Phi} = 0 \ . \tag{8}$$

III. THE BASIC VARIABLES

Now we intend to study the scalar mode of first-order metric perturbations around spatially flat FRW spacetime when the holonomy corrections is taken into account in the semi-classical framework of loop quantum cosmology. To derive the cosmological perturbation equations we adopt the Hamiltonian approach which has been developed in the effective loop quantum cosmology with inverse triad corrections [10, 12]. We summarize the basic idea and steps as follows.

In loop quantum gravity, instead of the spatial metric q_{ab}, a densitized triad E^a_i is primarily used, which satisfies $E^a_i E^b_i = q^{ab} detq$. Moreover, in the canonical formulation the space-time metric is given by

[1] However we must point out that, as a matter of fact, $\Phi = \Psi$ is a consequence of equations of motion, which can also be seen in this paper

$$ds^2 = -N^2 d\eta^2 + q_{ab}(dx^a + N^a d\eta)(dx^b + N^b d\eta) \ , \tag{9}$$

where N and N^a are lapse function and shift vector respectively.

By comparing the above equation with the FRW metric (1), the background variables, $\bar{N}$, N^a and $\bar{E}^a_i$, can be expressed as respectively

$$\bar{N} = \sqrt{\bar{p}}; \bar{N}^a = 0; \bar{E}^a_i = \bar{p}\delta^a_i \ , \tag{10}$$

where the background variables are denoted with a bar, which describe smoothed out, spatial averaged quantities. Another background variable, the extrinsic curvature components $\bar{K}^i_a$, can be derived from the following relation

$$\bar{K}_{ab} = \frac{1}{2\bar{N}}(\dot{\bar{q}}_{ab} - 2D_{(a}\bar{N}_{b)}) = \dot{a}\delta_{ab} \ . \tag{11}$$

where D is the covariant spatial derivation. Thus, the extrinsic curvature can be expressed as

$$\bar{K}^i_a = \frac{\bar{E}^b_i}{\sqrt{|det(\bar{E}^c_j)|}}\bar{K}_{ab} = \frac{\dot{\bar{p}}}{2\bar{p}}\delta^i_a =: \bar{k}\delta^i_a \ . \tag{12}$$

In equation (12), we have defined the background extrinsic curvature as $k =: \frac{\dot{\bar{p}}}{2\bar{p}} = \frac{\dot{a}}{a}$, which can also be obtained from the background equations of motion [12]. Therefore, in classical FRW background, the extrinsic curvature is nothing but the conformal Hubble parameter $\mathbb{H}$. However, in the effective loop quantum cosmology, the relation between the extrinsic curvature and the conformal Hubble parameter will change due to quantum gravity corrections, which we will see in the next section.

The canonical perturbed variables can be related to the perturbed metric variables by comparing the perturbed metric (2) with the canonical one(9). It turns out that the perturbed triad is given by

$$\delta E^a_i = -2\bar{p}\Psi\delta^a_i \ , \tag{13}$$

and the perturbed lapse function is

$$\delta N = \bar{N}\Phi. \tag{14}$$

We note that, due to only considering diagonal metric perturbation, the first-order shift vector $\delta N^{a(1)}$ is vanishing as well as the background shift vector. Therefore, if one intends to preserve the character of the non-vanishing diffeomorphism constraint, it is necessary to expand the shift vector to the second-order $\delta N^{a(2)}$. As shown in the above, the extrinsic curvature components can be diagonal, thus it can be expanded as

$$K_a^i = \bar{K}_a^i + \delta K_a^i = \bar{k}\delta_a^i + \delta K_a^i \ . \tag{15}$$

The perturbed extrinsic curvature will be derived from the equation of motion in the following. We can assume that δE_i^a and δK_a^i do not have homogeneous modes, namely

$$\int_\Sigma \delta E_i^a \delta_a^i d^3x = 0, \int_\Sigma \delta K_a^i \delta_i^a d^3x = 0 \ . \tag{16}$$

And the homogeneous mode is defined by

$$\bar{p} = \frac{1}{3V_0}\int_\Sigma E_i^a \delta_a^i d^3x, \bar{k} = \frac{1}{3V_0}\int_\Sigma K_a^i \delta_i^a d^3x \ , \tag{17}$$

where we integrate over a bounded region of coordinate size $V_0 = \int_\Sigma d^3x$. Then we can construct the Poisson brackets of the background and perturbed variables [13],

$$\{\bar{k}, \bar{p}\} = \frac{8\pi G}{3V_0}, \{\delta K_a^i(x), \delta E_j^b(y)\} = 8\pi G \delta_j^i \delta_a^b \delta^3(x-y) \ . \tag{18}$$

In addition, we point out that the similar conditions will be required in the perturbed lapse δN, the scalar field $\delta\varphi$ and conjugate momentum $\delta\pi$ such that

$$\int_\Sigma \delta N d^3x = 0, \int_\Sigma \delta\varphi d^3x = 0, \int_\Sigma \delta\pi d^3x = 0 \ , \tag{19}$$

which is used in expanding the Hamiltonian constraint. While the homogeneous mode of the scalar field and its conjugate momentum is

$$\bar{\varphi} = \frac{1}{V_0}\int_\Sigma \varphi d^3x, \bar{\pi} = \frac{1}{V_0}\int_\Sigma \pi d^3x \ . \tag{20}$$

Therefore, the Poisson brackets of the background and perturbed variables of scalar field is

$$\{\bar{\varphi}, \bar{\pi}\} = \frac{1}{3V_0}, \{\delta\varphi(x), \delta\pi(y)\} = \delta^3(x-y) \ . \tag{21}$$

IV. THE COSMOLOGICAL PERTURBATION THEORY WITH HOLONOMY CORRECTIONS

Now we turn to the derivation of the cosmological perturbation theory in the effective loop quantum cosmology with holonomy corrections. For more details on the Hamilton cosmological perturbation theory, we refer to Ref.[10, 12].

Thanks to the holonomy corrections, in the isotropic and homogeneous models, the exact effective Hamiltonian can be obtained at the phenomenological level by simply replacing the background Ashtekar connection $\gamma\bar{k}$ by $\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}$, where γ is the Barbero-Immirzi parameter. The parameter $\bar{\mu}$ depends on the quantization scheme and may be a function of $\bar{p}$. In Ref. [4], the so-called $\bar{\mu}$ scheme has been pushed forward and elaborated. Recently, it has also been shown that, for the flat FRW universe, $\bar{\mu}$ scheme is the only consistent choice [24]. In the $\bar{\mu}$ scheme,

$$\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}} \ , \tag{22}$$

where $\Delta = 2\sqrt{3}\pi\gamma l_{PL}^2$. However, when the inhomogeneities are taken into account, it is no longer true. To study the effects of holonomy corrections on inhomogeneous perturbations, we similarly substitute the appearance of k in the classical Hamiltonian by a general form $\frac{\sin m\bar{\mu}\gamma\bar{k}}{m\bar{\mu}\gamma}$ where m is an integer. In the context of vector modes [14] and tensor modes [15], due to the requirement of the anomaly cancellation, we can fix the parameter m. Although in the context of the scalar modes, the anomaly-free constraint algebra with holonomy corrections is still absent, at the phenomenological level, we can fix the parameter m in a similar manner. Therefore, one can write down the expressions for the gravitational Hamiltonian density $\mathcal{H}_G^h = \mathcal{H}_G^{h(0)} + \mathcal{H}_G^{h(1)} + \mathcal{H}_G^{h(2)}$ with

$$\begin{aligned}
\mathcal{H}_G^{h(0)} &= -6\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2\sqrt{\bar{p}} \ , \\
\mathcal{H}_G^{h(1)} &= -4\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)\sqrt{\bar{p}}\delta_j^c\delta K_c^j - \frac{1}{\sqrt{\bar{p}}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2\delta_c^j\delta E_j^c + \frac{2}{\sqrt{\bar{p}}}\partial_c\partial^j\delta E_j^c \ , \\
\mathcal{H}_G^{h(2)} &= \sqrt{\bar{p}}\delta K_c^j\delta K_d^k\delta_k^c\delta_j^d - \sqrt{\bar{p}}(\delta K_c^j\delta_j^c)^2 - \frac{2}{\sqrt{\bar{p}}}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)\delta E_j^c\delta K_c^j - \frac{1}{2\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2\delta E_j^c\delta E_k^d\delta_c^k\delta_d^j \\
&\quad + \frac{1}{4\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2(\delta E_j^c\delta_c^j)^2 - \frac{\delta^{jk}}{2\bar{p}^{3/2}}(\partial_c\delta E_j^c)(\partial_d\delta E_k^d) \ ,
\end{aligned} \tag{23}$$

where the superscript "h" represents the holonomy corrections and the corresponding classical expressions can be found in Ref.[12, 13]. We now only consider the scalar field as the matter source. Its Hamiltonian density expands as $\mathcal{H}_M = \mathcal{H}_M^{(0)} + \mathcal{H}_M^{(1)} + \mathcal{H}_M^{(2)}$. Since the matter is free from the holonomy corrections, the expressions of scalar field Hamiltonian density, $\mathcal{H}_M = \mathcal{H}_\pi + \mathcal{H}_\nabla + \mathcal{H}_\varphi$, expanding up to the second order, are as the classical cases [12, 13],

$$\mathcal{H}_\pi^{(0)} = \frac{\bar{\pi}_{\bar{\varphi}}^2}{2\bar{p}^{3/2}}, \mathcal{H}_\nabla^{(0)} = 0, \mathcal{H}_\varphi^{(0)} = \bar{p}^{3/2}V(\bar{\varphi}), \tag{24}$$

$$\mathcal{H}_\pi^{(1)} = \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - \frac{\bar{\pi}^2}{2\bar{p}^{3/2}}\frac{\delta_c^j\delta E_j^c}{2\bar{p}}, \mathcal{H}_\nabla^{(1)} = 0, \mathcal{H}_\varphi^{(1)} = \bar{p}^{3/2}(V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi + V(\bar{\varphi})\frac{\delta_c^j\delta E_j^c}{2\bar{p}}), \tag{25}$$

and

$$\begin{aligned}
\mathcal{H}_\pi^{(2)} &= \frac{1}{2}\frac{\delta\pi^2}{\bar{p}^{3/2}} - \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}}\frac{\delta_c^j\delta E_j^c}{2\bar{p}} + \frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}}(\frac{(\delta_c^j\delta E_j^c)^2}{8\bar{p}^2} + \frac{\delta_c^k\delta_d^j\delta E_j^c\delta E_k^d}{4\bar{p}^2}) \ , \\
\mathcal{H}_\nabla^{(2)} &= \frac{1}{2}\sqrt{\bar{p}}\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi \ , \\
\mathcal{H}_\varphi^{(2)} &= \frac{1}{2}\bar{p}^{3/2}V_{,\bar{\varphi}\bar{\varphi}}(\bar{\varphi})\delta\varphi^2 + \bar{p}^{3/2}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi\frac{\delta_c^j\delta E_j^c}{2\bar{p}} \\
&\quad \bar{p}^{3/2}V(\bar{\varphi})(\frac{(\delta_c^j\delta E_j^c)^2}{8\bar{p}^2} - \frac{\delta_c^k\delta_d^j\delta E_j^c\delta E_k^d}{4\bar{p}^2}) \ .
\end{aligned} \tag{26}$$

A. The background equations

In the isotropic and homogeneous FRW background, the diffeomorphism constraint vanishes. Therefore background equations are generated only by the background Hamiltonian constraint, which can be expressed as

$$H^{h(0)}[\bar{N}] = \frac{1}{16\pi G}\int_\Sigma d^3x\bar{N}[\mathcal{H}_G^{(0)} + 16\pi G(\mathcal{H}_\pi^{(0)} + \mathcal{H}_\varphi^{(0)})] \ . \tag{27}$$

Thus the explicit expression for the background Hamiltonian constraint is

$$-\frac{3}{8\pi G}\sqrt{\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \frac{\bar{\pi}^2}{2\bar{p}^{3/2}} + \bar{p}^{3/2}V(\bar{\varphi}) = 0 \ . \tag{28}$$

Then, by means of Poisson bracket, we can derive the equation of motion for the gravitational variables $\bar{k}$ and $\bar{p}$.

$$\dot{\bar{k}} = \{\bar{k}, H^{h(0)}[\bar{N}]\} = -[\frac{1}{2}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2] + 4\pi G[-\frac{\bar{\pi}^2}{2\bar{p}^2} + \bar{p}V(\bar{\varphi})] \ . \tag{29}$$

$$\dot{\bar{p}} = \{\bar{p}, H^{h(0)}[\bar{N}]\} = 2\bar{p}(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}) \ . \tag{30}$$

Similarly, the equation of motion for scalar field $\bar{\varphi}$ and its conjugate momentum field $\bar{\pi}$ can also be derived as

$$\dot{\bar{\varphi}} = \{\bar{\varphi}, H^{h(0)}[\bar{N}]\} = \frac{\bar{\pi}}{\bar{p}} \ . \tag{31}$$

$$\dot{\bar{\pi}} = \{\bar{\pi}, H^{h(0)}[\bar{N}]\} = -\bar{p}^2 V_{,\bar{\varphi}}(\bar{\varphi}) \ . \tag{32}$$

Note that in above Poisson brackets, we have used the relation $\bar{N} = \sqrt{\bar{p}}$. Substituting the relation (31) into the constraint equation (28) gives rise to the corrected Friedmann equation

$$(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 = \frac{8\pi G}{3}[\frac{1}{2}\dot{\bar{\varphi}}^2 + \bar{p}V(\varphi)] \ . \tag{33}$$

At the same time, equation (29) is just the corrected Raychaudhuri equation

$$\dot{\bar{k}} + [\frac{1}{2}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2] = 4\pi G[-\frac{\dot{\bar{\varphi}}^2}{2} + \bar{p}V(\bar{\varphi})] \ . \tag{34}$$

In the classical limit, $\bar{\mu} \to 0$, above two equations can be reduced to the Friedmann and Raychaudhuri equation in the standard cosmology. Finally, the Klein-Gordon equation can be derived from Eqs. (31), (32) and (30)

$$\ddot{\bar{\varphi}} + 2(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\dot{\bar{\varphi}} + \bar{p}V_{,\bar{\varphi}}(\bar{\varphi}) = 0 \ . \tag{35}$$

In addition, from the equation of motion (30), one can find that the extrinsic curvature $\bar{k}$ is related to the conformal Hubble parameter $\mathbb{H}$ by

$$\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} = \frac{\dot{\bar{p}}}{2\bar{p}} =: \mathbb{H} \ . \tag{36}$$

Therefore, due to the holonomy corrections, the conformal Hubble parameter $\mathbb{H}$ is not simply equal to the extrinsic curvature $\bar{k}$ but receives corrections. For consistency, in our next derivation we will continuously use the extrinsic curvature $\bar{k}$ rather than the conformal Hubble parameter. Only at the end, we will use the conformal Hubble parameter $\mathbb{H}$ instead of $\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}$ in the perturbation equations.

B. The perturbed equations

In this subsection, we will derive the cosmological perturbation equation with holonomy corrections. Firstly we will derive the equations of motion of perturbed variables. In the canonical formulation, the equation of motion of any phase space function f is determined by Poisson bracket, $\dot{f} = \{f, H\}$. Here H is the total Hamiltonian, which is a sum of the Hamiltonian constraint $H[N]$ and the diffeomorphism constraint $D[N^a]$, $H = H[N]+D[N^a]$. Since the zero-order and first-order shift vectors vanish, the diffeomorphism constraints is identically satisfied up to the second-order. Thus, the equations of motion of the perturbed variables are only generated by the Hamiltonian constraint. The perturbed Hamiltonian constraint up to the second-order is written as $\tilde{H}^h[N] = \tilde{H}^h[\bar{N}] + \tilde{H}^h[\delta N]$ with

$$
\begin{aligned}
\tilde{H}^h[\bar{N}] &= \frac{1}{16\pi G}\int_\Sigma d^3x\bar{N}[\mathcal{H}_G^{(2)} + 16\pi G(\mathcal{H}_\pi^{(2)} + \mathcal{H}_\nabla^{(2)} + \mathcal{H}_\varphi^{(2)})] \ , \\
\tilde{H}^h[\delta N] &= \frac{1}{16\pi G}\int_\Sigma d^3x\delta N[\mathcal{H}_G^{(1)} + 16\pi G(\mathcal{H}_\pi^{(1)} + \mathcal{H}_\varphi^{(1)})] \ . \qquad (37)
\end{aligned}
$$

Note that we have used the conditions that the perturbed variables do not have homogeneous modes as described in Eq.(16) and (19). As well, we input the boundary condition requiring that the integration over the boundary vanishes, namely

$$
\int_\Sigma \bar{N}[\mathcal{H}_G^{h1} + 16\pi G(\mathcal{H}_\pi^{1} + \mathcal{H}_\varphi^{1})] = 0, \int_\Sigma \delta N[\mathcal{H}_G^{h0} + 16\pi G(\mathcal{H}_\pi^{0} + \mathcal{H}_\varphi^{0})] = 0 \ . \qquad (38)
$$

Therefore, the equations of motion of perturbed variables are generated only by the second order part of Hamiltonian constraints. Thus, we can arrive at the equation of motion of the perturbed variables by means of the Poisson bracket

$$\begin{aligned}
\delta \dot{K}^i_a &\equiv \{\delta K^i_a, \tilde{H}^h[\bar{N}] + \tilde{H}^h[\delta N]\} \\
&= \frac{\bar{N}}{\bar{p}^{3/2}}[-\bar{p}(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\delta K^i_a - \frac{1}{2}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 \delta E^d_k \delta^k_a \delta^i_d + \frac{1}{4}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2(\delta E^d_k \delta^k_d)\delta^i_a + \frac{1}{2}\delta^{ik}\partial_a\partial_d \delta E^d_k] \\
&\quad -\frac{1}{2}\frac{\delta N}{\sqrt{\bar{p}}}(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2\delta^i_a + \frac{1}{\sqrt{\bar{p}}}\partial_a\partial^i \delta N \\
&\quad +4\pi G\frac{\bar{N}}{\bar{p}^{3/2}}[-\frac{\bar{\pi}\delta\pi}{\bar{p}}\delta^i_a + \frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^2}(\frac{1}{2}\delta E^d_k \delta^k_d \delta^i_a + \delta E^d_k \delta^k_a \delta^i_d) + \bar{p}^2 V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi\delta^i_a \\
&\quad +\bar{p}V(\varphi)(\frac{1}{2}\delta E^d_k \delta^k_d \delta^i_a - \delta E^d_k \delta^k_a \delta^i_d)] + 4\pi G\delta N[-\frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{5/2}} + \sqrt{\bar{p}}V(\bar{\varphi})]\delta^i_a,
\end{aligned} \tag{39}$$

$$\begin{aligned}
\delta \dot{E}^a_i &\equiv \{\delta E^a_i, \tilde{H}^h[\bar{N}] + \tilde{H}^h[\delta N]\} \\
&= \frac{\bar{N}}{\sqrt{\bar{p}}}[-\bar{p}\delta K^j_c \delta^c_i \delta^a_j + \bar{p}(\delta K^j_c \delta^c_j)\delta^a_i + (\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\delta E^a_i] + 2\delta N(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\sqrt{\bar{p}}\delta^a_i,
\end{aligned} \tag{40}$$

$$\delta\dot{\varphi} \equiv \{\delta\varphi, \tilde{H}^h[\bar{N}] + \tilde{H}^h[\delta N]\} = \frac{\bar{N}}{\bar{p}^{3/2}}(\delta\pi - \bar{\pi}\frac{\delta E^c_j \delta^j_c}{2\bar{p}}) + \frac{\delta N}{\bar{p}^{3/2}}\bar{\pi}\ , \tag{41}$$

$$\delta\dot{\pi} \equiv \{\delta\pi, \tilde{H}^h[\bar{N}] + \tilde{H}^h[\delta N]\} = \frac{\bar{N}}{\bar{p}^{3/2}}[\bar{p}^2\nabla^2\delta\varphi - \bar{p}^3 V_{,\bar{\varphi}\bar{\varphi}}\delta\varphi - \frac{1}{2}\bar{p}^2 V_{,\bar{\varphi}\bar{\varphi}}\delta E^c_j \delta^j_c]\ . \tag{42}$$

Furthermore, using Eqs.(13) and (30), we can obtain the perturbed extrinsic curvature δK^i_a from equation (40),

$$\delta K^i_a = -\delta^i_a[\dot{\Psi} + (\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})(\Psi + \Phi)]\ . \tag{43}$$

Similarly, using Eq.(13), equations (41) and (42) can be respectively reexpressed as

$$\delta\dot{\varphi} = \frac{\delta\pi}{\bar{p}} + \frac{\bar{\pi}}{\bar{p}}(3\Psi + \Phi)\ . \tag{44}$$

$$\delta\dot{\pi} = \bar{p}\nabla^2\delta\varphi - \bar{p}^2 V_{,\bar{\varphi}\bar{\varphi}}\delta\varphi + 3\bar{p}^2 V_{,\bar{\varphi}}\Psi\ . \tag{45}$$

Now, we derive the Hamiltonian's equation using the equation of motion of δK^i_a. Collecting the expressions δE^a_i (13), δK^i_a (43) , δN(14), and equations (31), (44), one can obtain

$$
\begin{aligned}
&\{\ddot{\Psi}\ +\ (\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})(2\dot{\Psi}+\dot{\Phi})+[(\cos 2\bar{\mu}\gamma\bar{k}-\frac{1}{2})\dot{\bar{k}}+(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})^2+\frac{\dot{\bar{\mu}}}{\bar{\mu}}(\bar{k}\cos 2\bar{\mu}\gamma\bar{k}-\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}) \\
&\qquad -\frac{1}{2}\bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2](\Psi+\Phi)+\bar{p}V(\bar{\varphi})(\Phi-\Psi)\}\delta^i_a+\partial_a\partial^i(\Phi-\Psi) \\
&\qquad = 4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi}-\bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi)\ . \qquad (46)
\end{aligned}
$$

When deriving this equation, we have used the relation

$$
4\pi G\dot{\bar{\varphi}}^2=(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2-\dot{\bar{k}}-\bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2\ , \qquad (47)
$$

which can be obtained from the corrected Friedmann equation (33) and Raychaudhuri equation (34). From equation (46), we can read the off-diagonal equation

$$
\partial_a\partial^i[\Phi-\Psi]=0\ , \qquad (48)
$$

which implies $\Phi=\Psi$. Therefore, in the following derivation, we will identify Φ with Ψ. Then the diagonal equation gives

$$
\begin{aligned}
&\ddot{\Phi}\ +\ 3(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\dot{\Phi}+[(2\cos 2\bar{\mu}\gamma k-1)\dot{\bar{k}}+2(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})^2+2\frac{\dot{\bar{\mu}}}{\bar{\mu}}(\bar{k}\cos 2\bar{\mu}\gamma\bar{k}-\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}) \\
&\qquad -\bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2]\Phi=4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi}-\bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi)\ . \qquad (49)
\end{aligned}
$$

Subsequently, we will consider the diffeomorphism constraint equation. In order to have non-vanishing diffeomorphism constraint, we must expand the shift vector N^a up to second-order, $\delta N^{a(2)}$. Therefore, the perturbed diffeomophism constraint with holonomy corrections is

$$
D[N^c]=\frac{1}{8\pi G}\int_\Sigma d^3x\delta N^{c(2)}[\bar{p}\partial_c(\delta^d_k\delta K^k_d)-\bar{p}(\partial_k\delta K^k_c)-(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\delta^k_c(\partial_d\delta E^d_k)+8\pi G\bar{\pi}\partial_c\delta\varphi]\ . \qquad (50)
$$

The diffeomorphism constraint equation can be obtained by varying the diffeomorphism constraint with respect to the shift perturbation:

$$
8\pi G\frac{\delta D[\delta N^{c(2)}]}{\delta(\delta N^{c(2)})}=\bar{p}\partial_c(\delta^d_k\delta K^k_d)-\bar{p}(\partial_k\delta K^k_c)-(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\delta^k_c(\partial_d\delta E^d_k)+8\pi G\bar{\pi}\partial_c\delta\varphi=0\ . \qquad (51)
$$

Using the expressions δE^a_i (13), δK^i_a (43) and equation (31), the above equation reduces to

$$\partial_c[\dot{\Phi} + (\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\Phi] = 4\pi G\dot{\bar{\varphi}}\partial_c\delta\varphi \ . \tag{52}$$

Finally, we will derive the Hamiltonian constraint equation. We note that after the variation with respect to the background lapse $\bar{N}$, the constraint equation will be second-order and can be neglected. So one can obtain the Hamiltonian constraint equation by only varying the perturbed lapse δN

$$\begin{aligned} \frac{\delta \tilde{H}^h[N]}{\delta(\delta N)} \ = \ & \frac{1}{16\pi G}[-4\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma\bar{k}}\sqrt{\bar{p}}\delta K^i_a\delta^a_i - (\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma\bar{k}})^2\frac{1}{\sqrt{\bar{p}}}\delta E^a_i\delta^i_a + \frac{2}{\sqrt{\bar{p}}}\partial_a\partial^i\delta E^a_i] \\ & +\frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - (\frac{\bar{\pi}^2}{2\bar{p}^{3/2}} - \bar{p}^{3/2}V(\bar{\varphi}))\frac{\delta E^a_i\delta^i_a}{2\bar{p}} + \bar{p}^{3/2}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi \\ & = 0 \ . \end{aligned} \tag{53}$$

Substituting the expressions δE^a_i (13), δK^i_a (43) and equation (31) into the above equation yields the Hamilton constraint equation

$$\nabla^2\Phi - 3(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\dot{\Phi} - [\dot{\bar{k}} + 6(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})^2 - 4(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2]\Phi = 4\pi G[\dot{\bar{\varphi}}\dot{\delta\varphi} + \bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi] \ . \tag{54}$$

In addition, using Eqs.(44) and (45), with the help of the background equations (30), (31) and (32), the perturbed Klein-Gordon equation can be expressed as

$$\delta\ddot{\varphi} + 2(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma})\delta\dot{\varphi} - \nabla^2\delta\varphi + \bar{p}V_{,\bar{\varphi}\bar{\varphi}}\delta\varphi + 2\bar{p}V_{,\bar{\varphi}}\Phi - \dot{\bar{\varphi}}(\dot{\Phi} + 3\dot{\Psi}) = 0 \ . \tag{55}$$

Now, we replace $\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}$ by Hubble parameter $\mathbb{H}$ in the perturbation equations (54), (49) and (52) such that these equations can be reexpressed as

$$\nabla^2\Phi - 3\mathbb{H}\dot{\Phi} - [\dot{\bar{k}} + 6\mathbb{H}^2 - 4(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2]\Phi = 4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi} + \bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi) \ , \tag{56}$$

$$\ddot{\Phi} + 3\mathbb{H}\dot{\Phi} + [2\dot{\mathbb{H}} - \dot{\bar{k}} + 2\mathbb{H}^2 - \bar{p}\frac{\partial}{\partial\bar{p}}(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2]\Phi = 4\pi G(\dot{\bar{\varphi}}\dot{\delta\varphi} - \bar{p}V_{,\bar{\varphi}}(\bar{\varphi})\delta\varphi) \ , \tag{57}$$

$$\partial_a(\dot{\Phi} + \mathbb{H}\Phi) = 4\pi G\dot{\bar{\varphi}}\delta\varphi_{,a} \ . \tag{58}$$

Obviously, in the classical limit, $\bar{\mu} \to 0$, all the equations above reduce to the classical cosmological perturbations (3), (4) and (5) respectively. Combing these equations, one can obtain the following second order differential equation for Φ

$$\ddot{\Phi} - \nabla^2\Phi + (6\mathbb{H} + 2\bar{p}\frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}})\dot{\Phi} + [2\dot{\mathbb{H}} + 8\mathbb{H}^2 - 4(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 + \frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}}\mathbb{H}]\Phi = 0 \ . \tag{59}$$

In addition, using the relation between the extrinsic curvature $\bar{k}$ and the conformal Hubble parameter $\mathbb{H}$ (36), one can obtain

$$(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma})^2 = \frac{1 - \sqrt{1 - 4(\bar{\mu}\gamma)^2\mathbb{H}^2}}{2(\bar{\mu}\gamma)^2} \ . \tag{60}$$

Therefore, the second order differential equation (59) becomes

$$\ddot{\Phi} - \nabla^2\Phi + (6\mathbb{H} + 2\bar{p}\frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}})\dot{\Phi} + [2\dot{\mathbb{H}} + 8\mathbb{H}^2 - 4(\frac{1 - \sqrt{1 - 4(\bar{\mu}\gamma)^2\mathbb{H}^2}}{2(\bar{\mu}\gamma)^2}) + \frac{V_{,\bar{\varphi}}(\bar{\varphi})}{\dot{\bar{\varphi}}}\mathbb{H}]\Phi = 0 \ . \tag{61}$$

When using the Klein-Gordon equation (35), the above second order differential equation can be further rewritten as

$$\ddot{\Phi} - \nabla^2\Phi + 2(\mathbb{H} - \frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}})\dot{\Phi} + 2[\dot{\mathbb{H}} - \mathbb{H}\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}} + 2\mathbb{H}^2 - 2(\frac{1 - \sqrt{1 - 4(\bar{\mu}\gamma)^2\mathbb{H}^2}}{2(\bar{\mu}\gamma)^2})]\Phi = 0 \ . \tag{62}$$

Up to now, we have completed the derivation of the cosmological perturbation equations in the effective loop quantum cosmology with holonomy corrections. Furthermore, we can also introduce the Mukhanov-Sasaki variable $\upsilon = \frac{a}{\dot{\bar{\varphi}}}\Phi$. Then the cosmological perturbation equation (62) reduces to

$$\ddot{\upsilon} - \nabla^2\upsilon + [(\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}})^{\cdot} - (\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}})^2 + \dot{\mathbb{H}} - \mathbb{H}^2 + 4S_h]\upsilon = 0 \ , \tag{63}$$

where we have denoted $S_h = \mathbb{H}^2 - \frac{1-\sqrt{1-4(\bar{\mu}\gamma)^2\mathbb{H}^2}}{2(\bar{\mu}\gamma)^2}$, which results from the holonomy corrections in the presence of the metric perturbation.

V. DISCUSSION

The effects of quantum gravity on structure formation, generally called trans-Planckian issues, have been investigated intensively (for example, we can refer to [25]). In loop quantum cosmology, the analogous issues have also been investigated in Ref.[9, 16–19]. However, in Ref.[19], they assume that after a super-inflation phase, the universe underwent a normal inflation stage. Then they find that the loop quantum effects can hardly lead to any imprint in the primordial power spectrum. Although in Ref.[9] the scale invariant spectrum was obtained and the holonomy effects also leave their imprint on the power spectrum, only the holonomy effects from a fixed background were taken into account. In this paper, along the Hamiltonian approach we have derived the cosmological perturbation equation for scalar modes in longitudinal gauge in the presence of holonomy corrections. In the presence of metric perturbation, we find that holonomy effects influence both background and perturbations, which contribute a non-trivial sector S_h. Therefore, the holonomy effects will affect the power spectrum such that it is possible that the quantum gravity effects will leave their imprint on the cosmic microwave background observed today. In the future work, we will investigate analytically and numerically the characters of power spectrum in the presence of holonomy corrections, which might open a window to test the loop quantum gravity effects.

In addition, in momentum space, the cosmological perturbation equation (63) can be written as

$$\ddot{\upsilon} - [\kappa^2 - 4(\mathbb{H}^2 - \frac{1-\sqrt{1-4(\bar{\mu}\gamma)^2\mathbb{H}^2}}{2(\bar{\mu}\gamma)^2}) - m_{eff}^2]\upsilon = 0 \ , \tag{64}$$

where κ denotes the momentum and $m_{eff}^2 = (\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}})^{\cdot} - (\frac{\ddot{\bar{\varphi}}}{\dot{\bar{\varphi}}})^2 + \dot{\mathbb{H}} - \mathbb{H}^2$. Therefore, the cosmological perturbation equation (64) can be effectively viewed as imposing such a modified dispersion relation at quantum gravity phenomenological level. Obviously, in such a modified dispersion relation, both the energy and momentum are bounded. Here, we point out that, in Ref. [26], Y. Ling *et. al* have also proposed a bounded modified dispersion relation, motivated by the isotropic homogenous effective loop quantum cosmology with holonomy corrections. Although both are bounded, they are also very different, implying we can not simply use the background corrections instead of perturbation corrections. In the future work, we will furthermore discuss the implications of such two modified dispersion relations.

Our present paper is the first step towards studying the holonomy corrected cosmological perturbation equations in the presence of metric perturbation. Since constraints are modified, the form of gauge invariant variables should change as well. Therefore, it is necessary to study the perturbations with different gauges or in a gauge invariant manner in this formalism, which is under progress.

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