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LUKE-WARM DARK MATTER: BOSE-CONDENSATION OF ULTRA-LIGHT PARTICLES

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ABSTRACT

We discuss the thermal evolution and Bose condensation of ultra-light scalar particles with Compton wavelength of galactic scales. We find an upper bound of 1.5 K for the dark matter temperature from the WMAP constraints on the amount of hot dark matter for a ΛCDM model. Agglomerations of these particles can form stable halos and naturally prohibit small scale structure, which may be favored by observations of dark matter distributions near the centers of galaxies. We present numerical as well as approximate analytical solutions of the Friedmann-Klein-Gordon equations and study the cosmological evolution of this scalar field dark matter from the early universe to the era of matter domination. Today, the particles in the ground state mimic presureless matter, while the excited state particles are radiation like.

1. INTRODUCTION

Most matter in the universe is non-luminous. The observed flatness of the galactic rotation curves indicates the presence of dark matter halos around galaxies. Observations of the cosmic microwave background anisotropies (Spergel *et al.* 2007) combined with large-scale structure and type Ia supernova luminosity data (Reiss *et al.* 1998, 2004, 2005; Perlmutter *et al.* 1999) constrain cosmological parameters finding that visible matter contributes only about 4% of the energy density of the universe, as opposed to 22% being dark matter and 74% dark energy. More recently, a clear separation between the center of baryonic matter and the total center of mass was observed in the Bullet cluster (Clowe *et al.* 2006) and later in other galaxy cluster collisions (Bradac *et al.* 2008). These observations reinforced the claim that dark matter is indeed composed of weakly interactive massive particles and is not a modification of gravity.

In the past few decades numerous dark matter candidates have been suggested including WIMPs, axions, supersymmetric particles, Kaluza-Klein particles, and various spin zero bosons (See Kamionkowski (2007); Bertone *et al.* (2005) for some reviews on the subject.) Fundamental spin zero particles are the simplest class of dark matter candidates and have been extensively studied. They are represented by scalar fields and play an important role in particle physics models (Peccei & Quinn 1977; Torres *et al.* 2000). These particles could form gravitationally stable structures such as boson stars, soliton stars, and galactic halos. Compact objects like boson stars and soliton stars could have formed from scalar particles through some type of Jeans instability mechanism (Seidel & Suen 1991; Urena-Lopez 2002; Alcubierre *et al.* 2003). Their stability and gravitational wave signatures have been studied numerically by many authors (Seidel & Suen 1990; Balakrishna *et al.* 2006, 2008).

Bosonic halos in which scalar particles Bose-condense and form gravitationally stable structures are supported against collapse by Heisenberg's uncertainty principle like boson stars. A Bose-Einstein condensate is described by the simple Klein-Gordon wave equation. Structure formation on scales smaller than the spreading of an individual boson (the Compton wavelength of one particle) is forbidden by quantum mechanics (Hu *et al.* 2000). Halos formed from ultralight scalars with Compton wavelength of galactic scales thus do not lead to over-abundance of dwarf galaxies unlike cold dark matter simulations with heavier bosons (Navarro *et al.* 1996; Salucci *et al.* 2003).

Scalar field galactic halos that explain the flatness of the rotation curves have been widely studied (Schunck & Liddle 1997; Guzman & Urena-Lopez 2003; Arbey *et al.* 2001). A non-thermal analysis of the cosmological behavior of ultralight bosonic halos was performed by Arbey *et al.* (2002).

Urena-Lopez (2009) pointed out that scalars field particles can Bose-condense at finite temperatures resurrecting previous work on relativistic Bose-Einstein condensation by Parker & Zhang (1991, 1993). A condensate is considered relativistic when the temperature of the condensate is significantly larger than the mass of one boson. Parker & Zhang (1993) discuss inflationary expansion driven by a relativistic Bose-Einstein condensate that then evolves into a radiation dominated universe.

In this paper we perform a thermal analysis of the post-inflationary cosmological behavior of scalar field dark matter formed from ultra-light bosons. We use the quantum field theory formalism of Parker & Fulling (1974) and extend the analysis that Hu (1982) used for a description of finite temperature effects in the early universe. The bosons are described by a complex scalar field to provide a conserved charge.

We assume the scalar particles decouple after inflation in the early universe. For ultralight particles ($m \sim 10^{-22} - 10^{-23}$ eV), the condensate is very pure with a high critical temperature. The particles in the ground state are Bose-condensed behaving as cold dark matter today, while the bosons in excited states behave like radiation. We constrain the radiation density of these particle to contribute less than 10% to the current observed radiation density leading to a luke-warm dark matter temperature today of $T_{DM} \lesssim 1.5$ K.

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In §2 we provide a brief review of relativistic Bose-condensation at high temperatures followed by a discussion of the temperature at which the scalars could decouple. In §3 we follow the cosmological evolution of the scalar field and describe possible solutions to the Friedmann-Klein-Gordon equation. We use units where $\hbar = c = 1$.

2. BOSE-EINSTEIN CONDENSATION AT FINITE TEMPERATURE

We consider a system of ultralight ($m \sim 10^{-23}$ eV) relativistic bosons represented by complex scalar fields. The condition for a relativistic condensate is that the temperature of the condensate $T \gg m$, which is certainly true up to the present day (Urena-Lopez 2009).

In the case of a complex field, there is a conserved charge, which is required for traditional Bose-Einstein condensation (BEC)[1]. The charge density is defined as the excess of particles n over anti-particles $\bar{n}$:

$$q = n - \bar{n}. \tag{1}$$

For the excited states, this charge density is (Mukhanov 2005)

$$q_{\rm ex} = g\frac{\mu T^2}{3}, \tag{2}$$

where g is the number of degrees of freedom of the system ($g = 1$ for scalar particles) and the chemical potential $\mu \leq m$. The maximum $q_{\rm ex} = mT^2/3$ occurs for $\mu = m$. For ultra-light bosons, the excess of bosons over anti-bosons in the excited states is small compared to the number density. Any new particles added to the system when $\mu = m$ will then have to condense to the ground state.

The critical temperature below which condensation occurs is found in terms of the charge density of the dark matter particles (Urena-Lopez 2009; Mukhanov 2005)

$$T < T_c = \sqrt{\frac{3q}{m}}\ . \tag{3}$$

When $T < T_c$ the majority of the bosons will condense to the ground state. In the ground state the particles will behave like non-relativistic matter, while the particles in excited states will remain highly relativistic.

Assuming that BEC occurs and that most particles are in the ground state, a first approximation to the total dark matter density is

$$\rho_{DM} \approx nm. \tag{4}$$

The density of dark matter today ρ^0_{DM} is

$$\rho^0_{DM} \approx 23\%\rho_c, \tag{5}$$

where $\rho_c \approx 4.19 \times 10^{-11} eV^4$. So,

$$n \approx \frac{\rho_{DM}}{m} \approx 10^{12} eV^3, \tag{6}$$

which leads to a very high critical temperature of

$$T_c \approx 1.7 \times 10^{17} eV \sim 10^{21}\ \text{K}. \tag{7}$$

The high critical temperature suggests that the condensate is very pure today with most bosons in the ground state. Note that the required charge density is very high implying the necessity of a mechanism that would produce such a large asymmetry of scalar particles over anti-particles.

The matter in the excited states is relativistic and contributes to the energy density of radiation (it can be considered hot dark matter). It is therefore constrained by WMAP observations (Spergel *et al.* 2007). A reasonable estimate for the amount of radiation from unknown sources is $\lesssim 10\%$ of the Cosmic Microwave Background (CMB) energy density

$$\rho^{\rm ex}_{DM} \approx 0.1\rho_{CMB} \approx 4.8 \times 10^{-6}\rho_c, \tag{8}$$

where we assumed a simple ΛCDM model[2].

Both $\rho^{\rm ex}_{DM}$ and ρ_{CMB} have the same temperature dependence $\propto T^4$ and so

$$T_{\rm DM} \lesssim 0.1^{1/4} \times T_{CMB} \approx 1.5K. \tag{9}$$

To raise the temperature of the neutrinos and photons relative to the scalar field, we can postulate the presence of beyond the Standard Model particles to add extra degrees of freedom in the thermal bath. In the beginning all particles are in thermal equilibrium and have the same temperature: $T_\phi = T_\nu = T_\gamma = T^{\pm}_e$. The scalar particles decouple and their entropy is separately conserved. The total entropy of the other components, which includes photons, electron-positron plasma, and neutrinos is also conserved. The beyond Standard Model particles ζ annihilate, dumping energy into and raising the temperature of the other components but not our already-decoupled scalar bosons. Similarly, neutrinos decouple then the electrons and positrons annihilate when the temperature drops below (~ 1 MeV) (Mukhanov 2005), raising the photon temperature above neutrinos and scalar bosons.

From conservation of entropy

$$\left(\frac{s_\gamma + s_{e^\pm} + s_\nu + s_\zeta}{s_\nu}\right)\left(\frac{s_\nu}{s_\phi}\right) = C, \tag{10}$$

where C is a constant. After the bosons decouple, but before ζs annihilate $T_\nu = T_\phi$. The constant can determined from the number of degrees of freedom of each species (Mukhanov 2005)

$$C = 1 + \frac{7}{8} \times \frac{1}{2}(2 \times 2 + 6) + g_\zeta = 10.75 + g_\zeta, \tag{11}$$

where g_ζ is the number of degrees of freedom associated with the new particles ζ. After ζs annihilate

$$10.75 \times \frac{s_\nu}{s_\phi} = 10.75 + g_\zeta. \tag{12}$$

The entropy density $s_\gamma \propto T^3_\gamma$ and $s_\nu \propto T^3_\nu$. So,

$$T_\phi = \left(\frac{10.75}{10.75 + g_\zeta}\right)^{1/3} T_\nu. \tag{13}$$

Assuming the temperatures of neutrinos and bosons today are $T_\nu = 1.95$ K and $T_\phi \approx 1.5$ K respectively gives

[1] Recently, Sikivie and Yang showed that dark matter axions can form a BEC as well.

[2] We are using the cosmological parameters that fit the 5 year WMAP data to the ΛCDM model from http://lambda.gsfc.nasa.gov

$g_\zeta = 12.85$. The extra particles need about ~ 13 additional degrees of freedom.

3. COSMOLOGICAL EVOLUTION

The density evolution of the universe must closely follow the standard ΛCDM model at times later than nucleosynthesis, at $a \approx 10^{-10}$. During radiation domination, our scalar field must be a subdominant contribution to the density of the universe, and during matter domination it must be a replacement for dark matter. As shown below, the excited states are a subdominant contribution to the density. The macroscopically-occupied ground state has $\rho \propto a^{-6}$ at early times and must be constrained to be less dense than the density of radiation for at least all times after nucleosynthesis.

In the sections below, we first derive equations in a general background, as well as expressions for density and pressure, then specialize to the epochs of radiation and matter domination. We also present numerical solutions to the equations which confirm our approximations in earlier sections and show that the initial conditions can be adjusted to satisfy the cosmological constraints and plot the resulting evolution numerically.

3.1. *Evolution Equations*

The line element in an expanding universe can be written as the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right). \tag{14}$$

The Klein-Gordon equation

$$\Box\Phi - m^2\Phi = 0$$

is derived from a Lagrangian density of the form

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - m^2\Phi). \tag{15}$$

The density and pressure can be defined in the usual way

$$\begin{aligned} \rho &= \frac{1}{2}\left(\partial_t\Phi^\dagger\partial_t\Phi + \partial_j\Phi^\dagger\partial^j\Phi + m^2\Phi\Phi^\dagger\right), \\ p &= \frac{1}{2}\left(\partial_t\Phi^\dagger\partial_t\Phi + \partial_j\Phi^\dagger\partial^j\Phi - m^2\Phi\Phi^\dagger\right), \end{aligned} \tag{16}$$

where Greek indices vary between 1 and 4 and the index j varies between 1 and 3.

Following Hu (1982), we perform a series of variable transformations to expose the conformal properties of the scalar field equation. We introduce a conformal time coordinate defined by $dt = ad\tau$. We also make the substitution $\Psi = a\Phi$. The metric is conformally static

$$ds^2 = a(\tau)^2\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right). \tag{17}$$

We can now rewrite the Klein-Gordon equation using the flat space operator $\tilde{\Box} \equiv -\partial_\tau^2 + \partial_x^2 + \partial_y^2 + \partial_z^2$ and remembering that the d'Alembertian $\Box = (\sqrt{-g})^{-1}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$

$$\frac{1}{a^3}\tilde{\Box}\Psi + \frac{a''}{a^4}\Psi - \frac{m^2}{a}\Psi = 0\,, \tag{18}$$

where $'$ denotes the derivative with respect to τ. The last two terms in Eq. (18) break conformal invariance. The invariance could be restored, as in Hu (1982), by setting the mass to zero and adding a term proportional to the four-dimensional Ricci scalar. However, in this case the field would no longer be minimally coupled. We choose to treat the terms that break conformal invariance as a perturbation.

The solution to the scalar field equation can be decomposed into modes (Parker & Fulling 1974)

$$\Psi(\mathbf{x},\tau) = \int d^3\vec{k}A_{\vec{k}}\psi_k(\tau)e^{i\vec{k}\cdot\vec{x}} + \text{H.c.}, \tag{19}$$

where H.c. denotes the Hermitian conjugate and ψ_k satisfies

$$\frac{d^2\psi_k}{d\tau^2} + \left[k^2 - \frac{a''}{a} + a^2m^2\right]\psi_k = 0\,. \tag{20}$$

The conserved current in mode k can be written as

$$J_{0k} = \frac{1}{i}\left(\psi_k\partial_\tau\psi_k^\star - \psi_k^\star\partial_\tau\psi_k\right). \tag{21}$$

The canonical commutation relation of the field Φ and its conjugate momentum Π leads to the usual commutation relations

$$[A_{\vec{k}}, A_{\vec{k}'}] = 0, \quad [A_{\vec{k}}, A^\dagger_{\vec{k}'}] = \delta(\vec{k},\vec{k}'), \tag{22}$$

when the conserved current is chosen to be $J_{0k} = 1$ for particles and $J_{0k} = -1$ for anti-particles. The operator $A_{\vec{k}}$ corresponds to physical particles and the number density of particles is defined to be $n =< A^\dagger_{\vec{k}}A_{\vec{k}} >$ (Parker & Fulling 1974).

The commutation relations are automatically satisfied if we take ψ_k of the form

$$\psi_{\vec{k}}(\tau) = \frac{1}{\sqrt{2\omega_k}}e^{-i\int^\tau \omega_k d\tau'}. \tag{23}$$

Each mode is now characterized by its eigenfunction ω_k given by

$$\begin{aligned} -\frac{1}{2}(a^2H)^2\omega_k\frac{d^2\omega_k}{da^2} + \frac{3}{4}(a^2H)^2\left(\frac{d\omega_k}{da}\right)^2 \\ -\omega_k^2 aH\frac{d(a^2H)}{da} + \frac{\omega_k}{2}(a^2H)\frac{d(a^2H)}{da}\frac{d\omega_k}{da} - \omega_k^4 \\ +(k^2 + a^2m^2)\omega_k^2 = 0. \end{aligned} \tag{24}$$

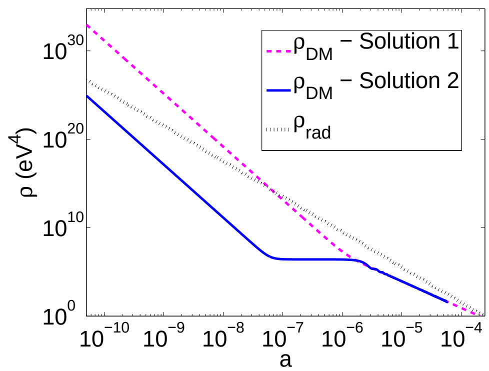
To concomitantly solve the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho \tag{25}$$

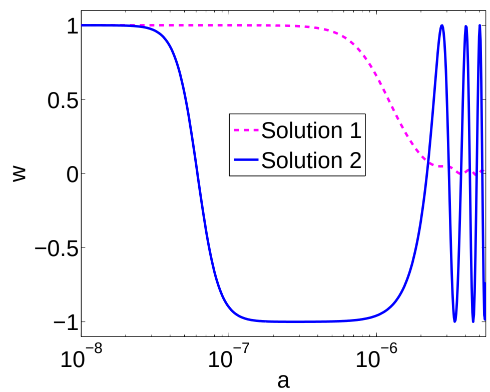
we approximate the Hubble parameter in different epochs power laws of the form $H = a'/a^2 = H_0a^{-n}$. Here $\rho = \rho_{\rm rad} + \rho_\Lambda + \rho_m$, the radiation energy density $\rho_{\rm rad} \propto a^{-4}$, the matter density $\rho_m \approx \rho_\Phi \propto a^{-3}$ is dominated by dark matter and the dark energy term ρ_Λ = constant in a ΛCDM model. Thus, the exponent is $n = 2$ during the radiation domination era and $n = 3/2$ during the matter domination era.

3.2. *Radiation domination*

In the radiation domination regime ($n = 2$), the scalar field Eq. (20) reduces to the flat space wave equation with an effective mass that varies with the scale factor



(a)



(b)

FIG. 1.— The dark matter density ρ_{DM} (along with $\rho_{\rm rad}$) and $w = p_{DM}/\rho_{DM}$ are displayed as a function of scale factor a.

$$\frac{d^2\psi_k}{d\tau^2} + \left(k^2 + a^2m^2\right)\psi_k = 0\ . \tag{26}$$

The mass term is the only perturbation from conformal invariance (Since $a'' = 0$.) We extend the analysis of Hu (1982) to determine the average density in excited states. Modes with $k \gg am$ are effectively massless, and for these modes $\omega_k \approx \sqrt{k^2 + a^2m^2}$. We use Eq. (16) to write the density in state k

$$\rho_k = \frac{1}{2}\left[\frac{H^2}{2\omega_k a^2} + \frac{(\omega_k')^2}{8a^4\omega_k^3} + \frac{H\omega_k'}{2a^3\omega_k^2} + \frac{\omega_k}{a^4}\right]. \tag{27}$$

The total density in the excited states is approximately

$$\begin{aligned}\rho_{\rm ex} &= \frac{1}{2\pi^2}\int_0^\infty k^2dk\frac{\rho_k}{\exp[\omega_k/(Ta)]-1} \\ &\approx \frac{1}{2\pi^2}\int_0^\infty k^2dk\left(\frac{H^2}{2a^2\omega_k} + \frac{\omega_k}{a^4}\right)\frac{1}{\exp[\omega_k/(Ta)]-1} \\ &= \frac{T^4\pi^2}{30} + \frac{T^2H^2}{24} - \frac{m^2}{24}\frac{T^2}{a^2} + ...\end{aligned} \tag{28}$$

When $T \gg H$, the T^4 term dominates the density and the excited states behave like radiation.

For the ground state ($k = 0$), Eq. (24) can be rewritten as

$$-\frac{y}{2}\frac{d^2y}{dx^2} + \frac{3}{4}\left(\frac{dy}{dx}\right)^2 - y^4 + x^2y^2 = 0, \tag{29}$$

where y and x are dimensionless variables defined by

$$\omega_0 = \sqrt{H_{0r}my}, \quad a = \sqrt{\frac{H_{0r}}{m}}x, \tag{30}$$

where $H = H_{0r}a^{-2}$ with $H_{0r} \approx 1.4 \times 10^{-35}$ eV. Note that $x = 1$ ($a \approx 10^{-6}$), which corresponds to $H = m$, is the transition to matter-like behavior for these particles. When $x \ll 1$ (or $H \gg m$), we can neglect the x^2y^2 term. Now Eq. (29) has an exact solution of

$$y(x) = \frac{C_0}{1 + C_0^2(x - x_0)^2}, \tag{31}$$

where C_0 and x_0 are constants. When $x_0 > 1$, y is approximately constant. When $x_0 \lesssim 1$, the typical behavior is that y for small x, peaks at $x = x_0$ with a height C_0, and then falls off as $C_0^{-1}x^{-2}$ (the higher C_0, the narrower the peak and hence the transition between the constant and x^{-2} behaviors is more abrupt). Initially, $y =$ constant and the density ρ_0 and pressure p_0 for the ground state are $\propto a^{-6}$. When $x \gg x_0$ and if $x \gg 1/|C_0|$ then $y = C_0^{-1}x^{-2}$. The pressure and the density then have two terms

$$\begin{aligned}\rho_0 &= \frac{H_{0r}^{3/2}}{4a^6m^{1/2}C_0} + \frac{m^{5/2}C_0}{2H_{0r}^{3/2}} \\ p_0 &= \frac{H_{0r}^{3/2}}{4a^6m^{1/2}C_0} - \frac{m^{5/2}C_0}{2H_{0r}^{3/2}},\end{aligned} \tag{32}$$

where the a^{-6} term dominates at early times. The density transitions to a cosmological constant with $p_0 = -\rho_0$ when $a \approx H_{0r}^{1/2}C_0^{-1/3}2^{-1/6}m^{-1/2}$.

3.3. *Matter Domination*

In the matter domination regime ($n = 3/2$), Eq. (20) becomes

$$\frac{d^2\psi_k}{d\tau^2} + \left(k^2 - \frac{H_{0m}^2}{2a} + a^2m^2\right)\psi_k = 0, \tag{33}$$

where $H = H_{0m}a^{-3/2}$ with $H_{0m} \approx 7.8 \times 10^{-34} eV$.

Using Eq. (23) we obtain an equation for ω_k:

$$\begin{aligned}&-\frac{\omega_k a}{2}\frac{d^2\omega_k}{da^2} - \frac{\omega_k}{4}\frac{d\omega_k}{da} + \frac{3a}{4}\left(\frac{d\omega_k}{da}\right)^2 - \frac{\omega_k^4}{H_{0m}^2} \\ &+\omega_k^2\left[\left(\frac{k}{H_{0m}}\right)^2 - \frac{1}{2a} + \left(\frac{am}{H_{0m}}\right)^2\right] = 0.\end{aligned} \tag{34}$$

For the ground state ($k = 0$), this equation has an exact solution

$$\omega_0 = \frac{am}{C_1\sin[4ma^{3/2}/(3H_{0m}) + \alpha] + C_2}, \tag{35}$$

where C_1, C_2 and the phase α are constants with the constraint $C_2^2 - C_1^2 = 1$. This solution is in agreement with Arbey *et al.* (2002). When $C_1 = 0$ the solution reduces to $\omega_0 = am$. Solutions with non-zero C_1 oscillate around the $\omega_0 = am$ solution. Eq. (35) can also be written in terms of t as

$$\omega_0 = \frac{am}{C_1\sin(2mt + \alpha) + C_2}. \tag{36}$$

The oscillations have a period of π/m ($\sim$ a few years for $m = 10^{-23}$ eV). The pressure averages to zero on cosmological timescales causing the ground state scalar field particles to behave like pressureless matter.

3.4. *Numerical Solution*

We also solve Eq. (24) numerically including the effect of both radiation and matter in the Hubble parameter,

$$H = \sqrt{H_{0r}^2 a^{-4} + H_{0m}^2 a^{-3}} \ . \tag{37}$$

The numerical solutions are fully specified by the value of the field and its first derivative at a given a, as well as an overall scaling of the density. In both cases, the overall scaling of the density was chosen to match the observed cosmological density of cold dark matter.

Two representative solutions are shown in Fig. 1. The initial conditions in terms of the x and y variables introduced above are $y(10^{-8}) \approx 0.48$ and $y'(10^{-8}) = 0.49$ for solution 1, and $y(10^{-8}) \approx 5 \times 10^3$ and $y'(10^{-8}) \approx 5. \times 10^7$ for solution 2. Fig. 1(a) shows the ground state density as a function of scale factor, along with the radiation density $\rho_{\rm rad}$. Fig. 1b displays $w = p_{DM}/\rho_{DM}$. The density at early times decays like a^{-6} and is determined by the mass of the scalar particle and the required density in the ground state at late times. Solution 1 requires a large density at early times which is not compatible with the standard ΛCDM model. Solution 2 has a phase where the ground state density is constant. This allows the initial density to be much lower and therefore compatible with the standard model up to the the approximate time of nucleosynthesis ($a \approx 10^{-10}$). The final density is the same, but w oscillates at late times around the pressureless $w = 0$ solution.

4. CONCLUSION

In this paper we propose a complex scalar field dark matter cosmological model with particles of temperature ≈ 1.5 K today. This temperature is close to the temperatures of photons and neutrinos. Our particles decouple before neutrinos to avoid affecting nucleosynthesis.

We assume dark matter is formed from ultralight particles with a Compton wavelength of galactic scales, which naturally do not exhibit small scale structure avoiding the over-abundance of dwarf galaxies (Hu *et al.* 2000). Urena-Lopez (2009) has shown that for $m < 10^{-14}$ eV condensation always occurs. Most of our ultralight particles are condensed to the ground state. We find that the particles left in the excited states are radiation-like today, while those in the ground state behave like pressureless matter. The temperature of the dark matter is independent of the particle mass and is constrained by cosmological observations that limit the density of radiation from unknown sources in the universe (we take this be be $\lesssim 10$ % of ρ_{CMB}).

We have solved the Klein-Gordon equation in radiation and matter domination regimes and studied the behavior of ground state and excited state particles. When the Compton wavelength of the particle is larger than the cosmological horizon ($H > m$) the dark matter density starts with a dependence on the scale factor of the form $\rho_{DM} \propto a^{-6}$ and then switches to a cosmological-constant behavior. This is consistent with Arbey *et al.* (2002). The density of the excited states remains radiation-like $\rho_{\rm DM\ ex} \propto a^{-4}$ until $T \sim H$ ($a \sim 10^{-32}$) and is sub-dominant to the density of the known radiation. When $H < m$ the scalars in the ground state act as non-relativistic matter $\rho_{DM} \propto a^{-3}$ and the excited states behave like radiation.

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REFERENCES

Alcubierre, M. *et al.* (2003) Class. Quantum Grav. 20, 2883.
Arbey, A., Lesgourgues, J. & Salati, P. (2001) Phys. Rev. D64, 123528.
Arbey, A., Lesgourgues, J. & Salati, P. (2002) Phys. Rev. D65, 083514, astro-ph/0112324.
Balakrishna, J., Seidel, E. & Suen, W.-M. (1998) Phys. Rev. D58, 104004.
Balakrishna, J. *et al.* (2006) Class. Quant. Grav. 23, 2631, gr-qc/0602078.
Balakrishna, J. *et al.* (2008) Phys. Rev. D77, 024028. 0710.4131.
Bertone, G., Hooper, D. & Silk, J. (2005) Phys. Rept. 405, 279, hep-ph/0404175.
Bradac, M. *et al.* (2008) ApJ687, 959.
Clowe, D. *et al.* (2006) ApJ648, L109.
Guzman, F. S. (2004), Phys. Rev. D70, 044033, gr-qc/0407054.
Guzman, F. S. & Urena-Lopez (2003), L. A. Phys. Rev. D68, 024023.
Hu, B. (1992) Physics Letters 108B, 19.
Hu, W., Barkana, R. & Gruznikov, A. (2000) Phys. Rev. Lett.85, 1158.
Kamionkowski, M. arXiv:0706.2986.
Mukhanov, V. “Physical Foundations of Cosmology”, Cambridge University Press, 2005.
Navarro, J. F. *et al.* (1996), ApJ462, 563 (1996).
Parker, L. & Fulling, S. A. (1974), Phys. Rev. D9, 341.
Parker, L. and Zhang, Y. (1991) Phys. Rev. D44, 2421.
Parker, L. and Zhang, Y. (1993) Phys. Rev. D47, 416.
Peccei, R. D. & Quinn, H. (1977) Phys. Rev. Lett.38, 1440. Phys. Rev. D16, 1791.
Perlmutter, S. *et al.* (1999), ApJ517, 565.
Riess, A. G. *et al.* (1998) ApJ116, 1009. (2004) ApJ607, 665, (2005) ApJ627, 579.
Salucci, P., Walter, F. & Borriello, A. (2003), Astron. & Astrophys. 409, 53.
Schunck, F. E. & Liddle, A. R. (1997) Phys. Lett. B404, 25.
Seidel, E. & Suen (1990), W.-M. Phys. Rev. D42, 384.
Seidel, E. & Suen, W.-M. (1991) Phys. Rev. Lett.66, 1659.
Spergel, D. N. *et al.* (2007) ApJ Suppl. 170, 377, astro-ph/0603449.
Torres, D. F., Capozziello, S., & Lambiase, G. (2000) Phys. Rev. D62, 104012.
Urena-Lopez, L. A. (2002) Class. Quantum Grav. 19, 2617.
Urena-Lopez, L. A. (2009) JCAP 01, 014, arXiv:0806.3093.