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Post-Inflationary Evolution via Gravitation

N. C. Tsamis†

Department of Physics, University of Crete
GR-710 03 Heraklion, HELLAS.

R. P. Woodard*

Department of Physics, University of Florida
Gainesville, FL 32611, UNITED STATES.

ABSTRACT

We study a class of non-local, purely gravitational models which have the correct structure to reproduce the leading infrared logarithms of quantum gravitational back-reaction during the inflationary regime. These models end inflation in a distinctive phase of oscillations with slight and short violations of the weak energy condition and should, when coupled to matter, lead to rapid reheating. By elaborating this class of models we exhibit one that has the same behaviour during inflation, goes quiescent until the onset of matter domination, and induces a small, positive cosmological constant of about the right size thereafter.

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† e-mail: tsamis@physics.uoc.gr
* e-mail: woodard@phys.ufl.edu

• **Introduction:** During the inflationary era infrared gravitons are produced out of the vacuum because of the accelerated expansion of spacetime. The interaction stress among the gravitons produced – an inherently non-local effect – can lead to a non-trivial quantum gravitational back-reaction on inflation [1]. In a previous paper [2] we proposed a phenomenological model which can provide evolution beyond perturbation theory. In one sentence, we constructed an *effective* conserved stress-energy tensor $T_{\mu\nu}[g]$ which modifies the gravitational equations of motion: [1]

$$G_{\mu\nu} \;\equiv\; R_{\mu\nu} \,-\, \frac{1}{2} g_{\mu\nu}\, R \;=\; -\Lambda\, g_{\mu\nu} \,+\, 8\pi G\, T_{\mu\nu}[g] \;\; . \tag{1}$$

and which, we hope, contains the most cosmologically significant part of the full effective quantum gravitational equations.

Our physical *ansatz* consisted of parametrizing $T_{\mu\nu}[g]$ as a "perfect fluid":

$$T_{\mu\nu}[g] \;=\; (\rho + p)\, u_\mu\, u_\nu \,+\, p\, g_{\mu\nu} \;\; , \tag{2}$$

with the gravitationally induced pressure given as the following functional of the metric tensor:

$$p[g](x) \;=\; \Lambda^2\, f[-\epsilon\, X](x) \qquad , \qquad X \;\equiv\; \frac{1}{\Box} R \;\; , \tag{3}$$

where the function f grows without bound and satisfies:

$$f[-\epsilon\, X] \;=\; -\epsilon\, X \,+\, O(\epsilon^2) \;\; , \tag{4}$$

and where the scalar d'Alembertian:

$$\Box \;\equiv\; \frac{1}{\sqrt{-g}}\, \partial_\mu \Big(g^{\mu\nu} \sqrt{-g}\, \partial_\nu \Big) \;\; , \tag{5}$$

is defined with retarded boundary conditions. The induced energy density $\rho[g]$ and 4-velocity $u_\mu[g]$ were determined, up to their initial value data, from stress-energy conservation:

$$D^\mu\, T_{\mu\nu} \;=\; 0 \;\; . \tag{6}$$

[1] Hellenic indices take on spacetime values while Latin indices take on space values. Our metric tensor $g_{\mu\nu}$ has spacelike signature and our curvature tensor equals: $R^\alpha{}_{\beta\mu\nu} \equiv \Gamma^\alpha{}_{\nu\beta,\mu} + \Gamma^\alpha{}_{\mu\rho}\, \Gamma^\rho{}_{\nu\beta} - (\mu \leftrightarrow \nu)$. The initial Hubble constant is $3H_0^2 \equiv \Lambda$. We restrict our analysis to scales $M \equiv (\Lambda/8\pi G)^{\frac{1}{4}}$ below the Planck mass $M_{\rm Pl} \equiv G^{-\frac{1}{2}}$ so that the dimensionless coupling constant $\epsilon \equiv G\Lambda$ of the theory is small.

The 4-velocity was chosen to be timelike and normalized:

$$g^{\mu\nu}\, u_\mu u_\nu = -1 \qquad \Longrightarrow \qquad u^\mu\, u_{\mu;\nu} = 0 \ \ . \tag{7}$$

The homogeneous and isotropic evolution [2] of this model – using a combination of numerical and analytical methods – revealed the following basic features [3] :

01. After the onset and during the era of inflation, the source $X(t)$ grows while the curvature scalar $R(t)$ and Hubble parameter $H(t)$ decrease.

02. Inflationary evolution dominates roughly until we reach a critical point X_{cr} defined by:

$$1 - 8\pi G\Lambda\, f[-G\Lambda\, X_{cr}] \ \equiv \ 0 \ \ . \tag{8}$$

03. The epoch of inflation ends close to but before the universe evolves to the critical time. This is most directly seen from the deceleration parameter since initially $q(t=0) = -1$ while at criticality $q(t = t_{cr}) = +\frac{1}{2}$.

04. Oscillations in $R(t)$ become significant as we approach the end of inflation; they are centered around $R = 0$, their frequency equals:

$$\omega \ = \ G\Lambda H_0 \sqrt{72\pi\, f'_{cr}} \ \ , \tag{9}$$

and their envelope is linearly falling with time.

05. During the oscillations era, although there is net expansion, the oscillations of $H(t)$ take it to small negative values for small time intervals – a feature conducive to rapid reheating; those of $\dot{H}(t)$ take it to positive values for about half the time; and, those of $a(t)$ are centered around a linear increase with time.

Since we shall be concerned with the various post-inflationary phases to which the universe evolved, and since all these phases are characterized by constant $\varepsilon \equiv -\dot{H}H^{-2} = 1 + q$, it is convenient to provide expressions for some basic quantities describing such spacetimes. In particular, in terms of their initial values, the time evolution of the scale factor, Hubble parameter

[2] The line element in co-moving coordinates is $ds^2 = -dt^2 + a^2(t)\, d\vec{x}\cdot d\vec{x}$. In terms of the scale factor a, the Hubble parameter equals $H(t) = \dot{a}\, a^{-1}$ and the deceleration parameter equals $q(t) = -a\, \ddot{a}\, \dot{a}^{-2}$.

[3] In [2], our analytical results were obtained for any function f satisfying (4) and growing without bound, our numerical results for the choice: $f(x) = \exp(x) - 1$.

and Ricci scalar are, respectively:

$$\varepsilon \equiv -\frac{\dot{H}(t)}{H^2(t)} \quad \Longrightarrow \quad a(t) = a_{in} \Big[1 + \varepsilon\, H_{in}(t - t_{in}) \Big]^{\frac{1}{\varepsilon}} \; , \tag{10}$$

$$H(t) = \frac{H_{in}}{1 + \varepsilon\, H_{in}(t - t_{in})} \; , \tag{11}$$

$$R(t) = 6\dot{H} + 12H^2 = 6\,(2 - \varepsilon)\, H^2(t) \; . \tag{12}$$

We shall also need the temporal component of the Ricci tensor:

$$R_{00} = +3\, q\, H^2 = -3\,(1 - \varepsilon)\, H^2 \; . \tag{13}$$

Moreover, the first two time derivatives of the curvature scalar are:

$$\dot{R} = -12\varepsilon\,(2 - \varepsilon)\, H^3 \; , \tag{14}$$

$$\ddot{R} = +36\varepsilon^2\,(2 - \varepsilon)\, H^4 \; , \tag{15}$$

so that:

$$\Box R = -\ddot{R} - 3H\dot{R} = +36\varepsilon\,(1 - \varepsilon)(2 - \varepsilon)\, H^4 \; . \tag{16}$$

Notice that R vanishes for radiation ($\varepsilon = 2$) and $\Box R$ for radiation as well as inflation ($\varepsilon = 0$).

- **Two Problems:** The homogeneous and isotropic evolution described by the simple model briefly reviewed in the introduction does not give a completely satisfactory end to inflation. The oscillations that occur after the end of inflation are not a problem, but the average expansion $a(t) \propto t$ is unacceptably rapid. At that rate there would be no reheating and the late time universe would be cold and empty. Nonetheless, the same is true for scalar-driven inflation if one ignores the possibility for energy to flow from the inflaton into ordinary matter. We believe that energy will flow from the gravitational sector of our model into ordinary matter to create a radiation dominated universe, just as it is thought to do for scalar-driven inflation.

An amazing possibility arises if this process can be shown to occur: *our quantum gravitational correction cancels the bare cosmological constant and then becomes dormant during the epoch of radiation domination.* To see this, suppose the deceleration parameter has the pure radiation value of $q(t) = +1$ for times $t > t_r$. This case corresponds to $\varepsilon = 2$ and (10-12) give:

$$q = +1 \quad \Longrightarrow \quad a(t) = a_r \Big[1 + 2H_0(t - t_r) \Big]^{\frac{1}{2}} \; , \tag{17}$$

$$H(t) \;=\; \frac{H_r}{1 + 2H_r(t - t_r)} \;\; , \tag{18}$$

$$R(t) \;=\; 0 \;\; , \tag{19}$$

Our simple source $X(t)$ obeys the differential equation $\Box X = R$, so for $t > t_r$ it must be a linear combination of its two homogeneous solutions:

$$\forall t \;>\; t_r \quad \Longrightarrow \qquad \Box X \;=\; 0 \quad \Longrightarrow$$

$$X(t) \;=\; X_r \;+\; \dot{X}_r \int^t dt' \left[\frac{a_r}{a(t')} \right]^3 \;=\; X_r \;-\; \frac{\dot{X}_r}{H_r} \, \frac{1}{\sqrt{1 + 2H_r(t - t_r)}} \;\; . \tag{20}$$

The only solution consistent with $q = +1$ is: [4]

$$X_r \;=\; X_{cr} \qquad , \qquad \dot{X}_r \;=\; 0 \;\; . \tag{21}$$

Having $X(t)$ approach X_{cr} within the context of a hot, radiation dominated universe would be a great success for our model, but the eventual transition to matter domination poses enormous problems. The onset of matter domination is really a gradual process but let us simplify the exposition by considering a sudden change from $q = +1$ to $q = +\frac{1}{2}$ at some time $t_m \gg t_r$. During this matter dominated epoch, for which $\varepsilon = \frac{3}{2}$, expressions (10-12) become:

$$q = +\frac{1}{2} \quad \Longrightarrow \qquad a(t) \;=\; a_m \Bigl[1 + \frac{3}{2} H_m(t - t_m) \Bigr]^{\frac{2}{3}} \;\; , \tag{22}$$

$$H(t) \;=\; \frac{H_m}{1 + \frac{3}{2} H_m(t - t_m)} \;\; , \tag{23}$$

$$R(t) \;=\; \frac{3H_m^2}{\Bigl[1 + \frac{3}{2} H_m(t - t_m) \Bigr]^2} \;\; . \tag{24}$$

where H_m and a_m are $H(t_m)$ and $a(t_m)$, respectively, computed from the radiation dominated geometry (17-18). The resulting change in the source

[4]For all power laws, such as radiation, the pressure and density fall like t^{-2}. Therefore, the second of the two homogeneous solutions cannot be present, implying that $\dot{X}_r = 0$, since its time dependence is $t^{-\frac{1}{2}}$ and could not sustain radiation. Neither can the first homogeneous solution sustain radiation unless the constant X_r eliminates the cosmological constant $3H_0^2$ and, hence, equals X_{cr}.

$X(t)$ is:

$$q = +\frac{1}{2} \quad \Longrightarrow$$
$$\Delta X(t) \;\equiv\; X(t) - X_{cr} \;=\; -\frac{4}{3}\ln\Big[1 + \frac{3}{2}H_m(t - t_m)\Big] \;+\; O(1) \;\; . \qquad (25)$$

To understand what is wrong with the change (25) caused by matter domination, it is useful to recall our *ansatz* for the quantum gravitationally induced pressure:

$$p[g](x) \;=\; \Lambda^2 \, f[-G\Lambda \, X](x) \;\; . \qquad (26)$$

In the context of this ansatz there are two major problems with (25):

01. The sign problem. It derives from the function $f(x)$ in (26) being monotonically increasing and unbounded. Hence, pushing $X(t)$ below $X_{cr} \ll 0$ results in positive total pressure, whereas observation implies negative pressure during the current epoch [3, 4]. Note that we cannot alter this feature of $f(x)$ without sacrificing the very desirable ability of the model to cancel an arbitrary bare cosmological constant.

02. The magnitude problem. In one sentence, the magnitude of the total pressure produced by (25) is vastly too large. The problem arises from the factors of the bare cosmological constant Λ in our ansatz (26). The total pressure $p_{\rm tot}$ is the sum of the classical contribution and our ansatz (26):

$$p_{\rm tot} \;=\; -\frac{\Lambda}{8\pi G}\Big\{1 - 8\pi G\Lambda \, f\Big[-G\Lambda\,(X_{cr} + \Delta X)\Big]\Big\} \qquad (27)$$
$$\simeq\; -\frac{\Lambda}{G} \times (G\Lambda)^2 \, f'_{cr} \, \Delta X \;\; . \qquad (28)$$

Comparing with the currently observed value $p_{\rm now}$ of the pressure:

$$p_{\rm now} \;\simeq\; -\frac{3}{8\pi G}\, H^2_{\rm now} \;\; , \qquad (29)$$

gives:

$$\frac{p_{\rm tot}}{p_{\rm now}} \;\simeq\; \Big(\frac{G\Lambda\, H_0}{H_{\rm now}}\Big)^2 f'_{cr}\,\Delta X \;\simeq\; 10^{86} \times f'_{cr} \times \Delta X \; , \qquad (30)$$

where we have assumed $H_0 \sim 10^{13}$ GeV and $H_{\rm now} \sim 10^{-33}$ eV. The derivative f'_{cr} is unity for the linear model and of order $(G\Lambda)^{-1} \sim 10^{12}$ for the exponential model, so we expect f'_{cr} to be at least of order one and possibly much greater.

There is no way of addressing either problem without generalizing our ansatz (26) for the pressure. This necessarily takes us away from what can be motivated by explicit computation during the de Sitter regime.

- **Decreasing the Magnitude:** The magnitude problem arises because the constant Λ in (26) is about the square of the inflationary Hubble parameter rather than its late time descendant that could be 55 orders of magnitude smaller. Solving the problem entails replacing one of these factors of Λ by some dynamical scalar that changes as time evolves in a way that also preserves the original relaxation mechanism. The latter requirement rules out any tampering with the factor of Λ^2 that multiplies $f[-G\Lambda\, X]$ in (26). It is safer to make the factor of Λ in the argument of the function f dynamical and move it to the right of the $\square^{-1}$:

$$-G\Lambda\, X \;=\; -\frac{G\Lambda}{\square}\, R \qquad \longrightarrow \qquad -\frac{G\Lambda}{\square}\Bigl(R \times \frac{S}{\Lambda}\Bigr) \;\equiv\; -G\Lambda \times Y[g] \;\; , \tag{31}$$

where S is an appropriate scalar quantity. The idea is for (31) to approach $-G\Lambda \times X_{cr}$ during inflation and then freeze in to this value during radiation domination, throughout which the scalar falls off so that subsequent evolution is driven by an acceptably small source. There are many possibilities for the scalar S in (31), which we shall always normalize so as to make the ratio $S\Lambda^{-1}$ give unity for de Sitter spacetime.

One might think $S = \frac{1}{4}R$ can work, but numerical simulations show that it responds too quickly to the slowing geometry. Instead of inflation ending, the scale factor approaches an accelerating, power law expansion – that is, $a(t) \propto t^s$ with $s > 1$. It is instructive to understand why this happens by perturbing the pressure around the critical value which cancels the bare cosmological constant. The relevant FRW equation of motion becomes:

$$\begin{aligned} -\,2\dot{H} - 3H^2 \;&=\; -\Lambda + 8\pi G\, p && (32)\\ &=\; -\Lambda \Biggl\{ 1 - 8\pi G\Lambda\, f\Bigl[-G\Lambda (X_{cr} + \Delta Y) \Bigr] \Biggr\} && (33)\\ &\approx\; -8\pi (G\Lambda)^2\, f'_{cr}\, \Lambda \times \Delta Y \quad . && (34)\end{aligned}$$

When $S = \frac{1}{4}R$, the function ΔY obeys:

$$S = \frac{1}{4}\,R \quad \Longrightarrow \qquad \Box \Delta Y \;=\; \frac{R^2}{4\Lambda} \;\; . \tag{35}$$

Then, for $a(t) \propto t^s$ we have:

$$\begin{aligned} a(t) \;\propto\; t^s \;\;\Longrightarrow \qquad R \;&=\; \frac{6s\,(2s-1)}{t^2} \\ \Longrightarrow \qquad \Delta Y \;&=\; \frac{Y_1}{t^{3s-1}} \;+\; \frac{3s^2(2s-1)^2}{2(s-1)\Lambda t^2} \;\; , \end{aligned} \tag{36}$$

where Y_1 is an integration constant. For $s > 1$ the homogeneous solution proportional to Y_1 falls off faster than the inhomogeneous solution. Neglecting the homogeneous solution and substituting into (34) gives an algebraic equation for the power s, which is easy to solve for $G\Lambda \ll 1$:

$$\begin{aligned} \frac{(s-1)(s-\frac{2}{3})}{s(s-\frac{1}{2})^2} \;&=\; 16\pi (G\Lambda)^2\, f'_{cr} \quad \Longrightarrow \\ s \;&\approx\; \frac{1}{16\pi (G\Lambda)^2\, f'_{cr}} \;\gg\; 1 \quad , \quad G\Lambda \ll 1 \;\; . \end{aligned} \tag{37}$$

A more non-local scalar, which responds less rapidly to the slowing geometry, can be formed from derivatives of X:

$$\frac{S}{\Lambda} \;=\; \frac{-g^{\mu\nu}\,\partial_\mu X[g]\,\partial_\nu X[g]}{\frac{16}{3}\Lambda} \quad \longrightarrow \quad \frac{\dot{X}^2}{\frac{16}{3}\Lambda} \qquad , \qquad X[g] \;\equiv\; \frac{1}{\Box}\,R \;\; . \tag{38}$$

This gives an end to inflation for relatively large values of $G\Lambda$ but still goes over to a power law with the asymptotic form (37) for small $G\Lambda$. The reason is obvious from the form of $\dot{X}$ for FRW:

$$\dot{X} \quad \longrightarrow \quad -\frac{1}{a^3(t)} \int_0^t dt'\; a^3(t')\, R(t') \;\; . \tag{39}$$

If R simply vanished after some time, then $\dot{X}$ would decay subsequently like $a^{-3}(t)$. Hence the lag to changes in R is of the order of a few Hubble times and inflation will only be ended for unrealistically large values of $G\Lambda$ such that the critical time t_{cr} falls within this period.

We also tried scalars formed by using inverses of other operators like the conformal d'Alembertian:

$$\square_c \;\equiv\; \square - \frac{1}{6} R \quad \longrightarrow \quad -\frac{1}{a^2}\frac{d}{dt}\, a\, \frac{d}{dt}\, a \ . \tag{40}$$

and the Paneitz operator [6, 7]:

$$D_P \;\equiv\; \square^2 + 2 D_\mu \Bigl(R^{\mu\nu} - \frac{1}{3} g^{\mu\nu} R \Bigr) D_\nu \quad \longrightarrow \quad \frac{1}{a^3}\frac{d}{dt}\, a\, \frac{d}{dt}\, a\, \frac{d}{dt}\, a \frac{d}{dt} \ . \tag{41}$$

However, the result was always power law expansion like (37) for $G\Lambda \ll 1$.

To avoid the magnitude problem and still end inflation requires that we evaluate the dynamical scalar far back in the past. One way of achieving this is to use the integral curves $\chi_\mu[g](x)$ of a timelike 4-velocity field $V^\mu[g](x)$. We can construct such a 4-velocity field by taking the gradient of the invariant 4-volume $\mathcal{V}[g](x)$ of the past light-cone from the point $x^\mu = (t, \vec{x})$ back to our initial value surface at $x'^\mu = (t, \vec{x}')$:

$$\mathcal{V} \;\equiv\; \int_0^t dt' \int d^3x' \, \sqrt{-g(x')}\, \theta\Bigl(-\sigma(x; x') \Bigr) \ . \tag{42}$$

Here $\sigma(x; x')$ is $\frac{1}{2}$ times the square of the geodesic length from x^μ to x'^μ [8]. Because the volume of the past light-cone grows as one evolves into the future, its gradient is guaranteed to be timelike in any geometry. We define $V^\mu[g](x)$ as the normalized gradient of $\mathcal{V}[g](x)$:

$$V^\mu[g](x) \;\equiv\; \frac{-g^{\mu\nu}\, \partial_\nu \mathcal{V}}{\sqrt{-g^{\alpha\beta}\, \partial_\alpha \mathcal{V}\, \partial_\beta \mathcal{V}}} \ . \tag{43}$$

Now construct the integral curves $\chi^\mu[g](\tau, x)$ – as a functional of the metric and an ordinary function of the parameter τ and a coordinate point x^μ – so that they obey the conditions:

$$\frac{\partial \chi^\mu}{\partial \tau} \;=\; -V^\mu(\chi) \qquad \text{and} \qquad \chi^\mu(0, x) \;=\; x^\mu \ . \tag{44}$$

Our physical requirement on the scalar S is to evaluate it at *any* time t^* such that inflation is still dominant. Suppose $\tau^*[g](x)$ corresponds to this time as we follow the integral curves from x^μ back to the initial value surface. Then, we might define the scalar S at point x^μ in expression (31) to equal:

$$S(x) \;=\; \frac{1}{4} R[\chi^\mu(\tau^*; x)] \ . \tag{45}$$

The modification we have outlined would not change the initial value problem, is defined for any geometry, and seems to be invariant. It also solves the magnitude problem while preserving the general features of the way the original ansatz (26) ends inflation. For an FRW geometry and for the value of $\tau^*[g](x)$ associated with, e.g. $\frac{9}{10}$ of the time from x^μ back to the initial value surface, the functional $Y[g](x)$ becomes:

$$Y[g](x) \quad \longrightarrow \quad -\int_0^t dt' \, \frac{1}{a^3(t')} \int_0^{t'} dt'' \, a^3(t'') \, R(t'') \times \frac{R(\frac{1}{10}t'')}{4\Lambda} \quad . \tag{46}$$

Figure 1 shows the Hubble parameter versus time for this model with $G\Lambda = \frac{1}{100}$. Any of the less non-local models we tried evolved to the power law solution (37) for this small a value of $G\Lambda$. The duration of the plot corresponds to 350 initial Hubble times. One can see that the period of oscillation lengthens towards the end because $\frac{1}{4\Lambda}R(\frac{1}{10}t)$ decreases, but $H(t)$ still oscillates with decreasing amplitude and the oscillations still drop below zero. For the enormously smaller values of $G\Lambda$ relevant to primordial inflation $\frac{1}{4\Lambda}R(\frac{1}{10}t)$ would be effectively constant during the oscillatory phase, so the period of oscillation would also be constant, just as in the simple model (26).

- **Changing the Sign:** We turn now to the sign problem. It arises because the Ricci scalar (12) is positive during both inflation ($\varepsilon = 0$) and matter domination ($\varepsilon = \frac{3}{2}$):

$$q \;=\; -1 \qquad \Longrightarrow \qquad R \;=\; +12H^2 \quad , \tag{47}$$

$$q \;=\; +\frac{1}{2} \qquad \Longrightarrow \qquad R \;=\; +3H^2 \quad . \tag{48}$$

What we need is a source which changes sign from inflation to matter domination, and is still zero (or very small) during radiation domination. There are again many possibilities but a simple one that works is to change the source from the form motivated by (45):

$$Y[g](x) \;=\; \frac{1}{\Box}\left[R(x) \times \frac{R(\chi(\tau^*,x))}{4\Lambda} \right] \quad , \tag{49}$$

to the following form:

$$Y_\alpha[g](x) \;=\; \frac{1}{4\Lambda}\, \frac{1}{\Box} \Big[R(x)\, R\Big(\chi(\tau^*,x)\Big) \;+\; \alpha\, \Box R(x) \Big] \quad . \tag{50}$$

The term proportional to α vanishes for both de Sitter inflation ($\varepsilon = 0$) and pure radiation domination ($\varepsilon = 2$), so it should make little difference until the onset of matter domination. For $t \gg t_m$ and for the choice of $\tau^*[g](x)$ already discussed, the various dynamical scalars in (50) give ($\varepsilon = \frac{3}{2}$): [5]

$$t \gg t_m \quad \Longrightarrow \qquad R(t) \;=\; +3H^2(t) \quad , \quad R(\tfrac{1}{10}t) \;=\; +100 \times 3H^2(t) \;\; , \tag{51}$$

$$\Box R(t) \;=\; -\frac{27}{2} H^4(t) \;\; . \tag{52}$$

where we also used (16). There exists a reasonable range of α for which the sign gets reversed.

- **An Even Simpler Ansatz:** Although expression (50) resolves the sign problem, it introduces a new parameter α to the theory and, more importantly, it adds initial value data since the second term $\alpha \Box R$ contains fourth order derivatives of the metric. [6]

Both issues can be avoided by adopting the following geometrically motivated form for the source:

$$Y_S[g](x) \;=\; \frac{1}{\Lambda} \frac{1}{\Box} \Biggl\{ R(x) \;\times \\ \Bigl[-V^{\mu}\Bigl(\chi(\tau^*, x)\Bigr) \; V^{\nu}\Bigl(\chi(\tau^*, x)\Bigr) \; R_{\mu\nu}\Bigl(\chi(\tau^*, x)\Bigr) \Bigr] \Biggr\} \;\; , \tag{53}$$

which – for an FRW geometry and for the value of $\tau^*[g](x)$ associated with, e.g. $\frac{9}{10}$ of the time from x^{μ} back to the initial value surface – becomes:

$$Y_S[g](x) \quad \longrightarrow \quad \int_0^t dt' \, \frac{1}{a^3(t')} \int_0^{t'} dt'' \; a^3(t'') \; R(t'') \times \frac{R_{00}(\frac{1}{10}t'')}{\Lambda} \quad , \tag{54}$$

where we have used that for these particular spacetimes:

$$FRW \quad \Longrightarrow \qquad V^{\mu}[g](x) \;\equiv\; \frac{-g^{\mu\nu} \, \partial_{\nu} \mathcal{V}}{\sqrt{-g^{\alpha\beta} \, \partial_{\alpha} \mathcal{V} \, \partial_{\beta} \mathcal{V}}} \;=\; \delta^{\mu}_{\;\,0} \;\; . \tag{55}$$

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[5] Because during matter domination $H(t) = \frac{2}{3t}$ and $R(t) = 3H^2(t)$, it trivially follows that: $R(\frac{1}{10}t) = 10^2 R(t)$.

[6] Unlike the first term $\frac{1}{4}R^2$ which has the square of second order derivatives acting on the metric.

• **Late Time Acceleration:** The ansatz (53) and its FRW form (54) has essentially the same time evolution with that of the simple model (26) during inflation, vanishes during radiation domination, and the transition to matter domination can trigger a slow, very small decrease in $p[g]$. This, in turn, can uncover a tiny portion of the bare cosmological constant and eventually cause a phase of acceleration.

To see this behaviour quantitatively, note that:

$$\begin{aligned} R(t) \times R_{00}(\tfrac{1}{10}t) &= 0 \times 300H^2 = 0 \quad , \quad 10\, t_r < t < t_m && (56) \\ &= 3H^2 \times 300H^2 = 900H^4 \quad , \quad t_m < t < 10\, t_m && (57) \\ &= 3H^2 \times 150H^2 = 450H^4 \quad , \quad 10\, t_m < t < t_{\rm now} && (58) \end{aligned}$$

where we have used (12-13).

In view of (57-58), to compute the source $Y[g](x)$ given by (53):

$$Y_S[g](t) \; = \; -\frac{1}{\Lambda} \frac{1}{\Box} \left[R(t)\, R_{00}(\tfrac{1}{10}t) \right] \; \equiv \; X_{cr} + \Delta Y_S \ , \tag{59}$$

we must evaluate the action of the inverse differential operator $\Box^{-1}$ on $H^4(t)$; we might as well do so for any constant ε and for a generic lower limit of integration which we shall denote by T:

$$\begin{aligned} \frac{1}{\Box} H^4 &= -\int_T^t dt' \, \frac{1}{a^3(t')} \int_T^{t'} dt'' \; a^3(t'') \; H^4(t'') \\ &= -\frac{1}{3\,(1-\varepsilon)} \int_T^t dt' \, \left\{ H^3(t') - \frac{a^3(T)\, H^3(T)}{a^3(t')} \right\} \\ &= -\frac{1}{3\,(1-\varepsilon)} \left\{ -\frac{1}{2\,\varepsilon}\, H^2(t) + \frac{1}{2\,\varepsilon}\, H^2(T) + \frac{1}{3-\varepsilon}\, \frac{a^3(T)\, H^3(T)}{a^3(t)\, H(t)} \right. \\ &\qquad \left. -\frac{1}{3-\epsilon}\, H^2(T) \right\} \ . \end{aligned} \tag{60}$$

This is the full answer for any ε. Of the four terms present in (60), the first and third are sub-dominant at late times for matter domination ($\varepsilon = \frac{3}{2}$) because of their t^{-2} and t^{-1} behaviour, respectively. The remaining two terms in (60) are constant and give:

$$t \gg t_m \quad \Longrightarrow \quad \frac{1}{\Box} H^4 \; = \; -\frac{2}{9}\, H^2(T) \ . \tag{61}$$

Taking into account (58) and setting the lower limit of integration equal to $T = t_m$, the relevant part of the source (59) becomes:

$$t \gg t_m \quad \Longrightarrow \quad \Delta Y_S[g](t) \;=\; \frac{1}{\Lambda} \Big[200 \, H_m^2 \Big] \quad , \tag{62}$$

where $H_m \equiv H(t_m)$.

In terms of the new source $Y_S[g](t)$, the total pressure (27) becomes:

$$\begin{aligned} t \gg t_m \quad \Longrightarrow \quad p_{\rm tot} \;&=\; -\frac{\Lambda}{8\pi G} \Big\{ 1 - 8\pi G\Lambda \, f[-G\Lambda \, Y_S] \Big\} & (63) \\ &=\; -\frac{\Lambda}{8\pi G} \Bigg\{ 1 - 8\pi G\Lambda \, f\Big[-G\Lambda \,(X_{cr} + \Delta Y_S) \Big] \Bigg\} & (64) \\ &\simeq\; -G\Lambda^3 \, f'_{cr} \, \Delta Y_S & (65) \\ &\simeq\; -200 \, G\Lambda^2 \, f'_{cr} \, H_m^2 \quad , & (66) \end{aligned}$$

where in the last step we used (62). The ratio of the predicted total pressure (66) to the current total pressure (29) is:

$$\begin{aligned} t \gg t_m \quad \Longrightarrow \quad \frac{p_{\rm tot}}{p_{\rm now}} \;&\simeq\; \frac{200}{3} \, 8\pi (G\Lambda)^2 \times f'_{cr} \times \Big(\frac{H_m}{H_{\rm now}} \Big)^2 & (67) \\ &\simeq\; \frac{200}{3} \, 8\pi (G\Lambda)^2 \times f'_{cr} \times 10^{10} \quad . & (68) \end{aligned}$$

For the exponential model analyzed in [2], we have:

$$f(x) \;=\; e^x - 1 \quad \Longrightarrow \quad f'_{cr} \;=\; \frac{1}{8\pi G\Lambda} \quad , \tag{69}$$

so that the pressure ratio (68) takes the form:

$$t \gg t_m \quad \Longrightarrow \quad \frac{p_{\rm tot}}{p_{\rm now}} \;\simeq\; \frac{2}{3} \times 10^{12} \times G\Lambda \quad . \tag{70}$$

Physical values of the dimensionless coupling constant $G\Lambda = M^4 M_{Pl}^{-4}$ easily balance the factor 10^{12} so that we can achieve equality of the predicted total pressure $p_{\rm tot}$ to the observed total pressure and, hence, account for the current acceleration of the universe.

- **Epilogue:** We have presented an improved *ansatz* for the most cosmologically significant part of the effective field equations of quantum gravity with a positive cosmological constant. This improvement of the simple *ansatz* analyzed in [2] and reviewed in the introduction herein, allows us to expect a reasonable post-inflationary time evolution into the present.

Prominent among the list of topics for future work is perturbations [5]. We need to show that the dynamical scalar mode of our model dumps the energy of oscillations into matter to reheat the universe in a natural way. If this happens, the improved model presented here can make the quantum gravity sector go quiescent during a long epoch of conventional radiation domination. The subsequent transition to matter domination might even give rise to something like the current phase of acceleration without severe fine tuning.

Furthermore, we must derive and solve the equation for scalar perturbations, at least enough to compute the scalar power spectrum. In contradistinction, the equation for tensor perturbations remains unchanged and we need only use the expansion history $a(t)$ predicted by our model in order to compute the tensor power spectrum. Finally, one also needs that there be no long-range scalar force at late times.

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References

[1] N. C. Tsamis and R. P. Woodard,
Nucl. Phys. **B474** (1996) 235, **arXiv:**hep-ph/9602315;
Annals Phys. **253** (1997) 1, **arXiv:**hep-ph/9602316;
Nucl. Phys. **B724** (2005) 295, **arXiv:**gr-qc/0505115.

[2] N. C. Tsamis and R. P. Woodard, **arXiv:**0904.2368.

[3] A. G. Riess *et al.*, Astron. J. **116** (1998) 1009, **arXiv:**astro-ph/9805201.
S. Perlmutter *et al.*, Astrophys. J. **517** (1999) 565,
arXiv:astro-ph/9812133.

[4] Y. Wang and P. Mukherjee, Astrophys. J. **650** (2006) 1,
arXiv:astro-ph/0604051.
U. Alam, V. Sahni and A. A. Starobinsky, JCAP **0702** (2007) 011,
arXiv:astro-ph/0612381.

[5] N. C. Tsamis and R. P. Woodard *(preprint in preparation)*

[6] S. Deser, Phys. Lett. **B479** (2000) 315, **arXiv:**hep-th/9911129;
Nucl. Phys. Proc. Suppl. **88** (2000) 204.

[7] S. Deser and R. P. Woodard, Phys. Rev. Lett. **99** (2007) 111301,
arXiv:0706.2151.

[8] B. S. DeWitt and R. W. Brehme, Annals Phys. **9** (1960) 220.

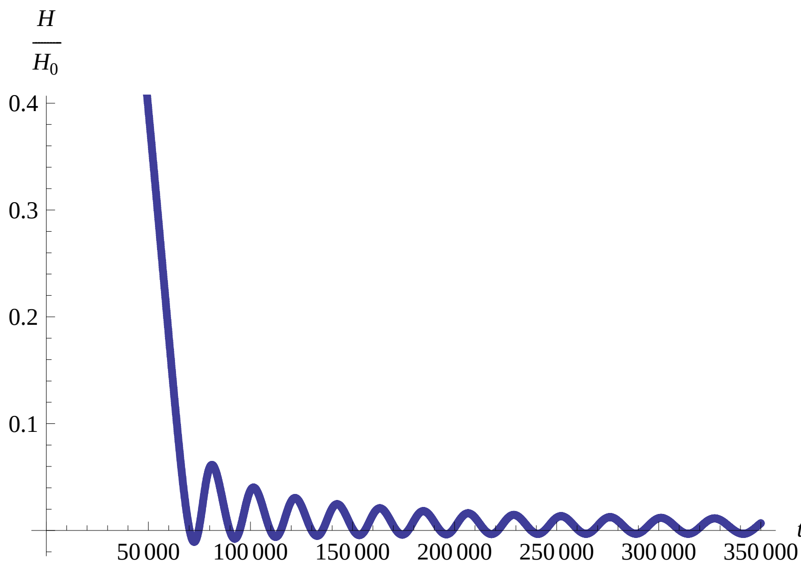

Figure 1: The Hubble parameter (in units of H_0) versus time (in units of $\frac{1}{1000}H_0$) for $p = \Lambda^2 f[-G\Lambda\, Y]$ with $G\Lambda = \frac{1}{100}$ and Y given in (46).