

On Non-Existence of Lightlike Hypersurfaces of Indefinite Kenmotsu Space Form

Nesip Aktan

Abstract

In this paper, lightlike hypersurfaces of indefinite Kenmotsu space form are studied. Some characterizations of non-existence of lightlike hypersurfaces of indefinite Kenmotsu space form are given.

Key Words: Lightlike hypersurface, Kenmotsu space form

1. Introduction

It is well known that in a semi-Riemannian manifold there are three causal types of submanifolds: spacelike, timelike and lightlike, depending on the character of the induced metric on the tangent space. In the third case, due to the degeneracy of the metric, basic differences occur between the study of lightlike submanifolds and classical theory of Riemannian and semi-Riemannian submanifolds (see [8] and [13]). Let M be a lightlike hypersurfaces of a semi-Riemannian manifold. The primary difference in studying the differential geometry of M consists in that the orthogonal vector bundle TM^\perp to the tangent bundle TM becomes a distribution of rank 1 on M (see [8], page 81).

There exist few papers dealing with lightlike hypersurfaces(see [1], [8], [9], [11], [14]). Duggal and Bejancu, discussed the Cauchy Riemann Lightlike submanifolds of an indefinite Kaehler manifold in ([8], chapter 6) and concluded that there exist no totally umbilical lightlike real hypersurfaces of indefinite complex space forms $\overline{M}(c)$ with $c \neq 0$. Kang et al. study a lightlike hypersurface when the ambient manifold is an indefinite

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Sasakian manifold and prove that there exist no totally umbilical lightlike hypersurfaces of indefinite Sasakian space forms $\overline{M}(c)$ with $c \neq 1$ particularly in [11]. Güneş et al. study a lightlike hypersurfaces of a semi-Riemannian manifold and they show that a lightlike hypersurface is totally geodesic if and only if it is locally symmetric in [9]. Şahin and Güneş investigate non-existence of real lightlike hypersurfaces of indefinite complex space form in [14]. Aktan study lightlike hypersurface of indefinite cosymplectic manifolds and non-existence of lightlike hypersurface of indefinite Sasakian space form in [2] and [3].

In the present paper, non-existence of lightlike hypersurfaces of indefinite Kenmotsu space form are studied. The paper is organized as follows: In section 2, basic definition of indefinite Kenmotsu manifolds and indefinite Kenmotsu space form is given, which will be used in the preceding sections. In section 3, a decomposition of indefinite Kenmotsu manifolds is given. In section 4, basic formulas and definitions for the induced geometric objects on a lightlike hypersurface of a semi-Riemannian manifold are reviewed. In the last section lightlike hypersurfaces of indefinite Kenmotsu manifolds are introduced and some characterizations of non-existence of lightlike hypersurfaces in an indefinite Kenmotsu space form are given.

2. Indefinite Kenmotsu Manifolds

Let \overline{M} be an $(2m + 1)$ -dimensional differentiable manifold equipped with a triple (ϕ, ξ, η) , where ϕ is a $(1, 1)$ -tensor field, ξ is a vector field and η is a 1-form on \overline{M} such that

$$\eta(\xi) = 1, \quad \phi^2 = -I + \eta \otimes \xi, \tag{2.1}$$

which implies

$$\phi\xi = 0 \quad \eta \circ \phi = 0 \quad \text{rank}(\phi) = 2m. \tag{2.2}$$

If \overline{M} admits a semi-Riemann metric \overline{g} , such that

$$\overline{g}(\xi, \xi) = \varepsilon, \quad \varepsilon = \begin{cases} +1, & \text{if } \xi \text{ is spacelike} \\ -1, & \text{if } \xi \text{ is timelike} \end{cases}$$

$$\overline{g}(\phi X, \phi Y) = \overline{g}(X, Y) - \varepsilon \eta(X) \eta(Y), \quad \eta(X) = \varepsilon \overline{g}(X, \xi), \tag{2.3}$$

then \overline{M} is said to be indefinite almost contact metric manifold with almost contact metric structure $(\phi, \xi, \eta, \overline{g})$.

An indefinite almost contact metric manifold \overline{M} is said to be a indefinite Kenmotsu manifold if $\forall X, Y \in \Gamma(T\overline{M})$

$$(\overline{\nabla}_X \phi) Y = -\overline{g}(\phi X, Y)\xi + \varepsilon\eta(Y)\phi X \quad (2.4)$$

$$\overline{\nabla}_X \xi = \phi^2 X = -X + \eta(X)\xi. \quad (2.5)$$

Throughout the this paper we may assume that $\varepsilon = 1$ without loss of generality.

An example of indefinite Kenmotsu manifold is locally a warped product $\overline{M} = (-\epsilon, \epsilon) \times_f N$, where N is an (indefinite) Keahler manifold and warping function is given by $f(t) = ce^t$ [12].

A plane section Π in $\Gamma(T\overline{M})$ is called a ϕ -section if there exists a vector $X \in \Gamma(T\overline{M})$ orthogonal to ξ such that $\{X, \phi X\}$ is an orthonormal basis of the plane section. The sectional curvature $\overline{K}(X; \phi X) = \overline{g}(\overline{R}(X, \phi X)\phi X, X)$ is called ϕ -sectional curvature. A Kenmotsu manifold \overline{M} with constant ϕ -sectional curvature c is said to be a Kenmotsu space form and is denoted by $\overline{M}(c)$.

The curvature tensor \overline{R} of an indefinite Kenmotsu space form $\overline{M}(c)$ is given by the same formulae as in case of positive definite metrics, i.e.,

$$\begin{aligned} \overline{R}(X, Y)Z = & \frac{c-3}{4} [\overline{g}(Y, Z)X - \overline{g}(X, Z)Y] \\ & \frac{c+1}{4} [\overline{g}(\phi Y, Z)\phi X - \overline{g}(\phi X, Z)\phi Y - 2\overline{g}(\phi X, Y)\phi Z \\ & +g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi \\ & +\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X] \end{aligned} \quad (2.6)$$

3. A Decomposition of Indefinite Kenmotsu Manifolds

Let be \overline{M} be a $(2m + 1)$ -dimensional indefinite Kenmotsu manifold, M be a hypersurface of \overline{M} and g be the induced tensor field by \overline{g} on M . We consider for any $p \in M$

$$T_p M^\perp = \{Y_p \in T_p \overline{M} : \overline{g}(X_p, Y_p) = 0, \forall X_p \in T_p M\}$$

and

$$\text{Rad}T_pM = T_pM \cap T_pM^\perp,$$

where T_pM is a hyperplane of the semi-Euclidean space $T_p\overline{M}$. We say that M is a lightlike hypersurface of \overline{M} if $\text{Rad}T_pM \neq \{0\}$ at any point $p \in M$. Thus, a hypersurface M of \overline{M} is lightlike if and only if TM^\perp is a distribution on M with rank 1.

The fundamental difference of the theory of lightlike hypersurfaces and classical theory of hypersurfaces of a semi-Riemannian manifold \overline{M} comes from the fact that, in the first case the normal bundle TM^\perp lies in the tangent bundle of a lightlike hypersurfaces.

If $E_p \in \text{Rad}T_pM$, then $E_p \in T_pM$ and $E_p \in T_pM^\perp$. Thus $\overline{g}(E_p, E_p) = 0$. Moreover $\overline{g}(\phi E_p, E_p) = 0$, and so ϕE_p is tangent to T_pM . Hence we get a distribution $\phi(TM^\perp)$ on M of rank 1. Now we choose a complementary distribution which is called screen distribution, $S(TM)$ to TM^\perp in TM , containing ϕE and E . Because of the screen distribution $S(TM)$ is non-degenerate, there exists a complementary orthogonal vector subbundle $S(TM^\perp)$ to $S(TM)$ in $T\overline{M}$ over M . Thus we have the orthogonal decomposition

$$T\overline{M} = S(TM) \perp S(TM^\perp). \quad (3.7)$$

Let U be a coordinate neighborhood of M and E be a basis of $\Gamma(TM^\perp|_U)$ satisfying the following conditions:

$$\overline{g}(N, E) = 1 \quad (3.8)$$

and

$$\overline{g}(N, N) = \overline{g}(N, W) = 0, \quad \forall W \in \Gamma(S(TM)). \quad (3.9)$$

Let $ltr(TM)$ denote one-dimensional vector subbundle of $T\overline{M}$ over M which is locally spanned by N . Then we have

$$S(TM^\perp) = TM^\perp \oplus ltr(TM). \quad (3.10)$$

One-dimensional vector subbundle $ltr(TM)$ is called a lightlike transversal vector bundle of M . We note that $ltr(TM)$ is not orthogonal to TM (see [8]). From (3.7) and (3.10), we have the following decomposition

$$\begin{aligned} T\overline{M} &= S(TM) \perp (TM^\perp \oplus ltr(TM)) \\ &= TM \oplus ltr(TM). \end{aligned} \quad (3.11)$$

Then N is orthogonal to ϕE and we have,

$$\bar{g}(\phi N, E) = -\bar{g}(N, \phi E) = 0, \quad \bar{g}(\phi N, N) = 0, \quad (3.12)$$

which means that ϕN is also tangent to M and belongs to $S(TM)$ and from (2.3)

$$\bar{g}(\phi N, \phi E) = 1. \quad (3.13)$$

Hence, $\phi(TM^\perp) \oplus \phi(\text{ltr}(TM))$ is a non-degenerate vector subbundle of $S(TM)$ of rank 2. Then there exists a non-degenerate distribution D on M such that

$$S(TM) = \{\phi(TM^\perp) \oplus \phi(\text{ltr}(TM))\} \perp D, \quad (3.14)$$

where $\xi \in \Gamma(D)$ and D are invariant distributions with respect to ϕ . Therefore, from (3.7), (3.11) and (3.14), we obtain following decompositions

$$TM = \{\phi(TM^\perp) \oplus \phi(\text{ltr}(TM))\} \perp D \perp TM^\perp$$

and

$$T\bar{M} = \{\phi(TM^\perp) \oplus \phi(\text{ltr}(TM))\} \perp D \perp \{TM^\perp \oplus \text{ltr}(TM)\}. \quad (3.15)$$

Let \bar{M} be an indefinite Kenmotsu manifold and M be its lightlike hypersurface. We consider the following distributions

$$D_0 = TM^\perp \perp \phi(TM^\perp) \perp D, \quad D' = \phi(\text{ltr}(TM)) \quad (3.16)$$

on M . Then D_0 is invariant under ϕ and

$$TM = D_0 \oplus D'. \quad (3.17)$$

Now we consider the local lightlike vector fields

$$\mathcal{U} = -\phi N, \quad \mathcal{V} = -\phi E. \quad (3.18)$$

We note that $\phi^2 N = -N$ holds. From (3.17) we have

$$\phi X = fX + u(X)N, \quad \text{for any } X \in \Gamma(TM), \quad (3.19)$$

where $u(X) = g(X, \mathcal{V})$ and f is a tensor field of type $(1, 1)$ defined on M .

4. The Induced Geometric Objects on a Lightlike Hypersurface

For the convenience of readers, we repeat the relevant material from [8] without proofs.

Let $(M, g, S(TM))$ be a lightlike hypersurface of semi-Riemannian manifold $(\overline{M}, \overline{g})$ and $\overline{\nabla}$ be the Levi-Civita connection on \overline{M} with respect to \overline{g} . Then by using the decomposition (3.11) we obtain

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad (4.20)$$

and

$$\overline{\nabla}_X V = -A_V X + \nabla_X^\perp V \quad (4.21)$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(ltr(TM))$, where $\nabla_X Y, A_V X \in \Gamma(TM)$ and $h(X, Y), \nabla_X^\perp V \in \Gamma(ltr(TM))$. It is easy to check that ∇ is a torsion free linear connection on M , h is a $\Gamma(ltr(TM))$ -valued symmetric $F(M)$ -bilinear form on $\Gamma(TM)$, A_V is a $F(M)$ -linear operator on $\Gamma(TM)$ and ∇^\perp is a linear connection on the vector bundle $ltr(TM)$.

Locally, suppose $\{E, N\}$ is a pair of sections on $U \subset M$. Then define a symmetric $F(U)$ -bilinear form B and a 1-form τ on U by

$$B(X, Y) = \overline{g}(h(X, Y), E), \quad \forall X, Y \in \Gamma(TM|_U) \quad (4.22)$$

and

$$\tau(X) = \overline{g}(\nabla_X^\perp N, E). \quad (4.23)$$

Thus (4.20), (4.21) locally become

$$\overline{\nabla}_X Y = \nabla_X Y + B(X, Y)N \quad (4.24)$$

and

$$\overline{\nabla}_X V = -A_N X + \tau(X)N, \quad (4.25)$$

respectively, where B, A_N , and ∇ are called the local second fundamental form, the shape operator and the induced linear torsion free connection. We call (4.24) and (4.25) as the formulas of Gauss and Weingarten of the lightlike hypersurfaces M , respectively.

Let denote P be the projection of TM on $S(TM)$. Local Gauss and Weingarten formulas are given by

$$\nabla_X PY = \nabla_X^* PY + C(X, PY)E \quad (4.26)$$

and

$$\nabla_X E = -A_E^* X - \tau(X)E, \quad (4.27)$$

where $\nabla_X^* PY$, $A_E^* X$ belongs to $S(TM)$ and C is a 1-form on U . Thus we have the equations

$$g(A_N X, PY) = C(X, PY), \quad \bar{g}(A_N X, N) = 0, \quad (4.28)$$

$$g(A_E^* X, PY) = B(X, PY), \quad \bar{g}(A_E^* X, N) = 0, \quad (4.29)$$

for any $X, Y \in \Gamma(TM)$.

We denote the curvature tensors associated with $\bar{\nabla}$ and ∇ by \bar{R} and R , respectively. \bar{R} and R are related by

$$\bar{R}(X, Y)Z = R(X, Y)Z + A_{h(X, Z)}Y - A_{h(Y, Z)}X + (\nabla_X h)(Y, Z) - (\nabla_Y h)(X, Z). \quad (4.30)$$

We note that the induced connection on M satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\zeta(Z) + B(X, Z)\zeta(Y), \quad \forall X, Y, Z \in \Gamma(TM), \quad (4.31)$$

where ζ is a differential 1-form locally defined on M , given by

$$\zeta(X) = \bar{g}(X, N), \quad \forall X \in \Gamma(TM). \quad (4.32)$$

When we say \mathcal{V} is a principle vector field, we mean a vector field satisfying the condition $A_N \mathcal{V} = k\mathcal{V}$, where k is real-valued function on the $\Gamma(TM)$.

5. Lightlike Hypersurfaces of Indefinite Kenmotsu Manifolds

Lemma 1 *Let M be a lightlike hypersurface of indefinite Kenmotsu space form $\bar{M}(c)$. Then the followings holds.*

i) The equation of Gauss of M is given by the following equality:

$$\begin{aligned} R(X, Y)Z &= \frac{c-3}{4} [\bar{g}(Y, Z)X - \bar{g}(X, Z)Y] \\ &\quad + \frac{c+1}{4} [\bar{g}(\phi Y, Z)fX - \bar{g}(\phi X, Z)fY - 2\bar{g}(\phi X, Y)fZ \\ &\quad + g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi \\ &\quad + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X] - B(X, Z)A_N Y + B(Y, Z)A_N X. \end{aligned} \quad (5.33)$$

ii) The equation of Codazzi of M is given by the following equality:

$$\begin{aligned}
 (\nabla_Y h)(X, Z) - (\nabla_X h)(Y, Z) &= \frac{c+1}{4} [\bar{g}(\phi Y, Z)u(X) \\
 &\quad - \bar{g}(\phi X, Z)u(Y) - 2\bar{g}(\phi X, Y)u(Z)] N
 \end{aligned}
 \tag{5.34}$$

for any $X, Y, Z \in \Gamma(TM)$.

Proof. Since \bar{M} is an indefinite Kenmotsu space form, we obtain from (2.6) and (4.30) that

$$\begin{aligned}
 R(X, Y)Z &= \frac{c-3}{4} [\bar{g}(Y, Z)X - \bar{g}(X, Z)Y] \\
 &\quad + \frac{c+1}{4} [\bar{g}(\phi Y, Z)\phi X - \bar{g}(\phi X, Z)\phi Y - 2\bar{g}(\phi X, Y)\phi Z \\
 &\quad + g(X, Z)\eta(Y)\xi - g(Y, Z)\eta(X)\xi \\
 &\quad + \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X] \\
 &\quad - A_{h(X, Z)}Y + A_{h(Y, Z)}X - (\nabla_X h)(Y, Z) + (\nabla_Y h)(X, Z)
 \end{aligned}
 \tag{5.35}$$

holds. Substituting (5.35) into (3.19) and comparing the tangential and transversal vector bundle parts of the both sides, see that (5.33) and (5.34) hold. \square

Lemma 2 Let M be a lightlike hypersurface of indefinite Kenmotsu space form $\bar{M}(c)$. Then

$$\begin{aligned}
 \bar{g}(R(X, E)Z, N) &= \frac{c-3}{4} [-\bar{g}(X, Z)] \\
 &\quad + \frac{c+1}{4} [-u(Z)\zeta(\phi X) - 2u(Z)\zeta(\phi Z) \\
 &\quad + \eta(X)\eta(Z)]
 \end{aligned}
 \tag{5.36}$$

holds.

Proof. The proof is obvious from Lemma 1. \square

Lemma 3 Let M be a lightlike hypersurface of an indefinite Kenmotsu manifold \bar{M} . Then, for any $X, Y, Z \in \Gamma(TM)$,

$$B(Y, \mathcal{U}) = C(Y, \mathcal{V})$$

holds.

Proof. By the definition of B , we obtain

$$\begin{aligned} B(Y, \phi N) &= \bar{g}(h(Y, \phi N), E) = \bar{g}(\bar{\nabla}_Y \phi N, E) \\ &= -\bar{g}(\bar{\nabla}_Y N, \phi E) + \bar{g}((\bar{\nabla}_Y \phi) N, E). \end{aligned}$$

By using (2.4) and (4.28), we have

$$B(Y, \phi N) = -\bar{g}(\bar{\nabla}_Y N, \phi E) = g(A_N Y, \phi E) = C(Y, \phi E).$$

This completes the proof. \square

Theorem 1 *There are no lightlike hypersurfaces of indefinite Kenmotsu space form $\bar{M}(c)$ ($c \neq -1$) with parallel second fundamental form.*

Proof. Suppose on contrary that there exists a lightlike hypersurface satisfying $c \neq -1$ and second fundamental form is parallel. Taking $Y = E$, $Z = \phi N$ in (5.34), we have

$$-\frac{3c+3}{4} [u(X) - 2\bar{g}(X, \phi E)] = 0,$$

and letting $X = \phi N$ in this equation, we deduce that

$$c = -1,$$

which is a contraction. Hence, the claim holds. \square

Theorem 2 *A screen distribution $S(TM)$ is parallel with respect to ∇ if and only if on each $U \subset M$ we have $C = 0$ [4].*

Theorem 3 *There are no lightlike hypersurface of indefinite Kenmotsu space form $\bar{M}(c)$ ($c \neq \frac{1}{3}$) with parallel screen distribution.*

Proof. Suppose contrary that $c \neq \frac{1}{3}$ and screen distribution is parallel. We obtain from (2.6) that

$$\bar{g}(\bar{R}(E, \phi N)\phi E, N) = \frac{3c-1}{4} \tag{5.37}$$

holds.

On the other hand, we know that (see, [4]) the equality

$$\begin{aligned} \bar{g}(\bar{R}(X, Y)PZ, N) &= \bar{g}(R(X, Y)PZ, N) = (\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) \\ &\quad + \tau(Y)C(X, PZ) - \tau(X)C(Y, PZ). \end{aligned} \quad (5.38)$$

is valid. From Theorem 2 and (5.38), we have

$$\bar{g}(\bar{R}(E, \phi N)\phi E, N) = 0, \quad (5.39)$$

using this equality together with (5.37) and (5.39), we obtain

$$c = \frac{1}{3},$$

This is the contradiction completing the proof. \square

Lemma 4 *Let M be a lightlike hypersurface of indefinite Kenmotsu manifold \bar{M} . If \mathcal{V} is a principle vector field, then*

$$B(\mathcal{V}, \mathcal{U}) = C(\mathcal{V}, \mathcal{V}) = 0.$$

Proof. Taking (2.4) and (4.24) into account,

$$\bar{\nabla}_X \mathcal{U} = -\bar{\nabla}_X \phi N = -\phi \bar{\nabla}_X N - (\bar{\nabla}_X \phi) N$$

or equivalently

$$\nabla_X \mathcal{U} + B(X, \mathcal{U})N = \phi A_N X - \tau(X)\phi N + \bar{g}(X, \mathcal{U})\xi. \quad (5.40)$$

Considering (3.19), we obtain by (5.40) that

$$\nabla_X \mathcal{U} + B(X, \mathcal{U})N = f A_N X + u(A_N X)N - \tau(X)\phi N + \bar{g}(X, \mathcal{U})\xi$$

and taking the transversal vector bundle parts of both sides of the above equation,

$$B(X, \mathcal{U}) = u(A_N X) = -g(A_N X, \phi E) = C(X, \mathcal{V}),$$

which proves the assertion. \square

Lemma 5 *Let M be a lightlike hypersurface of indefinite Kenmotsu space form $\overline{M}(c)$. Then the equation of Codazzi is given by the following equality:*

$$\begin{aligned} (\nabla_X A_N) Y - (\nabla_Y A_N) X &= \frac{c-3}{4} [\zeta(Y)X - \zeta(X)Y] \\ &\quad + \frac{c+1}{4} [\overline{g}(Y, \mathcal{U})\phi X - \overline{g}(X, \mathcal{U})\phi Y + 2\overline{g}(\phi X, Y)\mathcal{U} \\ &\quad + \zeta(X)\eta(Y)\xi - \zeta(Y)\eta(X)\xi] \\ &\quad + \tau(Y)A_N X - \tau(X)A_N Y \end{aligned}$$

Proof. By straight forward calculations, the desired equation follows. □

Now, we consider an orthonormal basis $\{z_1, \dots, z_{m-2}, \dots, z_{2m-4}, \xi, E, \phi E, \phi N\}$ of $\Gamma(TM)$ such that

$$\phi z_i = z_{m-2+i}, \phi z_{m-2+i} = -z_i \text{ and } \phi \xi = 0$$

for every $i = 1, \dots, m-2$ and $j = 1, \dots, n$.

Lemma 6 *Let M be a lightlike hypersurface of an indefinite Kenmotsu manifold \overline{M} . Then*

$$\begin{aligned} A_N U &= \sum_{i=1}^{2m-4} \frac{C(\mathcal{U}, z_i)}{\varepsilon_i} z_i + C(\mathcal{U}, \xi)\xi \\ &\quad + C(\mathcal{U}, \mathcal{U})\mathcal{V} + C(\mathcal{U}, \mathcal{V})\mathcal{U} \end{aligned} \tag{5.41}$$

and

$$A_N E = \sum_{i=1}^{2m-4} \frac{C(E, z_i)}{\varepsilon_i} z_i + C(E, \xi)\xi + C(E, \mathcal{U})\mathcal{V}, \tag{5.42}$$

where $\{\varepsilon_i\}$ is the signature of the basis $\{z_i\}$.

Proof. By the definition of lightlike hypersurface of an indefinite Kenmotsu manifold, we have

$$A_N \mathcal{U} = \sum_{i=1}^{2m-4} \lambda_i z_i + \gamma \xi + \beta_1 E + \beta_2 \phi E + \beta_3 \phi N.$$

From (4.28), we obtain $\lambda_i = \frac{1}{\varepsilon_i} C(\mathcal{U}, z_i)$, $\gamma = C(\mathcal{U}, \xi)$, $\beta_1 = 0$, $\beta_2 = -C(\mathcal{U}, \mathcal{U})$, $\beta_3 = -C(\mathcal{U}, \mathcal{V})$. Thus we derive (5.41). Similarly one can obtain (5.42). □

Theorem 4 *There are no lightlike hypersurfaces of indefinite Kenmotsu manifold $\overline{M}(c)$ ($c \neq \frac{1}{3}$) satisfying*

$$g((\nabla_E A_N)U, \mathcal{V}) = g((\nabla_U A_N)E, \mathcal{V})$$

and

$$B(U, U) = 0.$$

Proof. Letting $Y = U$ and $X = E$ in Lemma 5, we have

$$(\nabla_E A_N)U - (\nabla_U A_N)E = -\frac{3c-1}{4}U + \tau(U)A_N E - \tau(E)A_N U.$$

From (5.41) and (5.42), we obtain

$$\begin{aligned} (\nabla_E A_N)U - (\nabla_U A_N)E &= -\frac{3c-1}{4}U \\ &+ \tau(U) \left\{ \sum_{i=1}^{2m-4} \frac{C(E, z_i)}{\varepsilon_i} z_i \right. \\ &+ C(E, \xi)\xi + C(E, U)\mathcal{V} \left. \right\} \\ &- \tau(E) \left\{ \sum_{i=1}^{2m-4} \frac{C(U, z_i)}{\varepsilon_i} z_i + C(U, \xi)\xi \right. \\ &+ C(U, U)\mathcal{V} + C(U, \mathcal{V})U \left. \right\}. \end{aligned}$$

Thus, it follows by Lemma 4 that

$$g((\nabla_E A_N)U - (\nabla_U A_N)E, \mathcal{V}) = -\frac{3c-1}{4}g(U, \mathcal{V}) - \tau(E)g(U, U).$$

Hence, the proof is complete. □

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Nesip AKTAN
 Department of Mathematics,
 Faculty of Science and Arts,
 Afyon Kocatepe University,
 03200, Afyonkarahisar-TURKEY
 e-mail: naktan@aku.edu.tr

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