# A Robust Control Approach to Stabilization of Networked Control Systems with Short Time-varying Delays

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**Abstract** A robust control approach is proposed in this paper to solve the stabilization problem for networked control systems (NCSs) with short time-varying delays. By considering state feedback controllers, the closed-loop NCS is described as a discrete-time linear uncertain system model, and the uncertain part reflects the effect of the variation nature of the network-induced delays on the system dynamics. Then, the asymptotic stability condition for the obtained closed-loop NCS is derived, which establishes the quantitative relation between the stability of the closed-loop NCS and two delay parameters, namely, the allowable delay upper bound (ADB) and the allowable delay variation range (ADVR). Furthermore, design procedures for the stabilizing controllers are also presented. An illustrative example is finally given to demonstrate the effectiveness of the proposed method.

**Key words** Networked control system (NCS), time-varying delays, asymptotic stability, uncertainties, robust control

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Network-induced delay is one of the main problems in networked control systems (NCSs), and it is usually regarded as the major cause of deterioration of system performance and potential system instability. Due to varying network load and the scheduling policies in the networks and the nodes, the network-induced delay is typically varying. Compared with the constant delay, the time-varying or random delay is more difficult to deal with. For the past decade, various approaches have been presented in the existing literature to deal with the modeling, analysis, and synthesis problems for the NCSs with delays, such as the so-called queue mechanism approach[1], the stochastic system approach[2−3], the model predictive control approach[4−5], the time-delay system approach[6−8], the sampled-data system approach[9−10], the switched system approach[11], and the recently presented time-varying system approach[12].

The delay upper bound and the delay variation range are two important parameters that characterize the properties of the time-varying network-induced delays. Both may affect the stability and system performance of the NCSs. Therefore, it is necessary and of great significance to establish the relation between the stability of the NCSs and these two parameters. However, for the NCS with short (smaller than one sampling period) time-varying delays, only the sampled-data system approach, such as those in [9 − 10], and the time-varying system method in [12] are able to establish the relation between the stability of the NCSs and the delay upper bound. Besides, all the aforementioned approaches are unable to establish the relation between the stability of the NCS and the delay variation range. Therefore, new methods need to be explored to model and analyze the NCSs with short time-varying delays and to simultaneously establish the relation between the stability of the NCS and the delay upper bound and variation range. This motivates the present research.

In this paper, a robust control approach is developed to study the stabilizing controller design problem for the NCSs with short time-varying delays. A new uncertain system model is proposed to describe the considered NCSs, and the uncertainty of the delays is expressed as the uncertainty of the system matrices. Similar modeling methods have been presented in [13−14]. However, the uncertain matrix given in [13−14] is required to be unit norm-bounded, and some free parameters or numerical algorithms should be involved to satisfy the requirements. Besides, the uncertain system models presented in [13−14] are not suitable to establish the relation between the stability of the NCS and the delay upper bound for the NCSs with short delays. The uncertain system model presented in this paper is more general than those in [13−14]. Based on the presented system model, sufficient conditions are derived for the closed-loop NCS to be asymptotically stable. Moreover, the stability conditions also establish the relation between the asymptotic stability of the NCS and the delay upper bound and variation range. Design procedures for the state feedback stabilizing controllers are also presented. An illustrative example is finally provided to show the effectiveness of the proposed method.

## 1 Preliminaries

The structure of the considered NCS is shown in Fig. 1, where the continuous-time plant is described by the following linear time-invariant system model:

$$\dot{\boldsymbol{x}}(t) = A_p\boldsymbol{x}(t) + B_p\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ and $\boldsymbol{u}(t) \in \mathbf{R}^m$ are the system state and the control input, respectively, and $A_p$ and $B_p$ are two constant matrices. The discrete-time state feedback controller is of the form $\boldsymbol{u}(k) = K\boldsymbol{x}(k)$. Throughout the paper, the following assumptions are needed for the considered NCS.

**Assumption 1.** The sensor is time-driven and the sampling period is denoted by $h$. Both the controller and the actuator are event-driven.

**Assumption 2.** The unknown time-varying network-induced delay at time step $k$ is denoted by $\tau_k$ and $\tau_k = \tau_{sc}(k) + \tau_{ca}(k)$ is smaller than one sampling period and is upper bounded by $\tau_k \le \bar{\tau} = eh \le h$, where $0 \le e \le 1$, and $\tau_{sc}(k)$ and $\tau_{ca}(k)$ are the sensor-to-controller delay and the controller-to-actuator delay, respectively. There is no packet dropout in the networks.

The unknown time-varying network-induced delay $\tau_k$ can be represented as

$$\tau_k = \tau_0 + \Delta\tau_k \tag{2}$$

where $\tau_0$ is called the nominal part of $\tau_k$ and $\Delta\tau_k$ the un-

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certain part of $\tau_k$. Let $\tau_0 = \alpha_0\bar{\tau}$, $\alpha_1\bar{\tau} \leq \Delta\tau_k \leq \alpha_2\bar{\tau}$, where $\alpha_0$, $\alpha_1$, and $\alpha_2$ are known constants, and satisfy

$$0 \leq \alpha_0 \leq 1,\ \alpha_1 \leq 0,\ \alpha_0 + \alpha_1 \geq 0,\ \alpha_0 + \alpha_2 = 1 \quad (3)$$

Then, by (2) we have that $\tau_k$ varies over the interval $[(\alpha_0+\alpha_1)\bar{\tau},\ (\alpha_0+\alpha_2)\bar{\tau}]$ and the interval length $L_{\tau_k} = (\alpha_0+\alpha_2)\bar{\tau} - (\alpha_0+\alpha_1)\bar{\tau} = (\alpha_2-\alpha_1)\bar{\tau}$.

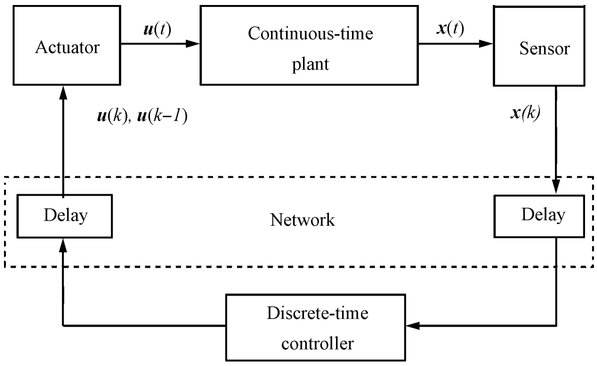


Fig. 1 The structure of NCS with short time-varying delays

**Remark 1.** The uncertain part $\Delta\tau_k$ that varies over the interval $[\alpha_1\bar{\tau},\ \alpha_2\bar{\tau}]$ captures the variation nature of the network-induced delay $\tau_k$. Note that $-\alpha_1 \neq \alpha_2$, and the delay range is not symmetric around the nominal value. If $L = \alpha_2 - \alpha_1$, then $L$ can be used to describe the variation range of the uncertain delay $\tau_k$, and we have by (3) that $0 \leq L \leq 1$. Moreover, the larger the $L$, the larger the delay variation range. For example, when $L = 0$, we have $\alpha_1 = \alpha_2$, which indicates that $\tau_k$ is a constant delay. When $L = 1$, the uncertain delay interval length $L_{\tau_k} = \bar{\tau}$, which indicates that the delay $\tau_k$ varies in full range.

In (2), the time-varying network-induced delay is separated into two parts, namely, the nominal part and the uncertain part, which is characterized by $e$ (determining the delay upper bound for a fixed sampling period) and the delay variation range $L$. In the next section, a new linear uncertain system model will be developed to describe the considered NCS with short time-varying delays. Such a system model will enable us to establish the relation between the stability of the NCS and the two parameters $e$ and $L$.

## 2 Modeling of the NCS

Sampling the system given by (1) with period $h$ and taking the network-induced delay $\tau_k$ into account, we obtain

$$\boldsymbol{x}(k+1) = A_0\boldsymbol{x}(k) + B_0(\tau_k)\boldsymbol{u}(k) + B_1(\tau_k)\boldsymbol{u}(k-1) \quad (4)$$

where $A_0 = \mathrm{e}^{A_p h}$ and

$$B_0(\tau_k) = \int_0^{h-\tau_k} \mathrm{e}^{A_p s}\mathrm{d}sB_p,\ B_1(\tau_k) = \int_{h-\tau_k}^{h} \mathrm{e}^{A_p s}\mathrm{d}sB_p$$

Denote $A_1 = \mathrm{e}^{A_p(h-\tau_0)}$ and $B_1 = \int_0^{h-\tau_0} \mathrm{e}^{A_p s}\mathrm{d}sB_p$. Then, by (2), we have that

$$\begin{aligned}B_0(\tau_k) = &\int_0^{h-\tau_k} \mathrm{e}^{A_p s}\mathrm{d}sB_p = \\ &\int_0^{h-\tau_0-\Delta\tau_k} \mathrm{e}^{A_p s}\mathrm{d}sB_p = \\ &\int_0^{h-\tau_0} \mathrm{e}^{A_p s}\mathrm{d}sB_p + \int_{h-\tau_0}^{h-\tau_0-\Delta\tau_k} \mathrm{e}^{A_p s}\mathrm{d}sB_p = \\ &B_1 + A_1\int_0^{-\Delta\tau_k} \mathrm{e}^{A_p s}\mathrm{d}sB_p = \\ &B_1 + A_1\Theta_0(\Delta\tau_k)B_p\end{aligned}$$

where $\Theta_0(\Delta\tau_k) = \int_0^{-\Delta\tau_k} \mathrm{e}^{A_p s}\mathrm{d}s$. Let $B_0 = \int_0^h \mathrm{e}^{A_p s}\mathrm{d}sB_p$. Since $B_0(\tau_k) + B_1(\tau_k) = B_0$, we have $B_1(\tau_k) = B_0 - B_0(\tau_k) = B_0 - B_1 - A_1\Theta_0(\Delta\tau_k)B_p$.

By the expressions of $B_0(\tau_k)$ and $B_1(\tau_k)$ and by applying the state feedback control law to system (4), we obtain the following closed-loop NCS

$$\boldsymbol{x}(k+1) = A_c\boldsymbol{x}(k) + B_c\boldsymbol{x}(k-1) \quad (5)$$

where

$$A_c = A_0 + B_1K + A_1\Theta_0(\Delta\tau_k)B_pK$$

$$B_c = (B_0 - B_1)K - A_1\Theta_0(\Delta\tau_k)B_pK$$

Denote $\sigma_{\max}(\Delta\tau_k)$ as the maximum singular value of the matrix $\Theta_0(\Delta\tau_k)$, and define $\sigma(\alpha_1,\alpha_2,e) = \sup_{\Delta\tau_k\in[\alpha_1\bar{\tau},\ \alpha_2\bar{\tau}]}\sigma_{\max}(\Delta\tau_k)$. Then, it can be seen that $\|\Theta_0(\Delta\tau_k)\|^2 < \sigma^2(\alpha_1,\alpha_2,e)$, where $\sigma(\alpha_1,\alpha_2,e)$ is a finite scalar since the delay is upper-bounded. Therefore, NCS (5) is essentially a discrete-time linear system with norm-bounded uncertainty. Based on the system model (5), we aim in the rest of this paper at solving the following problem.

**Problem 1.** Design a state controller of the form $\boldsymbol{u}(k) = K\boldsymbol{x}(k)$ such that the closed-loop NCS (5) is asymptotically stable, meanwhile establish the quantitative relation between the stability of the closed-loop NCS and two delay parameters, namely, the delay upper bound and the delay variation range bound.

**Remark 2.** In the proposed modeling method, the uncertainty of the network-induced delay is transformed into the uncertainty of the system matrices. Similar modeling approaches have been presented in [13−14]. In [13], several free parameters were required to be tuned to guarantee that the uncertain matrix is unit norm-bounded, and no effective algorithm on how to choose these parameters was given. In [14], the expression of the uncertain matrix was not explicitly given, and some numerical algorithms were required to ensure that the uncertain matrix is unit norm-bounded. Moreover, when the delay is shorter than one sampling the period, the relation between the stability of NCS and the allowable delay upper bound (ADB) was not established in both [13−14] based on the presented uncertain system models. The uncertain system model presented in this paper is more general than those in [13−14], and it contains the information of the delay upper bound and the variation range bound. Such a system model enables us to establish the quantitative relation between the asymptotic stability of the NCS and the two delay parameters by applying the robust control approach, which will be developed in the next section.

## 3 Stability analysis and stabilizing controller design

A sufficient condition for the existence of the state feedback stabilizing controllers for the NCS (4) is given in the following theorem.

**Theorem 1.** For given scalars $e$, $\alpha_0$, $\alpha_1$, and $\alpha_2$, if there exist matrices $R > 0$, $S > 0$, $V$, and scalars $\varepsilon > 0$ and $\mu > 0$ such that the following inequalities

$$\begin{bmatrix} -R+S & 0 & RA_0^{\rm T}+V^{\rm T}B_1^{\rm T} & V^{\rm T}B_p^{\rm T} \\ * & -S & V^{\rm T}(B_0-B_1)^{\rm T} & -V^{\rm T}B_p^{\rm T} \\ * & * & -R+\varepsilon A_1A_1^{\rm T} & 0 \\ * & * & * & -\mu I \end{bmatrix} < 0 \quad (6)$$

$$\sigma(\alpha_1,\alpha_2,e) < \delta \quad (7)$$

hold, then the NCS (4) controlled by $\boldsymbol{u}(k) = K\boldsymbol{x}(k)$ is asymptotically stable, where $\delta = 1/\sqrt{\varepsilon^{-1}\mu}$, and the controller gain matrix is given by $K = VR^{-1}$.

**Proof.** Choose the Lyapunov function $V(k) = \boldsymbol{x}^{\rm T}(k)P\boldsymbol{x}(k)+\boldsymbol{x}^{\rm T}(k-1)Q\boldsymbol{x}(k-1)$ for system (5). Then, we have by simple calculation that $\Delta V(k) = V(k+1)-V(k) = \boldsymbol{\xi}^{\rm T}(k)\Psi\boldsymbol{\xi}(k)$, where

$$\boldsymbol{\xi}(k) = [\boldsymbol{x}^{\rm T}(k)\ \boldsymbol{x}^{\rm T}(k-1)]^{\rm T}$$

$$\Psi = \begin{bmatrix} A_c^{\rm T} \\ B_c^{\rm T} \end{bmatrix} P[A_c\ B_c] + \begin{bmatrix} -P+Q & 0 \\ 0 & -Q \end{bmatrix}$$

$\Psi < 0$ guarantees that $\Delta V(k) < 0$, which implies by the Lyapunov stability theory that NCS (5) is asymptotically stable. By Schur complement, $\Psi < 0$ is equivalent to the following matrix inequality

$$\begin{bmatrix} -P+Q & 0 & A_c^{\rm T} \\ * & -Q & B_c^{\rm T} \\ * & * & -P^{-1} \end{bmatrix} < 0$$

which can be written as

$$\Psi_0 + \bar{D}\Theta_0(\Delta\tau_k)\bar{E} + \bar{E}^{\rm T}\Theta_0^{\rm T}(\Delta\tau_k)\bar{D}^{\rm T} < 0 \quad (8)$$

where

$$\Psi_0 = \begin{bmatrix} -P+Q & 0 & (A_0+B_1K)^{\rm T} \\ * & -Q & ((B_0-B_1)K)^{\rm T} \\ * & * & -P^{-1} \end{bmatrix}$$

$$\bar{D} = [0\ 0\ A_1^{\rm T}]^{\rm T},\ \bar{E} = [B_pK\ \ -B_pK\ \ 0]$$

Inequality (7) guarantees that $\Theta_0^{\rm T}(\Delta\tau_k)\Theta_0(\Delta\tau_k) < \sigma^2(\alpha_1,\alpha_2,e) < \delta^2$. Therefore, we have by Lemma 5.4.1 in [15] that inequality (8) is true if and only if there exists a scalar $\varepsilon > 0$ such that the following inequality

$$\Psi_0 + \varepsilon\bar{D}\bar{D}^{\rm T} + \varepsilon^{-1}\delta^2\bar{E}^{\rm T}\bar{E} < 0 \quad (9)$$

holds. Note that $\mu = \varepsilon\delta^{-2}$. It then follows from the Schur complement that (9) is equivalent to the following matrix inequality

$$\Xi = \begin{bmatrix} -P+Q & 0 & A_0^{\rm T}+K^{\rm T}B_1^{\rm T} & K^{\rm T}B_p^{\rm T} \\ * & -Q & K^{\rm T}(B_0-B_1)^{\rm T} & -K^{\rm T}B_p^{\rm T} \\ * & * & -P^{-1}+\varepsilon A_1A_1^{\rm T} & 0 \\ * & * & * & -\mu I \end{bmatrix} < 0 \quad (10)$$

Denote $R = P^{-1}$, $S = RQR$, and $V = KR$. Then, we obtain inequality (6) by pre- and post-multiplying $\Xi$ by $\text{diag}\{R,\ R,\ I,\ I\}$. □

Note that we need to estimate $\sigma(\alpha_1,\alpha_2,e)$ when applying the condition (7) to check the asymptotic stability of the closed-loop NCS (5). Estimation procedures for $\sigma(\alpha_1,\alpha_2,e)$ are given as follows.

For real matrix $A_p$, there always exists a non-singular matrix $T$ such that $A_p = T\Lambda_pT^{-1}$, where $\Lambda_p$ is the Jordan block of $A_p$, and

$$\Lambda_p = \text{diag}\{J_{d_1}(\lambda_1),\cdots,J_{d_r}(\lambda_r)\}$$

$$J_{d_i}(\lambda_i) = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix}_{d_i\times d_i}, \quad \sum_{i=1}^{r} d_i = n$$

So, we have

$$\Theta_0(\Delta\tau_k) = \int_0^{-\Delta\tau_k} \text{e}^{A_ps}\text{d}s = T\left(\int_0^{-\Delta\tau_k}\text{e}^{\Lambda_ps}\text{d}s\right)T^{-1} = T\Lambda_0(\Delta\tau_k)T^{-1} \quad (11)$$

where

$$\Lambda_0(\Delta\tau_k) = \text{diag}\left\{\int_0^{-\Delta\tau_k}\text{e}^{J_{d_1}(\lambda_1)s}\text{d}s,\ \cdots,\ \int_0^{-\Delta\tau_k}\text{e}^{J_{d_r}(\lambda_r)s}\text{d}s\right\}$$

$$\int_0^{-\Delta\tau_k}\text{e}^{J_{d_i}(\lambda_i)s}\text{d}s = \begin{bmatrix} \int_0^{-\Delta\tau_k}\text{e}^{\lambda_is}\text{d}s & \int_0^{-\Delta\tau_k}s\text{e}^{\lambda_is}\text{d}s & \int_0^{-\Delta\tau_k}\frac{s^2}{2!}\text{e}^{\lambda_is}\text{d}s & \cdots & \int_0^{-\Delta\tau_k}\frac{s^{d_i-1}}{(d_i-1)!}\text{e}^{\lambda_is}\text{d}s \\ & \int_0^{-\Delta\tau_k}\text{e}^{\lambda_is}\text{d}s & \int_0^{-\Delta\tau_k}s\text{e}^{\lambda_is}\text{d}s & \cdots & \vdots \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & \int_0^{-\Delta\tau_k}s\text{e}^{\lambda_is}\text{d}s \\ & & & & \int_0^{-\Delta\tau_k}\text{e}^{\lambda_is}\text{d}s \end{bmatrix} \quad (12)$$

$$\int_0^{-\Delta\tau_k}\frac{s^{d_i-1}}{(d_i-1)!}\text{e}^{\lambda_is}\text{d}s =$$

$$\begin{cases} (\frac{1}{\lambda_i})^{d_i}\left((-1)^{d_i}+\sum\limits_{k=1}^{d_i}(-1)^{k-1}\times \right. \\ \left.\frac{(-\lambda_i\Delta\tau_k)^{d_i-k}}{(d_i-k)!}\mathrm{e}^{-\lambda_i\Delta\tau_k}\right), \ \lambda_i\neq 0 \\ \frac{(-\Delta\tau_k)^{d_i}}{d_i!}, \qquad \lambda_i=0 \end{cases} \tag{13}$$

When $A_p$ is diagonalizable, $J_{d_i}(\lambda_i)$ is then a diagonal block in the form $J_{d_i}(\lambda_i) = \mathrm{diag}\{\lambda_i, \cdots, \lambda_i\}_{d_i\times d_i}$. We have in this case

$$\int_0^{-\Delta\tau_k}\mathrm{e}^{J_{d_i}(\lambda_i)s}\mathrm{d}s =$$

$$\mathrm{diag}\left\{\int_0^{-\Delta\tau_k}\mathrm{e}^{\lambda_i s}\mathrm{d}s, \cdots, \int_0^{-\Delta\tau_k}\mathrm{e}^{\lambda_i s}\mathrm{d}s\right\}_{d_i\times d_i} \tag{14}$$

where

$$\int_0^{-\Delta\tau_k}\mathrm{e}^{\lambda_i s}\mathrm{d}s = \begin{cases} \frac{1}{\lambda_i}(\mathrm{e}^{-\lambda_i\Delta\tau_k}-1), & \lambda_i\neq 0 \\ -\Delta\tau_k, & \lambda_i=0 \end{cases} \tag{15}$$

Now, by (11) $\sim$ (15) one can obtain some approximations of $\sigma(\alpha_1,\alpha_2,e) = \sup_{\Delta\tau_k\in[\alpha_1\bar\tau,\ \alpha_2\bar\tau]}\sigma_{\max}(\Delta\tau_k)$ by simply implementing one-dimensional search on the variable $\Delta\tau_k$ over the interval $[\alpha_1\bar\tau,\ \alpha_2\bar\tau]$.

**Remark 3.** It can be seen that the condition (7) is related to parameters $\alpha_1$, $\alpha_2$, and $e$. Moreover, the larger the $e$ and $L$ (note that $L=\alpha_2-\alpha_1$), the larger the $\sigma(\alpha_1,\alpha_2,e)$ will be, which indicates that the asymptotic stability of system (5) will not be guaranteed for some $e$ and $L$ which are large enough. Similar to the meaning of ADB, $L$ is said to be the allowable delay variation range bound (ADVRB) that guarantees the asymptotical stability of the closed-loop NCS. By the above analysis, it can be seen that Theorem 1 has established the quantitative relation between the asymptotic stability of the closed-loop NCS and the ADB $e$ and the ADVRB $L$. Moreover, for a fixed $L$, we can obtain the estimation of maximal allowable delay bound (MADB) $e_m$ by applying the following algorithm.

**Algorithm 1.**

**Step 1.** Set $e=1$ and $e_m=e$;

**Step 2.** Solve the following optimization problem

$$\begin{array}{ll} \min & \mu \\ \text{s.t.} & (6) \end{array} \tag{16}$$

and calculate $\delta$;

**Step 3.** Calculate $\sigma(\alpha_1,\ \alpha_2,\ e)$ and check whether the condition (7) is satisfied or not. If (7) is not satisfied, then go to Step 2 after decreasing $e$ to some extent; otherwise, set $e_m=e$ and exit.

## 4 Illustrative example

**Example.** Consider system (1) as in [9] and [16], where

$$A_p = \begin{bmatrix} -1 & 0 & -0.5 \\ 1 & -0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},\ \boldsymbol{B}_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Choose the sampling period $T=0.5\,\mathrm{s}$. Then, the corresponding discretized system model (4) with short time-varying network-induced delay is given by

$$\boldsymbol{x}(k+1) = \begin{bmatrix} 0.6065 & 0 & -0.2258 \\ 0.3445 & 0.7788 & -0.0536 \\ 0 & 0 & 1.2840 \end{bmatrix}\boldsymbol{x}(k) +$$

$$\int_0^{0.5-\tau_k}\mathrm{e}^{A_p s}\mathrm{d}sB_p\boldsymbol{u}(k) + \int_{0.5-\tau_k}^{1}\mathrm{e}^{A_p s}\mathrm{d}sB_p\boldsymbol{u}(k-1) \tag{17}$$

Suppose that the upper bound of the network-induced delays is $\bar\tau=0.2T$, the nominal value of the delays is $\tau_0=0.5\bar\tau$, and the uncertain parts of the delays vary over the interval $[-0.5\bar\tau,\ 0.5\bar\tau]$. Then, it can be seen that the delay variation range is $L=\alpha_2-\alpha_1=0.5-(-0.5)=1$, which indicates that the delays vary in full range. Direct computation gives $\sigma(-0.5,0.5,0.2)=0.0517$. Solving the linear matrix inequality (6), we obtain $\varepsilon=10.2405$, $\mu=1517.5$, and the controller gain matrix $\boldsymbol{K}=[0.0447\ \ 0.0131\ \ -1.8463]$. So, we have $\delta=1/\sqrt{\varepsilon^{-1}\mu}=0.0821$, and thus the condition (7) is satisfied. Choose $\boldsymbol{x}(0)=[-5;0;5]$. Then, the simulation result is shown in Fig. 2, which depicts the state trajectories of the resulting closed-loop system of (17) when applying the designed controller. Furthermore, by applying Algorithm 1, we obtain that a estimation of MADB which guarantees a feasible stabilizing controller is $e_m=0.28$.

Now, assume that the nominal value of the delays is $\tau_0=0.7\bar\tau$, and the uncertain parts of the delays vary over the interval $[-0.1\bar\tau,\ 0.3\bar\tau]$. Then, the delay variation range is $L=0.3-(-0.1)=0.4$. By applying Algorithm 1, we obtain that an estimation of MADB is $e_m=0.33$, which guarantees a feasible stabilizing controller. It can be seen from the calculation results that a larger delay variation range allows a smaller delay upper bound and vice versa.

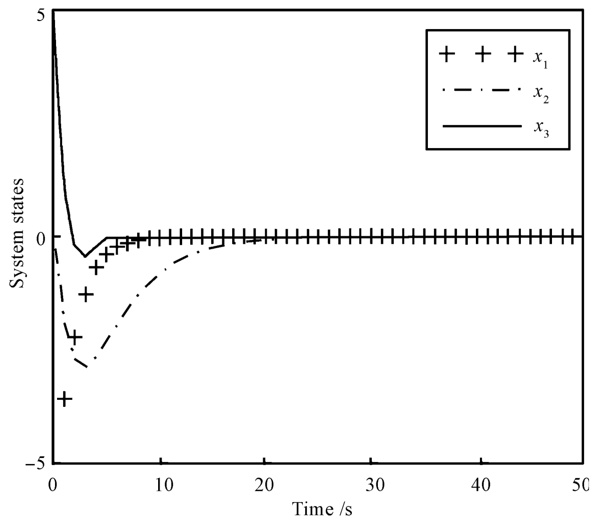


Fig. 2 The state trajectories

## 5 Conclusion

The stabilization problem has been studied in this paper for the NCSs with short time-varying delays. A new linear uncertain system model was proposed to describe the considered NCSs. The obtained asymptotic stability condition for the closed-loop NCSs establishes the quantitative relation between the delay upper bound and variation range bound. The presented system model is simple yet useful, and some system performance design, such as the $H_\infty$ control problem, can be investigated based on the proposed uncertain system model. Furthermore, the proposed method is also applicable to the NCSs with long delays, which may vary within one sampling period.

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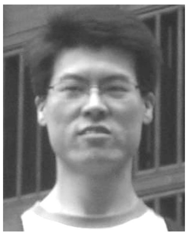

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