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Color Superconductivity for High Density Nuclear Matter*

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Astract: In this paper we built a relation between the thermodynamical theory of the phase transition and field theory. We emphasized that in the quantum field theory we have to introduce the order parameter fields. Then the discussion of the phase transition is closed to the creation of the Goldstone bosons. If we only discuss the first order transition, the Goldstone bosons fields are sufficient. If we want to discuss the second order transition, we have to discuss a set of fields that constructs a representation of a symmetry group. We also apply this concept to color superconductivity.

Key words: phase transition; order parameter; spontaneous breaking

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1 Introduction

Recently the study of Color Superconductivity phase of high density nuclear matter causes attention [1-5], since it is important to understand the high density nuclear matter. In this paper we recommend the necessary basic knowledge of color superconductivity and simply show our results.

2 Phase Transition

A first order phase transition is a point across which some thermodynamic variables (the density of a fluid, or the magnetization of a ferromagnet) changes discontinuously. These discontinuously changed quantities are called order parameters. In most circumstances, it is possible to change a second dynamical quantity in such a way that the competing states move closer together and two states become identical. Then the discontinuity in the order parameter disappears. This end point is called critical point or second order phase transition point.

To demonstrate this phenomenon, let us consider the ferromagnetic materials. Let us assume that the material has a preferred axis of magnetization. So that at low temperature the system will have its spins ordered either parallel or antiparallel to this axis. The total magnetization along this axis is an order parameter, and this fact usually is represented as M is order parameter.

In order to relate with the path integral of the quantum field theory, we use the free energy.

$$dA = -PdV - SdT - HdM . \qquad (1)$$

If V is fixed, the A can be considered a function of T, M. For this case it is convenient to consider the density of the free energy. Then we have

$$df = -s dT - H dm , \qquad (2)$$

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where $s=A/V$, $m=M/V$.

Let us concentrate our attention on the region of the critical point $T=T_c$, where $M\approx 0$ for $H=0$.

We can expand $f(T, m)$ as

$$f(T, m) = A(T) + B(T)m^2 + C(T)m^4 + \cdots. \tag{3}$$

Because the system has a symmetry under $M \to -M$, $f(T, m)$ only contain even powers of m. It is easy to find for $H=0$

$$2B(T)m + 4C(T)m^3 = 0 . \tag{4}$$

If $B(T)$ and $C(T)$ are positive, the only solution is $M=0$. However, if $C>0$ but B is negative below temperature T_c, we have a nontrivial solution as shown in Fig. 1. It is obvious that for $T<T_c$ the order parameter M changes discontinuously. The phase diagram in the H-T plane can be shown in Fig. 2.

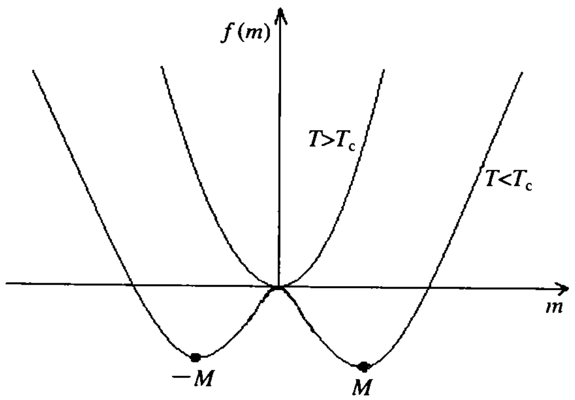

Fig. 1 The free energy density has the different function relation with the magnetization density at the different temperature.

More concretely the $B(T)$ and $C(T)$ is approximated for $T\approx T_c$ by $B(T)=b(T-T_c)$, $C(T)=C$. Then we have

$$M=\begin{cases} 0 & \text{for} \quad T>T_c \\ \pm\left[\frac{b}{2c}(T_c-T)\right] & \text{for} \quad T<T_c \end{cases} . \tag{5}$$

The critical point $T=T_c$ is called the point of the second order phase transition. In the following we shall discuss the region near the second order phase transition point.

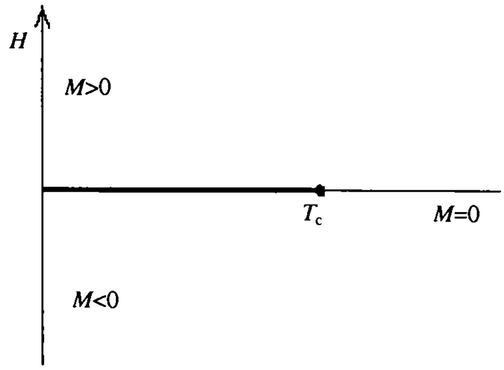

Fig. 2 The phase diagram is shown here.

We shall build the relation between the free energy and the path integral in the quantum field theory. At first, we consider the case of $T\to 0$. Here the partition function $\mathrm{Tr}(\exp(-\beta H))$ can be written as

$$\begin{aligned} &\mathrm{Tr}(\exp(-\beta H)) \\ &= \int D(q)\exp\left(-\int_{-\beta/2}^{\beta/2} L(q, q)\mathrm{d}\tau\right) \\ &= \exp(-\beta A) \end{aligned} \tag{6}$$

and

$$\mathrm{e}^{-\beta A} \xrightarrow[T\to 0]{} \int D(q)\mathrm{e}^{-S}, \tag{7}$$

where $\beta=1/(KT)$, q is a microscopic quantity corresponding to the statistical macroscopic quantity.

Eq. (7) means that

$$Z = \int D(q)\exp(-S) = \exp(-V\cdot\tau\cdot f), \tag{8}$$

where $V\cdot\tau$ is volume of 4 dimensions, f is the density of the free energy and $f=f(T=0, m)$.

In the case of the presence of the external fields, the path integral becomes

$$\begin{aligned} Z[J] &= \int D(q)\exp\{-[S-\int \mathrm{d}^4x q(x)J(x)]\} \\ &= \exp\{-\int \mathrm{d}^4x[f(x)-\langle q(x)\rangle J(x)] \\ &= \exp(W)\} = \mathrm{e}^{+W}. \end{aligned} \tag{9}$$

This equation is deduced from the assemble theory. Then we get

$$W = -\int \mathrm{d}^4x f(x) + \int \langle q(x)\rangle_J J(x)\mathrm{d}^4x \tag{10}$$

and

$$\Gamma = W - \int \langle q(x) \rangle_J J(x) \mathrm{d}^4 x = - \int f(\langle q(x) \rangle_J) \mathrm{d}^4 x \,. \tag{11}$$

It is obvious $\delta\Gamma/\delta\langle q(x)\rangle_J = -J(x)$ and Γ has stationary point for $J(x)=0$.

In the ferromagnet $m=\langle q(x)\rangle$ is the order parameter. For the translational case $\langle q(x)\rangle = m$ is independent of x. The phase transition happens when $\partial f(m)/\partial m=0$ has nonzero solution.

3 Spontaneously Broken Symmetries

The effective potential $V[\phi]$is defined by

$$\Gamma[\phi] = -\nu V[\phi] \,, \tag{12}$$

where ν is the four dimensional volume. The nonzero solution of the stable point of $V[\phi]$ spontaneously breaks the symmetry.

We can show that for each generator that is broken, there is a massless boson called Goldstone.

If $V[\phi]$ has a symmetry group $\exp(\mathrm{i}\varepsilon^a t^a)$, where t^a are the generators, we have

$$\sum_{nm} \frac{\partial V}{\partial \phi_n} t^a_{nm} \phi_m = 0 \,. \tag{13}$$

Differentiating with respect to ϕ_l we get

$$\sum_n \frac{\partial V}{\partial \phi_n} t^a_{nl} + \sum_{nm} \frac{\partial^2 V}{\partial \phi_n \partial \phi_l} t^a_{nm} \phi_m = 0 \,. \tag{14}$$

If the vacuum expectation value $\langle \phi_m \rangle = \Phi_m$ for $J_a = 0$ is not zero, we have

$$\sum_{nm} \left(\frac{\partial^2 V}{\partial \phi_n \partial \phi_l} \right)_{\phi=\Phi} t^a_{nm} \Phi_m = 0 \,. \tag{15}$$

Using $\left(\frac{\partial^2 V}{\partial \phi_n \partial \phi_l} \right)_{\phi=\Phi} = \triangle^{-1}_{nl}(0)$,
where $\triangle^{-1}_{nl}(0)$ is the propagator for ϕ_m at the 0 momentum, we get

$$\sum_{nm} \triangle^{-1}_{nl}(0) t^a_{nm} \Phi_m = 0 \,. \tag{16}$$

It is well known that $\triangle^{-1}_{nl}$ is the mass matrix. Then Eq. (16) means that mass matrix has zero eigenvalue.

Because of the existence of Goldstone bosons the order parameter has to be defined as a set of fields that is an irreducible representation of the symmetry group. These multiplets construct a linear representation in one phase and they construct a nonlinear representation in another phase. This implies that in order to study phase transition and define order parameter, it is necessary to consider Goldstone bosons and another massive boson (for example σ fields)

4 Superconductivity

The objects studied are many electrons and electromagnetic fields. Because of the interaction the cooper's pairs form in superconductor. In the field theory[6] we have to introduce the cooper's pairs field as order parameter field. In the superconductor phase, the vacuum expectation of the field operator of cooper's pairs has nonzero value. This breaks $U(1)$ symmetry. Consequently the electromagnetic gauge invariance is spontaneously broken. The minimal coupling for the electromagnetic interaction is

$$\partial_\mu \to \partial_\mu + \mathrm{i}eA_\mu \,. \tag{17}$$

The action of the electrons and photons will be invariant under gauge transformations with the form

$$A_\mu \to A_\mu + \partial_\mu \Lambda \,, \tag{18}$$

$$\Psi(x) \to \exp(-\mathrm{i}e\Lambda(x))\Psi(x) \,, \tag{19}$$

where the phases Λ and $2\pi/e+\Lambda$ are regarded identical. Because of the condensates of cooper's pairs $U(1)$ is broken and only the subgroup Z_2 left unbroken. The coset $U(1)/Z_2$ corresponds to a Goldstone boson. It can be introduced as

$$\Psi = \exp(-\mathrm{i}e\,\phi(x))\widetilde{\Psi}(x) \,, \tag{20}$$

where $\widetilde{\Psi}(x)$ has only symmetry group Z_2 and $\phi(x)$ is identified with $\phi(x)+\pi/e$.

When the region we discuss is far away from the critical point, the $\widetilde{\Psi}(x)$ can be integrated and its effect can be neglected. Then the Lagrangian for the Goldstone and electromagnetic fields may be written as

$$L_{ef} = -\frac{1}{4}\int d^3 x F_{\mu\nu}F^{\mu\nu} + L_S[A_\mu - \partial_\mu \phi] \ . \quad (21)$$

The electric current and charge density here are

$$J(x) = \frac{\delta L_S}{\delta A(x)} \ , \quad (22)$$

$$J^0(x) = -\frac{\delta L_S}{\delta A^0(x)} = -\frac{\delta L_S}{\delta \dot{\phi}(x)} \ , \quad (23)$$

$$A^0 = -A_0 ; \ \partial_0 = \partial t \ .$$

Since $L_S = -V_{eff}$ we determine the Vacuum by the minimum of V_{eff}. It is obvious that $A_\mu = 0$ and $\phi = 0$ is the minimum of V_{eff} which implies that trivial vacuum. But $A_\mu = \partial_\mu \phi$ is also minimum of V_{eff}. This implies that in the superconductor the electromagnetic field is a pure gauge.

$$A_\mu = \partial_\mu \phi \ . \quad (24)$$

This is known as the Meissner effect. If a magnetic field B penetrates the superconductor, the energy cost in allowing the magnetic field into the superconductor is of order of $B^2 L^5/\lambda^2$, where λ is some length depending on the nature of the material. On the other hand, the energy cost for expelling a magnetic field B from a volume L^3 is of order $B^2 L^3$. Hence a weak magnetic field will be expelled from a superconductor. The flux quantization is

$$\int_\omega B \cdot ds = \oint_c A \cdot d^3 X = \oint_c \nabla \phi \cdot d^3 X = \frac{n\pi}{e} \ , \quad (25)$$

where ω is an area in the superconductor and c is a closed curve that surrounds ω. Now we can show the superconductivity. Note that Eq. (23) can be interpreted as the statement that $-J^0$ is the canonical conjugate to ϕ. The Hamiltonian H is a functional of ϕ and $-J^0$. The canonical motion equation is

$$\frac{\delta H_S}{\delta(-J^0(x))} = \dot{\phi} \ . \quad (26)$$

Now the "voltage" $V(x)$ at any point is just the charge in an energy density per charge in the charge density at that point. Then Eq. (26) gives

$$V(x) = -\dot{\phi}(x) \ . \quad (27)$$

This means that a piece of superconducting wire that carries a steady current with time independent fields must have 0 voltage difference.

5 Color Superconductivity

We have known that the superconductivity is related to formation of the fermion pairs. According to the previous discussion the system of high quark density can be described by a field theory.

If we only consider the two flavors and omit the quark mass, the symmetry group is

$$SU(2)_L \otimes SU(2)_R \otimes SU(3)_C . \quad (28)$$

If the condensate takes the form

$$\langle q_i^{\alpha T} C \gamma^5 q_j^\beta \rangle \propto \epsilon_{ij} \epsilon^{\alpha\beta 3} , \quad (29)$$

where C is charge conjugacy matrix, the indices i and j denote the flavor, and the indices α, β denote the color. In this case the order parameter field operator can be taken as $q_i^{\alpha T} C \gamma^5 q_j^\beta$. In the superconductivity phase this composite field operators have nonzero vacuum expectation value. Then the symmetry group is broken to

$$SU(2)_L \otimes SU(2)_R \otimes SU(2)_C . \quad (30)$$

Now we consider three flavors there is a natural extension

$$\langle q_i^{\alpha T} C \gamma^5 q_j^\beta \rangle \propto \epsilon_{ijI} \epsilon^{\alpha\beta I} , \quad (31)$$

where I is a sum index.

This condensate corresponds to the spontaneous breaking of $SU(3)_{Color} \otimes SU(3)_L \otimes SU(3)_R$ down to subgroup $SU(3)$. The order parameter composite field operators have to take as

$$q_i^{\alpha T}(x) C \gamma^5 q_j^\beta(y) \ . \quad (32)$$

In the effective action we can study local and nonlocal condensates. The GCM bilocal field is a very excellent way to deal with this case. In the Euclidean space, the GCM action[7, 8] is taken as

$$S_E = \iint d^4 x d^4 y [q^+ \gamma^4 (\gamma^4 \partial_x^4 + \gamma^i \partial_x^i) \cdot \delta(x-y) q(y) + \frac{g^2 j_\mu^a(x) D_{\mu\nu}(x-y) j_\nu^a(y)}{2} \ . \quad (33)$$

In a finite density nuclear matter, we get the action as

$$S_E = \iint d^4x d^4y [q^+ \gamma^4 (\partial_x^4 - \mu_0) + \gamma^i \partial_x^i)\delta(x-y) q(y) + \frac{g^2 j_\mu^a(x) D_{\mu\nu}(x-y) j_\nu^a(y)}{2}. \quad (34)$$

After using fierz transformation, we change q^+qq^+q term in S_E to q^+q^+qq. If we only consider the infrared slavery the propagator of gluons can be written

$$D_{\mu\nu}(x-y) = \delta_{\mu\nu}\int 4\pi\alpha(q^2)\cdot \exp\left[\frac{iq\cdot(x-y)}{(2\pi)^4 g^2 q^2}\right] d^4q = \delta_{\mu\nu}\int 3\mu^2 \delta^4(q) \exp\left[\frac{iq\cdot(x-y)}{(2\pi)^4 16 g^2}\right] d^4q = \frac{3\mu^2}{(2\pi)^4 16 g^2}. \quad (35)$$

Then $j_\mu^a(x) j_\nu^a(y)$ can be written as

$$q_\mu^a(x) q_\nu^a(y) = q_L^+(y) M_L^0 q_L^+(x) q_L(y) M_L^0 q_L(x) + q_R^+(y) M_R^0 q_R^+(x) q_R(y) M_R^0 q_R(x) + q_R^+(y) M_1^0 q_L^+(x) q_L(x) M_1^0 q_R(y) - q_R^+(y) M_2^0 q_L^+(x) q_L(x) M_2^0 q_R(y). \quad (36)$$

In order to discuss the qq condensate we introduce the order parameter fields $B_L^0(x, y)$, $B_L^{0+}(x, y)$, $B_R^0(x, y)$, $B_R^{0+}(x, y)$ by using the path integral:

$$I_L = \int DB_L^0(y, x) DB_L^{0+}(x, y)\cdot \exp[-\iint B_L^{0+}(x, y) B_L^0(y, x) dx dy], \quad (37)$$

$$I_R = \int DB_R^0(y, x) DB_L^{0+}(x, y)\cdot \exp[-\iint B_R^{0+}(x, y) B_R^0(y, x) dx dy], \quad (38)$$

$$I_1 = \int DB_1^0(y, x) DB_1^{0+}(x, y)\cdot \exp[-\iint B_1^{0+}(x, y) B_1^0(y, x) dx dy], \quad (39)$$

$$I_2 = \int DB_2^0(y, x) DB_2^{0+}(x, y)\cdot \exp[-\iint B_2^{0+}(x, y) B_2^0(y, x) dx dy]. \quad (40)$$

After tedious algebraic calculation, we can get the effective action $S[\mu_0, B_L, B_R, B_1, B_2]$.

The condensates of the order parameter field can be obtained by variation principle. After solving this consistency equation, we obtain:

$$B_L(p) = \pm\left\{\frac{3\alpha[P^2 - (p^0 + \mu_0)^2]}{\beta g^2 \mu^2}\right\}^{1/2}, \quad (41)$$

$$B_L(p) = \pm B_R(p), \quad (42)$$

$$B_1(p) = \pm\left\{([(p^0 - \mu_0)^2 - P^2]\frac{g^2\mu^2}{12} - 9\alpha[P^2 - (p^0 + \mu_0)^2]^2/\beta)/([P^2 - (p^0 + \mu_0)^2] g^2 \mu^2)\right\}^{1/2}, \quad (43)$$

$$B_2(p) = -B_1(p), \quad (44)$$

where $g^2\mu^2 = 3\mu^2/(32(2\pi)^4)$, $\mu = 1$ GeV, P^2, p^0 is arbitrary parameter, and μ_0 is chemical potential.

$$\eta = \left\{\sqrt{\frac{2}{3}}\lambda^1, \sqrt{\frac{4}{3}}\lambda^2, \sqrt{\frac{2}{3}}\lambda^3, \sqrt{\frac{2}{3}}\lambda^4, \sqrt{\frac{4}{3}}\lambda^5, \sqrt{\frac{2}{3}}\lambda^6, \sqrt{\frac{4}{3}}\lambda^7, \sqrt{\frac{2}{3}}\lambda^8, \frac{2}{3}I\right\}, \quad (45)$$

where λ is a generator of $SU(3)$;

$$\zeta = \{I, \sigma^1, \sigma^2, \sigma^3\}, \quad (46)$$

where I is unit matrix, σ is Pauli matrix.

$$M_L^0 = \frac{1}{2\sqrt{2}}\eta^i \otimes \zeta^j \otimes \sigma^2 = M_R^0, \quad (47)$$

$$M^0 = \frac{1}{2}\eta^i \otimes \zeta^j \otimes \zeta^u, \quad (48)$$

where $\mathrm{Tr}(\sum M^0 \sum M^0) = -\alpha$; $\mathrm{Tr}(\sum M_L^0 \sum M_L^0 \sum M_R^0 \sum M_R^0) = \beta$.

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在高密度下核物质的色超导性*

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摘 要：建立了相变热力学理论和场论的关系. 强调在量子场论中必须引进序参量场，则相变的讨论就类似于 Goldstone bosons 的产生. 如果只讨论一级相变，Goldstone bosons 场就足够了；如果要讨论二级相变，则必须讨论一系列的场，这些场构成一个对称群的表示. 另外，也将这一思想用到色超导中.

关 键 词：相变；序参量；自发破缺

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