An ode to Phipps' jeep convoys

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The jeep problem was first solved by O. Helmer and N.J. Fine. But not much later, C.G. Phipps formulated a more general solution. He formulated a so-called convoy or caravan variant of the jeep problem and reduced the original problem to it.

We shall refine the convoy idea of Phipps and subsequently view a more general jeep problem, which we solve for jeep convoys as well as for a single jeep. In the last section we solve Maddex' jeep problem.

Key Words: The jeep problem; Phipps' jeep caravans.

1. INTRODUCTION

The original jeep problem is formulated as follows. Given a jeep that can carry one tankload of fuel and can travel one distance unit per tankload. The jeep is required to cross a desert d units wide. To do so, it may make depots of fuel in the desert. How much fuel is required at the border of the desert.

In case $x = 1\frac{1}{2}$, $2\frac{5}{6}$ tankloads suffice. The jeep can do the following steps.

1. Ride to $\frac{1}{6}$ with a full fuel tank. Dump $\frac{2}{3}$ tankload at $\frac{1}{6}$ and ride back to 0, where you arrive empty.

2. Ride to $\frac{1}{6}$ with a full fuel tank. Take $\frac{1}{6}$ tankload from the depot at $\frac{1}{6}$. Ride farther to $\frac{1}{2}$. Dump $\frac{1}{3}$ tankloads of fuel at $\frac{1}{2}$ and ride back to $\frac{1}{6}$. Take $\frac{1}{6}$ tankload form the depot at $\frac{1}{6}$ and ride back to 0, where you arrive empty.

3. Ride to $\frac{1}{6}$ with the remaining $\frac{5}{6}$ tankloads of fuel at position 0. Take the remaining $\frac{1}{3}$ tankload of fuel from position $\frac{1}{6}$ and ride farther to position $\frac{1}{2}$. Take the $\frac{1}{3}$ tankload of fuel from this position and ride to $1\frac{1}{2}$.
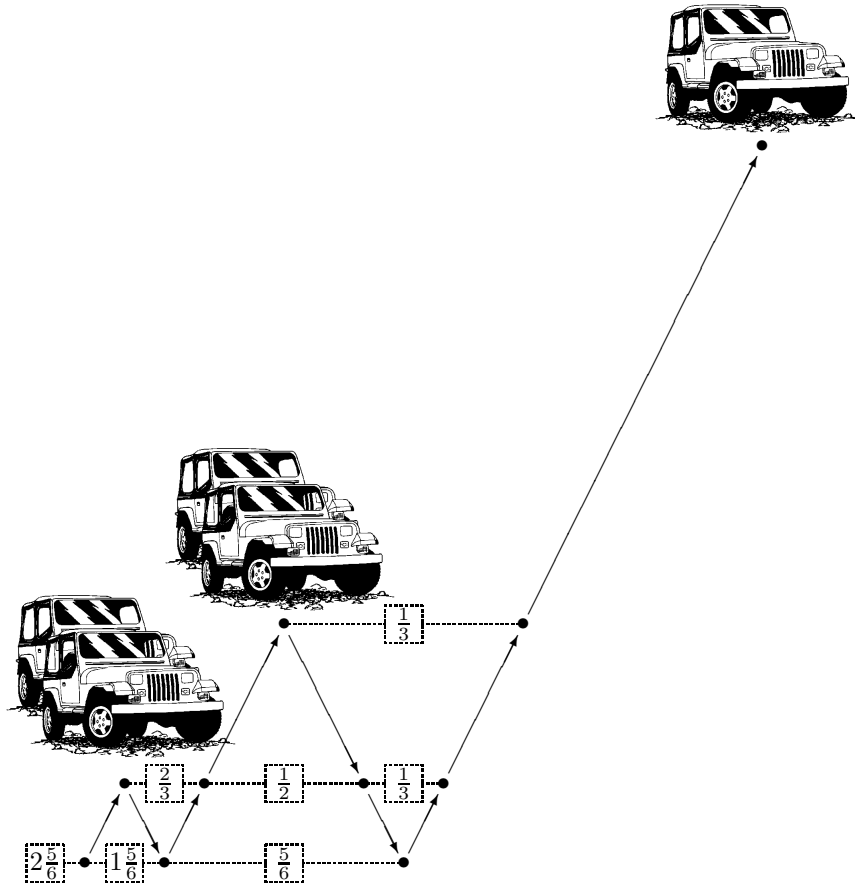
The above algorithm is illustrated in figure 1.

FIG. 1. An illustration of an algorithm to cross a desert of $1\frac{1}{2}$ units.

Both round trips are indicated by a double jeep (steps 1. and 2.) and a single jeep indicates the outward trip (step 3.). Single and double jeeps play an important role in the next sections.

Dealing with jeep problems, I decided that it is a better idea to let the jeeps ride from right to left, since solving jeep problems often requires to view things reversely. As a compromise, the jeep in figure 1 rides from below to above. The reader can turn the manuscript a quarter in the desired direction. The amount of fuel of depots is given in dashed boxes.

The jeep problem is solved in [7] and [9]. Additionally, O. Helmer solved the jeep problem in a series of two papers according to M. Pollack in [14].

In [1], the problem of crossing a desert of 2 units is considered, but the solution is essentially that of the general problem.

A variant of the jeep problem that was considered by O. Helmer as well is the so called round trip jeep problem, where the jeep has to return to the border of the desert after crossing the desert. An obtainable solution can be found in [9], except that some flaws are corrected one year later.

The above algorithm is *normal* in the sense that no abstract formulations are used and consists of two round trips (1. and 2.) and one outward trip (3.) into the desert. Instead of doing these trips after each other, one can also do them at the same time, provided 3 jeeps are available. This way we get a convoy formulation of the problem. C.G. Phipps reasoned that a round trip jeep can be seen as an outward trip jeep that consumes twice as much fuel, see [13] and [10]. We will discuss this view in section 2.

D. Gale solved the jeep variant with more jeeps involved, but did not use the convoy formulation of Phipps, since the argument that any jeep algorithm can be seen as a convoy algorithm with all jeeps traveling together seems to be quite incomplete, see [9]. That is why in [11], the authors refer to [9] for the solution of the round trip jeep problem. Since Phipps' ideas were brilliant, he deserves much better. That is why this article is titled as it is.

2. PHIPPS' CONVOY OF JEEPS

Consider the normal algorithm for crossing a desert of $1\frac{1}{2}$ units in section 1. If it is done with three jeeps such that all riding jeeps are at the same position all the time, we get a *normal convoy algorithm*.

We get the *forward convoy formulation* if we add the return trips of the round trip jeeps to the outward trips. In this formulation, round trip jeeps use twice as much fuel per unit as outward trip jeeps, since they pass each position between 0 and their farthest point from both directions.

So we can see the round trip of a normal jeep as a single trip with a *double jeep*: a jeep that can carry two tankloads of fuel and uses two tankloads per unit. The amount of fuel at some moment t in a double jeep is $o + (1 - r)$, where o is the amount of fuel in the corresponding moment in the outward part of the round trip and r is the amount of fuel in the corresponding moment in the return part of the outer trip. At the farthest position of a round trip, r equals o, so the double jeep finally keeps one tankload of fuel in its tank.

We get the following *forward convoy formulation* of the algorithm in section 1.

1. Ride to $\frac{1}{6}$ with two double jeeps and a single jeep, taking $4\frac{5}{6}$ tankloads of fuel from position 0. At $\frac{1}{6}$, 4 tankloads of fuel remain. Transfer fuel of

one double jeep to the other jeeps, such that the other jeeps get full fuel tanks.

2. Ride to $\frac{1}{2}$ with the full double jeep and the single jeep, leaving the other double jeep with 1 tankload at $\frac{1}{6}$. At $\frac{1}{2}$, 2 tankloads of fuel remain. Transfer fuel of the remaining double jeep to the single jeep, such that the single jeep gets a full tank.

3. Ride to $1\frac{1}{2}$ with the single jeep, leaving the remaining double jeep with 1 tankload at $\frac{1}{2}$.

Notice that the jeeps of the above forward convoy formulation correspond to the steps of the original formulation in section 1. In the forward convoy formulation, 2 tankloads more are used, but these tankloads are still in the double jeeps, since they finally keep one tankload of fuel each. So in fact, double jeeps that are used start with one tankload of additional fuel, which has to be restored finally.

It is not important where the fuel is at some moment that the convoy is progressing into the desert. Since the jeeps that are still in the convoy are all together, fuel can be exchanged as soon as one jeep gets empty. But for making a jeep algorithm without convoy from the above convoy algorithm, it is crucial that the jeeps are ordered, with the single jeep highest in order, such that fuel is only be transferred from lower to higher jeeps.

In the above algorithm, the double jeep first left is the lowest jeep. The order of jeeps corresponds to the order in time of the trips from 0 from the original jeep without convoy in section 1. If the jeeps can not be ordered as above, then in the corresponding normal algorithm for one jeep, the jeep must use fuel from a depot that is not carried yet to the depot, which is impossible.

If fuel is transferred to a double jeep, half of it is used in the outward part and the other half in the return part of the round trip in the corresponding formulation without a convoy.

Instead of questioning how much fuel is required to cross a desert, we can also question how far the jeep can get with some amount of fuel. Both problems are essentially the same.

The above forward convoy formulation can easily be generalized to arbitrary amounts of initial fuel at position 0. We get the following general convoy formulation.

1. Take one single jeep and $n_2 := \lceil x \rceil - 1$ double jeeps, where x is the initial amount of fuel at 0. Fill all jeeps completely with fuel, except the double jeep that is lowest in order: fill that jeep with $x - \lceil x \rceil + 2$ tankloads of fuel. The total amount of fuel is now $1 \cdot 1 + (\lceil x \rceil - 2) \cdot 2 + 1 \cdot (x - \lceil x \rceil + 2) = x + (\lceil x \rceil - 1) = x + n_2$.

2. If the single jeep is the only jeep that is remained in the convoy, then ride into the desert with that jeep until there is no fuel left. Otherwise,

ride into the desert with the convoy until the total amount of fuel of the convoy becomes an even integer. Transfer all fuel above one tankload of the lowest jeep that is still in the convoy to the other jeeps, which will get completely filled. Remove the lowest jeep from the convoy and repeat this step.

At the end, there are n_2 tankloads of fuel left: one tankload in each double jeep. So x tankloads of fuel are used.

Instead of using the forward convoy formulations as suggested by C.G. Phipps, we will use *backward convoy formulations* in the remaining of this article. The convoy is now riding from position d at the other side of the desert to position 0. The convoy starts with one single jeep, and riding back to 0, double jeeps are added to the convoy. The amount of fuel f in some jeep in the backward convoy formulation at some moment corresponds to the amount of emptiness of the same jeep at the corresponding moment in the forward convoy formulation (i.e. $1 - f$ tankloads of fuel for a single jeep and $2 - f$ tankloads for a double jeep).

The backward convoy formulation with $d = 1\frac{1}{2}$ is as follows:

1. Create a full single jeep at $1\frac{1}{2}$ and ride to $\frac{1}{2}$. The single jeep is now empty.

2. Create a double jeep at $\frac{1}{2}$ with one tankload of fuel. Transfer $\frac{1}{3}$ tankload of this fuel to the single jeep. Ride with both jeeps to $\frac{1}{6}$. Both jeeps are now empty.

3. Create a second double jeep with one tankload of fuel. Transfer $\frac{1}{3}$ tankload of fuel to the other double jeep and another $\frac{1}{3}$ tankload of fuel to the single jeep. Ride with all jeeps to 0.

We see that in the backward convoy formulation, fuel is transferred from lower to higher jeeps as well. This is because both time and amount of fuel are inverted.

Further, we see that in step 2. the relative amount of fuel in both jeeps is the same after the transfer (i.e. a third). Therefore, both jeeps are empty after the same amount of units from the transfer. However, in step 3., the absolute amounts of fuel becomes the same due to the transfers, but not the relative amounts of fuel. The single jeep gets relatively twice as much as the other jeeps. For that reason, the single jeep arrives at 0 with $\frac{1}{6}$ tankload of fuel, while both double jeeps arrive empty. Each jeep starts with one tankload of fuel at creation, so $3 - \frac{1}{6} = 2\frac{5}{6}$ tankloads are needed.

We generalize the backward convoy formulation now.

1. Create one single jeep at d and ride to 0.

2. If the convoy gets out of fuel, create a double jeep with one tankload of fuel. Distribute the tankload of fuel such that each jeep of the convoy

gets the same relative amount of fuel. Advance to 0 with the convoy and repeat this step.

We see that the backward convoy formulation is quite short. Further, we see that 2. of the above is in fact an event-handler of the algorithm. We shall formulate forthcoming algorithms by way of event-handlers, since it seems more natural to do so.

An algorithm that is similar to a forward or backward convoy formulation is called a *forward* or *backward convoy algorithm* respectively. So each forward or backward convoy formulation is a forward or backward convoy algorithm respectively, but a forward or backward convoy algorithm does not need to be derived from a normal (convoy) algorithm.

We call an algorithm in which one or more jeeps are involved a *Phipps algorithm* or an algorithm of Phipps' type, if each jeep only changes direction from backward to forward at position 0.

PROPOSITION 2.1. *A backward convoy algorithm can be formulated as a normal convoy algorithm of Phipps' type and vice versa.*

Proof. In this section, we reformulated a single jeep algorithm of Phipps' type as a backward convoy algorithm. More generally, we can reformulate any Phipps algorithm as such. This completes the vice versa part of this proposition.

Suppose we have a backward convoy algorithm. If we wish to reformulate it as a normal convoy algorithm, then we must specify how double jeeps are split in outward trips and return trips. Further, we must ensure that fuel can be exchanged between jeeps. This must be done by way of depots on the ground.

First, we formulate a normal convoy algorithm where jeeps may 'borrow' fuel from the ground. So some positions might contain a negative amount of fuel temporarily. After that, we remove the 'borrowing'.

We see a double jeep with $2r$ tankloads of fuel in a backward convoy algorithm as a return trip with r tankloads of fuel in the jeep's tank and an outward trip with $1 - r$ tankloads of fuel in the jeep's tank.

We organize the normal convoy algorithm with 'borrowing' as follows. First, all jeeps make their outward trip, each jeep leaving the convoy at the farthest position it reaches. After that, the farthest round trip jeep returns to 0, taking the other round trip jeeps with it along the way.

Notice that 'borrowing' might only be needed if a jeep dumps fuel in the return trip (to neutralize a negative depot) or a jeep does not return empty at position 0. The latter can be reduced to the former by demanding that jeeps that return at position 0 dump all their fuel after their return.

If fuel is dumped at position x in a return trip, the dumped fuel comes from a farther position of the desert. But all fuel originally comes from the

desert border, so there is no need to dump fuel in the return trips. This gives the desired result. ∎

3. AN OPTIMAL ALGORITHM FOR PHIPPS' JEEP PROBLEM

Phipps considered the problem of one jeep having to cross the desert, supported by 'helper jeeps'. The helper jeeps and possibly the crossing jeep as well may be obliged to return to the desert border. We consider the problem of n jeeps having to cross the desert, of which n_2 jeeps do and $n_1 = n - n_2$ jeeps do not have to return to the desert border eventually. The jeeps are supported by m helper jeeps of which m_1 jeeps do not need to return.

Phipps did not include depots to be filled in his algorithm explicitly, but remarked that such additions can be made. We formulate a so called extended backward convoy algorithm with such depots. In an *extended backward convoy algorithm*, we allow double jeeps with one tankload of fuel to disappear, in addition to the rules of a backward convoy algorithm. Such a disappearance corresponds to a change of direction from backward to forward at a position other than 0, as we will show in the next section. Single jeeps do not disappear in an extended convoy algorithm, which is not very amazing since such a disappearance would correspond to a jeep starting from that position instead of position 0. This is impossible within the context of Phipps' jeep problem.

ALGORITHM 3.1. Start with a convoy at position d with n jeeps initially, of which n_1 single jeeps and n_2 double jeeps, all with one tankload of fuel. Transfer fuel from the single jeeps to the double jeeps such that each jeep gets the same relative amount of fuel. If a depot has to be filled on position d, then call the handler of event 1 first. After that, ride to position 0 with the whole convoy.

Event 1: The convoy meets a position where a depot has to be filled.
Handler: Do the handler of event 2 as many times as required in order to get the amount of tank fuel larger than the amount of fuel the depot needs (without advancing to the desert border). After that, use fuel to fill the depot. Advance to the desert border, with each jeep having the same relative amount of fuel.

Event 2: The convoy runs out of fuel.
Handler: If the number of single jeeps is $n_1 + m_1$, then create a double jeep with one tankload of fuel. Otherwise, create a single jeep with one tankload of tank fuel. Distribute the tankload of fuel among the convoy such that each jeep gets the same relative amount of fuel and advance to the desert border.

In the rest of this section, we will prove that algorithm 3.1 is optimal. This seemed to be clear for Phipps, but D. Gale missed some arguments in Phipps' article. The largest problem is that jeeps may change direction from backward to forward at positions other than 0. Below we show that such changes of direction correspond to cancelations of double jeeps.

PROPOSITION 3.1. *Any algorithm with one or more jeeps can be formulated as an extended backward convoy algorithm.*

Proof. Call a round trip starting at one position p to a farther position q a *forward loop (from p to q)*. So exactly $2(q - p)$ units are traveled in a forward loop from p to q.

The reformulation is quite similar to that in proposition 2.1 in section 2. Suppose that we have a normal algorithm with a forward loop. Take a maximal forward loop, i.e. take the starting position of the forward loop minimal. Cancel the maximal loop from the algorithm and replace it by a one way trip of a double jeep from q to p, starting with one tankload of fuel at q. After the double jeep arrives at p, fuel is exchanged between the double jeep and the jeep from which the forward loop is canceled, such that the double jeep gets an amount of one tankload of fuel in its tank.

To show that this method works, we must show that after exchanging fuel between the double jeep and the jeep from which the forward loop is canceled, the amount of fuel of the latter jeep is the same as it was in the original algorithm. Suppose that the jeep started its maximal forward loop with o tankloads of fuel and ended it with r tankloads of fuel. In order to get the right amount of fuel, the jeep from which the forward loop is canceled should get rid of $o - r$ tankloads of fuel if $o > r$ and similarly should get $r - o$ tankloads of fuel if $o < r$. Since the amount of fuel of the double jeep is $1 - o + r$ just before exchanging fuel with the jeep from which the forward loop is canceled, the above is satisfied.

A problem of this construction is that fuel can not be exchanged, in the sense that some positions might contain a negative amount of fuel temporarily. This problem will disappear at the end of this reformulation, since then we have an extended backward convoy algorithm where each position is passed only once by the convoy, and therefore temporary underflows of fuel at positions can not occur.

Remove maximal forward loops in the above way until they do not exist any more. The remaining of the original algorithm is now in fact of Phipps' type and can be transformed to a backward convoy formulation as described in section 2. The above rides of double jeeps can be inserted in the backward convoy, which yields an extended convoy algorithm. ∎

THEOREM 3.1. *Algorithm 3.1 is optimal.*

Proof. Consider an optimal extended backward convoy algorithm S. We may assume that in S, jeeps are only added to the backward convoy if the backward convoy runs out of fuel, since postponing adding a jeep can only save fuel. Furthermore, single jeeps should be added first, since they use less fuel than double jeeps. This way we get algorithm 3.1. ▌

Now you read the above solution of the jeep problem, you could think the following: the backward convoy is funny, but the same results can be established without it with half as many pages. This might be true, but I think that the backward convoy is a concept that is worth being displayed extensively, for both scientific and historical reasons.

If there are depots with fuel to be used in the desert, then there are several complication that might play up. If the amount of fuel of the convoy does not get larger than one tankload, then nothing serious happens. But if the convoy gets more than one tankload, then there is enough fuel to cancel a double jeep, but this is only possible if there are more than n_2 double jeeps in the convoy.

For that reason, it is no longer true that single jeeps must be added first to the convoy and double jeeps after that. It is neither the case that the convoy should start with n_1 single jeeps: it should start with n jeeps of which at least n_2 double jeeps. With these adaptation, one can formulate an algorithm where the types of jeeps are undetermined, and thus we get a *nondeterministic algorithm*. But one can prove that such an algorithm is optimal, which means that it is optimal for some way of chosing jeep types, as long as the convoy is able to absorb all fuel to be used for all ways of chosing jeep types.

However, if some depot contains more fuel than the backward convoy can absorb, then things get really harder. An idea is to create new jeeps in order to enlarge the fuel capacity of the backward convoy. But such a backward convoy algorithm can not be transformed to a normal algorithm in general. The problem is that the jeeps cannot take advantage of the depot fuel before it is reached by some jeep. In a convoy formulation, time is in fact eliminated, whence this problem is not taken into account.

In [3], the case of a single jeep having to cross the desert with arbitrary depots of fuel to be used and to be filled is solved. The backward convoy is split in two parts there: one before and one after reaching the large depot of fuel. This problem is strongly related to Gale's round-trip problem, and generalizes Theorem 1 of [11]. In the same article, another problem is formulated, which will be discussed in the next section. This problem

involves a so-called *Dewdney jeep*. Other problems with Dewdney jeeps are studied in [3] as well.

In [11], another jeep problem is formulated as well. This jeep variant is also known as Klarner's camel-banana problem. Now, the jeep is a camel that needs to eat a banana every unit and can carry one banana on its back. It is solved in [2] and [15]. Only the outward case is considered in both references.

4. MADDEX' JEEP PROBLEM

Maddex' jeep problem is the following. Again, we have a jeep that must cross the desert. This jeep has a fuel tank of one tankload. In addition, it can carry B cans of C tankloads of fuel. It may make temporary depots, but such depots must be made of cans and only can fuel may be used to fill them. So the jeep's tank may be filled with fuel from a can, but not vice versa.

At the desert border, there is an unlimited amount of fuel, but there is only a finite number, say N, of cans. These cans can be filled at the desert border. We solve both the outward trip case and the round trip case of Maddex' problem, so there is only one jeep, a so-called *Dewdney jeep*, and there are no additional depots to be made or used in the desert.

Maddex' round trip jeep problem is not very hard. Let $D_N = (N \cdot C + 1)/2$. After leaving the fuel station, the jeep can ride only $N \cdot C + 1$ units before it must return to the fuel station, since the total capacity of all cans and the jeep's own fuel tank together is $N \cdot C + 1$. So the jeep cannot get farther than D_N. The following proposition shows that this upper bound can be achieved.

PROPOSITION 4.1. *With N cans, the jeep can dump a full can at D_{N-1}, without using fuel of this can, and eventually return to the fuel station, without leaving one of the other cans somewhere in the desert.*

Proof.   We show the case $C \leq 1$ first. By induction, it follows that the jeep can dump $N - 1$ cans at $D_{N-2}, D_{N-3}, \ldots, D_1, D_0$, in this order, without using the last can. After doing this, the jeep rides to D_{N-1} with the last can filled, using fuel of the other cans. At last, cans are retrieved in the order $D_0, D_1, \ldots, D_{N-3}, D_{N-2}$, which can be done in the same way as dumping full cans at these positions.

In case $C > 1$, the jeep cannot take a whole can in its tank, whence the above is not possible. But the proposition is still valid, since the jeep can take the can from which he takes fuel along with it, except in case it carries the can that is meant for D_{n-1}. But if it carries that can half a distance unit at a time, then it can use tank fuel to do so.

Inductively dump two cans at D_{N-2}, and the other cans at $D_{N-3}, \ldots,$ D_1, D_0. cans at D_{N-2} half a unit farther with fuel of the other cans and restore the used fuel. Carry the can at $D_{N-2} + \frac{1}{2}$ to D_{N-1} or another half a distance unit farther with the fuel of the other cans, etc. ∎

If $C > 1$ and $B \geq 2$, then cans do not need to be carried farther in steps of half a unit, which the reader may show. We formulate a pseudo-algorithm for the outward case of Maddex' jeep problem.

ALGORITHM 4.1.   Dump full cans at $D_{N-1}, D_{N-2}, \ldots, D_1, D_0$, eventually returning at the fuel station. Start from the fuel station with one tankload of tank fuel. Each time you meet a can, take the fuel of it and advance. Advance as a single jeep if the total amount of fuel of the jeep is $B \cdot C + 1$ at most. Otherwise, advance as a triple jeep until the amount of fuel becomes $B \cdot C + 1$ (which will be before the next can) and then advance as a single jeep again.

Now we have seen double jeeps, the triple jeeps in algorithm 4.1 should not be a problem. The remainder of this section is devoted to show the optimality of algorithm 4.1.

Let t be the last moment that the jeep is at the fuel station. Number the cans $1, 2, \ldots, N$ and let x_i be the position of can i at moment t. Without loss of generality, we assume that $x_N \leq x_{N-1} \leq \cdots \leq x_2 \leq x_1$. Let c_i be the amount of fuel of can i at moment t for all i.

LEMMA 4.1.   *If $1 \leq k \leq N$, then*

$$2 \sum_{i=1}^{k} \max\{x_i - D_{N-i}, 0\} \leq \sum_{i=1}^{k} (C - c_i)$$

Proof.   Notice that at least k times before moment t, the jeep must transport a can to x_k that will not return to a smaller position any more. Assume that for such a moment, there is another can that is transported to x_k later, after which it is used to partially refill the first can. If the second can does return to a smaller position than x_k after this refill, then we can interchange the roles of both cans during the refill. Therefore, we may assume that the second can does not return to a smaller position than x_k any more after the refill either.

When the jeep transports a can to x_k that will not return to a smaller position any more for the ith time, it can subsequently get as far as D_{N-i} by using fuel other than that from the i cans that will stay farther than x_k, but in order to reach x_i, $2 \max\{x_i - D_{N-i}, 0\}$ additional tankloads

of fuel are necessary. This fuel cannot be restored any more due to the above assumption. Taking the sum from 1 to k gives the desired result. ∎

After moment t, it is clear that the Dewdney jeep should ride into the desert and absorb all fuel it encounters, becoming a triple jeep just as in algorithm 4.1 when necessary. But the positions and amounts of fuel of the cans may be different. It is however equally expensive to ride as a triple jeep instead of a single jeep from D_{n-i} to x_i as to transport a can from D_{n-i} to x_i before moment t. So we get the following result.

THEOREM 4.1.   *The solution of Maddex' jeep problem with N cans is D_N in case of a round trip and the distance the Dewdney jeep reaches in algorithm 4.1 in case of an outward trip.*

If more jeeps need to make a round trip, then a distance larger than D_N can be crossed, even if there must be a moment that all jeeps are at the farthest position simultaneously. The reader may show this.

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