The Kyoto Economic Review 75(2): 151–160 (December 2006)

Increase in Risk and Comparative Static Analysis

Tatsuhiko Nariu[1] and Hideki Iwaki[1,2]

[1] *Kyoto University*
[2] *Corresponding author, Graduate School of Economics, Kyoto University.*
E-mail: iwaki@econ.kyoto-u.ac.jp.

In this paper we discuss signing of comparative statics analysis under uncertainty. We define change of risk by a mean preserving simple transformation (abbrev. MPST) of random variables. We show that the signs of the partial derivatives of pay-off functions do not uniquely determine the signs of comparative statics effects of an increase of risk in the sense of MPST.

Keywords: comparative statics, MPST (Mean Preserving Simple Transformation), uncertainty

JEL Classification Numbers: C00, D81

1. Introduction

The effect of risk change on economic decision making has been discussed by many researchers including Feder (1977), Kraus (1979), Katz (1981), Meyer and Ormiston (1983), and Cheng, Magill and Shafer (1987) since the seminal study by Rothschild and Stiglitz (1970). However, the signs of comparative static analysis in fields under uncertainty cannot generally be determined as long as we represent an increase of risk by a mean preserving spread. Recently, conditions to determine the signs of comparative static analysis have been developed in the form of new concepts regarding change of risk, and several researchers have obtained promising results.

Meyer and Ormiston (1989), and Ormiston (1992) in particular have defined change of risk by a deterministic transformation of random variables and have identified a sufficient condition to determine the signs of comparative static analysis. However, their research was restricted to cases where the decision variables were univariate. In this paper we discuss signing of comparative static analysis when the decision variables are bivariate. We define change of risk by a simple transformation of random variables while preserving their mean, that is, a “mean preserving simple transformation" (hereafter MPST).

This paper is organized as follows. In the next section we formulate a problem of decision making under uncertainty. We define an MPST as an increase of risk. Then we show that the signs of comparative static analysis of the MPST are determined under some general conditions whenever the decision variables are univariate. In Section 3 we show that we cannot determine the signs of the comparative static analysis if we expand the decision variables to bivariate ones under the same conditions as those given in Section 2. In Section 4 we discuss some special cases in which the signs can be determined. Finally, we summarize this paper in Section 5.

2. Formulation of a Problem

Let x and y be two scalar decision variables. Consider a maximization problem under uncertainty:

$$\max_{x,y} H = \max_{x,y} \mathbb{E}[u(Z(x,y,S))] = \max_{x,y} \int u(Z(x,y,s))\mathrm{d}F(s),$$

where Z denotes a payoff function and u denotes a utility function. S is a random variable. F is a cumulative distribution function of S. Throughout this paper we assume that $u_Z > 0$ and $u_{ZZ} < 0$. Further, we assume that $Z_s > 0$ without loss of generality.

If we assume the existence of interior solutions, the necessary and sufficient conditions of the maximization problem are given as follows:

$$H_i = \mathbb{E}[u_Z Z_i] = 0, \tag{1}$$

$$\begin{aligned} H_{ii} &= \mathbb{E}[u_Z Z_{ii} + u_{ZZ}(Z_i)^2] < 0, \ i = x,\ y, \\ \text{and } D &= H_{xx}H_{yy} - (H_{xy})^2 > 0, \end{aligned} \tag{2}$$

where $H_{xy} = \mathbb{E}[u_Z Z_{xy} + u_{ZZ} Z_x Z_y]$.

Throughout this paper, we consider a mean preserving simple transformation (MPST) of random variables as change of risk. The MPST is defined below:

Definition 1 (Definition of MPST). *Let $T(s)$ be a deterministic function of a variable s. A random variable $T(S)$ is defined as an MPST of a random variable S if the function $K(s) := T(s) - s$ satisfies*

(a) $\mathbb{E}[K(S)] = 0$,
(b) $K_s > 0$.

In the above definition, condition (a) means that expectation of the transformed random variable T is equivalent to that of the original random variable S, i.e., $\mathbb{E}[T(S)] = \mathbb{E}[S]$. However, it does not always mean that expectation of the payoff Z under each random variable coincides. If the payoff function is the product of the random variable S such that $Z(x,y,S) = g(x,y)S$, expectation of the payoff

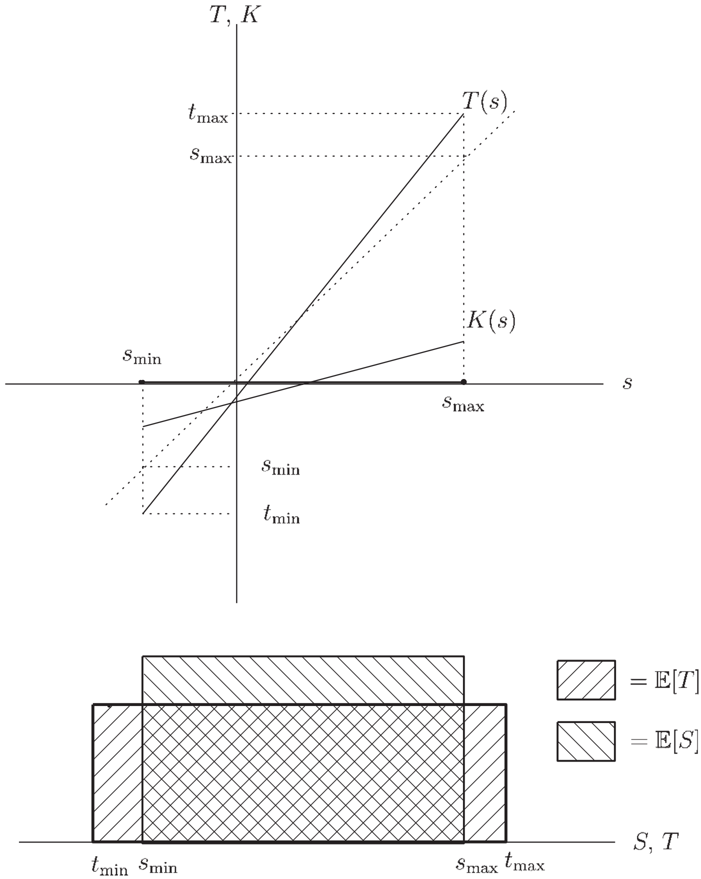

Figure 1 Properties of the MPST.

also coincides, since $\mathbb{E}[Z(x, y, S)] = g(x, y)\mathbb{E}[S] = g(x, y)\mathbb{E}[T] = \mathbb{E}[Z(x, y, T)]$. On the other hand, condition (b) means that $T(s)$ is a monotone function of s. Suppose that $[s_{\min}, s_{\max}]$ and $[t_{\min}, t_{\max}]$ are supports of the random variables S and T, respectively. Then inequalities $t_{\min} < s_{\min}$ and $s_{\max} < t_{\max}$ hold under conditions (a) and (b). This means that the support of the transformed random variable T is broader than that of the original random variable S. Hence the payoff under the transformed random variable is more divergent, since $Z_s > 0$.

Now, consider a transformation of the random variable such that $\tau(S, t) = S + tK(S)$, $0 \leq t \leq 1$. We note that if $t = 0$, $\tau = S$, and if $t = 1$, $\tau = T(S)$. Therefore, the effects of a marginal increment of risk on the F.O.C. of maximization is represented as

$$\begin{aligned} H_{it}|_{t=0} &= \mathbb{E}[(u_Z Z_{is} + u_{ZZ} Z_s Z_i)K] \\ &= \int u_Z Z_{is} K \mathrm{d}F - \int A u_Z Z_i Z_s K \mathrm{d}F, \quad i = x,\ y.^{1)} \end{aligned} \tag{3}$$

In the above equation, $A := -\frac{u_{ZZ}}{u_Z}$ denotes the coefficient of absolute risk aversion of Pratt (1964). As a result, the effects of a marginal increment of risk on the decision variables x and y are given as

$$\left.\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{t=0} = \frac{1}{D}(H_{ij} H_{jt}|_{t=0} - H_{jj} H_{it}|_{t=0}), \quad i, j = x, y,\ i \neq j, \tag{4}$$

by Cramer's formula.

We notice that, supposing $H_{xy} = 0$, the signs of $\left.\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{t=0}$, $i = x, y$, coincide with the signs of $H_{it}|_{t=0}$ by the S.O.C. of maximization, i.e., $H_{ii} < 0$, and $D > 0$. Next suppose that $H_{ij} < 0$, $i, j = x, y$, $i \neq j$ and $H_{it}|_{t=0} = 0$. Then the sign of $\left.\frac{\mathrm{d}j}{\mathrm{d}t}\right|_{t=0}$ coincides with that of $H_{jt}|_{t=0}$, note, however, that the sign of $\left.\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{t=0}$ is determined by the signs of H_{ij} and $H_{jt}|_{t=0}$.

Finally, suppose that $H_{xy} \neq 0$ and $H_{it}|_{t=0} \neq 0$, $i = x, y$. Then, if $H_{it}|_{t=0}$, $i = x, y$, have the same signs, such that $H_{it}|_{t=0} > 0 (< 0)$, $i = x, y$, and also if $H_{xy} \geq 0$, then $\left.\frac{\mathrm{d}i}{\mathrm{d}t}\right|_{t=0} > 0 (< 0)$; that is, the signs of the effects of an increase of risk on the decision variables are determined. On the other hand, if the signs of $H_{it}|_{t=0}$, $i = x, y$, are different such that $H_{xt}|_{t=0} > 0 (\leq 0)$ and $H_{yt}|_{t=0} < 0 (\geq 0)$, and also if $H_{xy} \leq 0$, then the signs of the comparative static analysis are determined to be $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} > 0 (\leq 0)$ and $\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=0} < 0 (\geq 0)$. In this way, the sign of H_{xy} is a sufficient condition to determine the signs of the comparative static analysis.

3. Main Results

In this section we show that, even if we put such a strict restriction on an increase of risk as the MPST, we cannot draw general conclusions about the effects of an increase of risk on decision variables. We must investigate the signs of $H_{it}|_{t=0}$, $i = x, y$ and H_{xy} in order to identify signs of comparative static analysis expressed by (4) even if the S.O.C. of maximization implies that $H_{ii} < 0$ and $D > 0$. To identify the signs, we make the following assumptions.

Assumption 1. $u_Z > 0, u_{ZZ} < 0$ *and* $A_Z \leq 0$.

Assumption 2. $Z_s > 0$ *and* $Z_{ss} \leq 0$.

Assumption 1 implies a gradual decrease of the coefficient of absolute risk aversion. This assumption is usually acceptable in economics. Further, if we note that $Z_s > 0$ does not lose generality and that many economic models have assumed that

1) See Ormiston (1992) for this treatment of an increase in risk.

$Z_{ss} = 0$, Assumption 2 is also not so restrictive. As for signs of $H_{it}|_{t=0}$, $i = x, y$, the next lemma holds.

Lemma 1. *Let the MPST represent an increase of risk. Then* $H_{it}|_{t=0} < (>)0$, $i = x, y$, *if* $Z_{is} > (<)0$ *and* $Z_{iss} \leq (\geq)0$. *Otherwise* $H_{it}|_{t=0} = 0$ *if* $Z_{is} = 0$.

Proof. At first we show $H_{it}|_{t=0} = 0$ if $Z_{is} = 0$. From the F.O.C. of maximization (1) and $u_Z > 0$, $Z_{is} = 0$ leads to $Z_i = 0$. Thus, if $Z_{is} = 0$, then $H_{it}|_{t=0} = 0$ from (3). We prove the remaining assertion of the lemma only in the case where $Z_{is} > 0$ and $Z_{iss} \leq 0$. Since the proof in the other case is quite similar, it is omitted. Let $\alpha(s) = u_Z(Z(s))Z_{is}(s)$[2]. According to the given assumptions, we can readily say that $\frac{\mathrm{d}\alpha(s)}{\mathrm{d}s} < 0$ for all s. From the definition of MPST, there exists s^K such that $K(s) \leq 0$ for $s \leq s^K$ and $K(s) > 0$ for $s > s^K$. Hence

$$\begin{aligned}\int \alpha K \mathrm{d}F &= \int^{s^K} \alpha(s)K(s)\mathrm{d}F(s) + \int_{s^K} \alpha(s)K(s)\mathrm{d}F(s)\\ &< \int^{s^K} \alpha(s^K)K(s)\mathrm{d}F(s) + \int_{s^K} \alpha(s^K)K(s)\mathrm{d}F(s)\\ &= \alpha(s^K)\int K(s)\mathrm{d}F(s).\end{aligned}$$

Since $\mathbb{E}[K(S)] = 0$ from the definition of MPST, we see that

$$\int u_Z Z_{is} K \mathrm{d}F = \int \alpha K \mathrm{d}F < 0. \tag{5}$$

Since $u_Z > 0$, $Z_{is} > 0$ for all s and $\mathbb{E}[u_Z Z_i] = 0$ from (1), there exists s^i such that $Z_i(s) \leq 0$ for $s \leq s^i$ and $Z_i(s) > 0$ for $s > s^i$. Define $M = \max\{s^K, s^i\}$. Since $A(s)u_Z(s)Z_i(s)Z_s(s)K(s) > 0$ for all $s > M$,

$$\int A u_Z Z_i Z_s K \mathrm{d}F > \int^M A u_Z Z_i Z_s K \mathrm{d}F. \tag{6}$$

Suppose $M = s^i$. Let $\beta(s) = A(s)u_Z(s)Z_i(s)Z_s(s)$. Since $\frac{\mathrm{d}\beta}{\mathrm{d}s} > 0$ for all $s \leq M$ under the given assumptions,

$$\begin{aligned}\int^M A u_Z Z_i Z_s K \mathrm{d}F &= \int^M \beta(s)K(s)\mathrm{d}F(s)\\ &= \int^{s^K} \beta(s)K(s)\mathrm{d}F(s) + \int_{s^K}^M \beta(s)K(s)\mathrm{d}F(s)\\ &> \int^{s^K} \beta(s^K)K(s)\mathrm{d}F(s) + \int_{s^K}^M \beta(s^K)K(s)\mathrm{d}F(s)\\ &= \beta(s^K)\int^M K\mathrm{d}F.\end{aligned}$$

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[2] Hereafter we suppress the arguments x, y of the function Z to simplify the notation.

Here, noting that $\int^M K\mathrm{d}F \leq 0$ by the definition of MPST and the definition of M, and that $\beta(s) \leq 0$ for all $s \leq M$, we get

$$\int^M Au_Z Z_i Z_s K\mathrm{d}F > 0. \tag{7}$$

On the other hand, suppose $M = s^K$. Let $\gamma(s) = A(s)Z_s(s)K(s)$. Then we can easily demonstrate that $\frac{\mathrm{d}\gamma}{\mathrm{d}s} > 0$ for all $s \leq M$. Hence

$$\begin{aligned}
\int^M Au_Z Z_i Z_s K\mathrm{d}F &= \int^M \gamma(s)Z_i(s)u_Z(s)\mathrm{d}F(s) \\
&= \int_{s^i}^M \gamma(s)Z_i(s)u_Z(s)\mathrm{d}F(s) + \int^{s^i} \gamma(s)Z_i(s)u_Z(s)\mathrm{d}F(s) \\
&> \int_{s^i}^M \gamma(s^i)Z_i(s)u_Z(s)\mathrm{d}F(s) + \int^{s^i} \gamma(s^i)Z_i(s)u_Z(s)\mathrm{d}F(s) \\
&= \gamma(s^i)\int^M Z_i(s)u_Z(s)\mathrm{d}F(s).
\end{aligned}$$

Since $\mathbb{E}[u_Z Z_i] = \int Z_i(s)u_Z(s)\mathrm{d}F(s) = 0$ from (1), $\int^M Z_i(s)u_Z(s)\mathrm{d}F(s) \leq 0$ by the definition of M. Further noting $\gamma(s) \leq 0$ for all $s \leq M$, we get

$$\int^M Au_Z Z_i Z_s K\mathrm{d}F > 0. \tag{8}$$

From (5), (6), (7) and (8) we can conclude that the lemma holds. □

This lemma provides a sufficient condition for identifying the signs of $H_{it}|_{t=0}$, $i = x, y$, under Assumption 1 and Assumption 2. Consider the case in which the decision variables are univariate and denoted by x; then the effects of an increase of risk (in the sense of MPST) on the variable x is given by

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} = -\frac{H_{xt}|_{t=0}}{H_{xx}}.$$

Therefore, we can deduce $\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} \leq (>)0$, since $H_{xt}|_{t=0} \leq (>)0$ by Lemma 1 under the assumption that $Z_{is} \geq (<)0$ and $Z_{iss} \leq (\geq)0$. In other words, we can determine the signs of the comparative static analysis. Even should decision variables be bivariate, if $H_{xy} = 0$ and if $Z_{is} \geq (<)0$ and $Z_{iss} \leq (\geq)0$, the signs of effects of an increase of risk on the decision variables (in the sense of MPST) i.e., $\frac{\mathrm{d}i}{\mathrm{d}t}|_{t=0}$, $i = x, y$, coincide with the signs of $H_{it}|_{t=0}$, $i = x, y$, and thus the signs of the comparative static analysis are determined. However, if $H_{xy} \neq 0$, the signs of the comparative statics depend on the sign of H_{xy}. To investigate this sign, the next lemma is useful.

Lemma 2. *Suppose the MPST represents an increase of risk. Then*

$$\mathbb{E}[Au_Z Z_x Z_y] \geq 0$$

if $Z_{is} \geq (\leq)0$ *for* $i = x, y$.

Proof. We first assume that $Z_{is} = 0$ holds. Then since $Z_i = 0$ from (1) and $u_Z > 0$, $\mathbb{E}[Au_Z Z_x Z_y] = 0$.

Next, we assume that $Z_{is} > 0$ holds. Since $u_Z(s) > 0$, $Z_{is}(s) > 0$, $i = x, y$ for all s and $\mathbb{E}[u_Z Z_i] = 0$ from (1), there exists s^i such that $Z_i(s) \leq 0$ for $s \leq s^i$ and $Z_i(s) > 0$ for $s > s^i$.

Define $m = \max\{s^x, s^y\}$. Since $A(s)u_Z(s)Z_x(s)Z_y(s) \geq 0$ for all $s \geq m$,

$$\mathbb{E}[Au_Z Z_x Z_y] = \int Au_Z Z_x Z_y \mathrm{d}F \geq \int^m Au_Z Z_x Z_y \mathrm{d}F. \tag{9}$$

Here we suppose $m = s^y$. Let $\delta(s) = A(s)Z_y(s)$. Since $\mathrm{d}\delta(s)/\mathrm{d}s > 0$ for $s \leq m$,

$$\begin{aligned}
\int^m Au_Z Z_x Z_y \mathrm{d}F &= \int^m \delta(s)u_Z(s)Z_x(s)\mathrm{d}F(s) \qquad (10)\\
&= \int^{s^x} \delta(s)u_Z(s)Z_x(s)\mathrm{d}F(s) \\
&\quad + \int_{s^x}^m \delta(s)u_Z(s)Z_x(s)\mathrm{d}F(s) \\
&> \int^{s^x} \delta(s^x)u_Z(s)Z_x(s)\mathrm{d}F(s) \\
&\quad + \int_{s^x}^m \delta(s^x)u_Z(s)Z_x(s)\mathrm{d}F(s) \\
&= \delta(s^x)\int^m u_Z Z_x \mathrm{d}F.
\end{aligned}$$

Noting $Z_x(s) \geq 0$ for $s \geq m$, we get $\int^m u_Z Z_x \mathrm{d}F \leq 0$ from (1). Further, since $\delta(s) \leq 0$ for all $s \leq m$, combining (9) with (10), we obtain $\mathbb{E}[Au_Z Z_x Z_y] > 0$.

Whenever $m = s^x$, we only have to exchange the subscripts x and y in the above argument to arrive at the same conclusion. We can also demonstrate the lemma when $Z_{is} < 0$ by an argument quite similar to the above. □

When $Z_{is} \geq (<)0$ and $Z_{iss} \leq (\geq)0$ hold for $i = x, y$, the signs of $H_{it}|_{t=0}$ coincide from Lemma 1. Furthermore, this lemma insists on $\mathbb{E}[Au_Z Z_x Z_y] \geq 0$ in this case.

When the signs of Z_{is}, $i = x, y$, differ between Z_{xs} and Z_{ys} the next lemma holds.

Lemma 3. *Let the MPST represent an increase of risk. Then*

$$\mathbb{E}[Au_Z Z_x Z_y] \leq 0$$

if $Z_{xs} \geq (\leq)0$ *and* $Z_{ys} \leq (\geq)0$.

Proof. The proof is quite similar to that of Lemma 2, so, it is omitted. □

On the basis of the above lemmas, we will discuss the signs of the comparative statics given by (4). As we saw in the preceding section, whenever $H_{xy} \neq 0$ and $H_{it}|_{t=0} \neq 0$ if the signs of $H_{it}|_{t=0}$ (do not) coincide and $H_{xy} > (<)0$, then the signs of the comparative statics can be determined. The sign of $H_{it}|_{t=0} \neq 0$ depends on the signs of Z_{is} and Z_{iss}.

Assume that the signs of $H_{it}|_{t=0} \neq 0\,(i = x, y)$ are the same when signs of Z_{is} and Z_{iss} are given. Then even if $Z_{xy} > 0$ holds, $H_{xy} > 0$ does not always hold, since

$$H_{xy} = \mathbb{E}[u_Z Z_{xy} + u_{ZZ} Z_x Z_y] = \mathbb{E}[u_Z Z_{xy}] - \mathbb{E}[A u_Z Z_x Z_y] \tag{11}$$

and $\mathbb{E}[A u_Z Z_x Z_y]$ can be non-negative from Lemma 2. Therefore, the effect of the MPST on the decision variables x, y can not be determined.

Similarly, if both signs of $H_{it}|_{t=0} \neq 0\,(i = x, y)$ are different when the signs of Z_{is} and Z_{iss} are given, $H_{xy} < 0$ does not always holds even if $Z_{xy} < 0$, since, by Lemma 3, $\mathbb{E}[A u_Z Z_x Z_y]$ can be non-positive. Hence we can derive the next proposition.

Proposition 1. *We cannot decide the effects of an increase of risk in the sense of MPST on the decision variables x, y only on the basis of the information of the signs of Z_{xy}, Z_{is}, and Z_{iss}, $i = x, y$, under Assumption 1 and Assumption 2.*

4. Some Special Cases

Through the proposition in the previous section, it becomes clear that we need more restrictive conditions than the signs of Z_{xy}, Z_{is}, and Z_{iss} to determine the effects of the MPST on the decision variables x and y under Assumption 1 and Assumption 2. In this section, we discuss some exceptions in which we can determine the effects without additional conditions. In both special cases, we assume a monopoly firm that is trying to maximize its utility through operating such decision variables as product price x and product quality y of the product.

4.1. Where the Pay-Off Function is a Multiplier of the Random Variables

Suppose the firm's profit function is represented as:

$$Z(x, y, s) = (x - C(y))Q(x, y)s, \quad Q_x < 0,\ Q_y > 0 \text{ and } C_y > 0,$$

where C and Q denote the variable cost and the sales volume, respectively. We assume $S > 0$ w.p.1 in this subsection.

Since

$$Z_x(s) = ((x - C)Q_x + Q)s, \quad Z_y(s) = ((x - C)Q_y - C_y Q)s,$$

noting $u_Z > 0$ and $S > 0$ w.p.1, we obtain

$$(x - C)Q_x + Q = (x - C)Q_y - C_y Q = 0$$

from the F.O.C. of maximization (1). Accordingly,

$$Z_x(s) = ((x - C)Q_x + Q)s = 0,\ Z_y(s) = ((x - C)Q_y - C_y Q)s = 0 \quad \forall\, s.$$

Further, these equations lead to

$$Z_{xs}(s) = (x - C)Q_x + Q = 0,\ Z_{ys}(s) = (x - C)Q_y - C_y Q = 0 \quad \forall\, s.$$

Therefore, we see that

$$H_{xt}|_{t=0} = H_{yt}|_{t=0} = 0$$

from (3). Hence, from (4), we can conclude that

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} = \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=0} = 0.$$

4.2. Where the Fixed Cost Depends Only on the Quality: Ibarra-Salazar (1995)

Suppose the firm's profit function is represented as:

$$Z(x, y, s) = (x - C)(Q(x, y) + s) - f(y),\ Q_x < 0,\ Q_y > 0 \text{ and } f_y > 0,$$

where f denotes the fixed cost and C is assumed to be a constant.

Since $Z_y = (x - C)Q_y - f_y$ does not depend on the random variable S, $Z_{ys} = 0$. This leads to $H_{yt}|_{t=0} = 0$ from Lemma 1.

On the other hand, since $Z_x = (x - C)Q_x + Q + s$, $Z_{xs} = 1 > 0$ and $Z_{xss} = 0$. This leads to $H_{xt}|_{t=0} < 0$ from Lemma 1. Further, since $Z_y = 0$ and $u_Z > 0$, $H_{xy} \leq (\geq)0 \iff Z_{xy} \leq (\geq)0$ from (11). Therefore, the effects of an increase of risk (in the sense of the MPST) on the decision variables x and y can be represented as:

$$\begin{aligned} \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} &= -\frac{1}{D}H_{yy}H_{xt}|_{t=0} < 0, \\ \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=0} &= \frac{1}{D}H_{xy}H_{xt}|_{t=0} \geq (\leq)0 \iff Z_{xy} \leq (\geq)0. \end{aligned}$$

5. Concluding Remarks

In this paper we discussed the effects of an increase of risk on decision making. We showed in Section 3 that we cannot generally determine the signs of the comparative static analysis only on the basis of the signs of Z_{xy}, Z_{is} and Z_{iss}, $i = x, y$. In Section 4 we considered some cases in which we can determine the signs of comparative static analysis only on the basis of these signs. However, these cases are exceptional. For example, suppose the firm's profit function is represented as:

$$Z(x, y, s) = (x - C(y))(Q(x, y) + s),\ Q_x < 0,\ Q_y > 0 \text{ and } C_y > 0.$$

In this case, we can readily show that $-H_{yt}|_{t=0} = C_y H_{xt}|_{t=0}$ from the F.O.C. of maximization (1). Substituting this equation in (4), we obtain

$$\begin{aligned} \left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{t=0} &= -\frac{1}{D}(C_y H_{xy} + H_{yy})H_{xt}|_{t=0}, \\ \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=0} &= \frac{1}{D}(H_{xy} + C_y H_{xx})H_{xt}|_{t=0}. \end{aligned}$$

Therefore, $\mathrm{d}x/\mathrm{d}t|_{t=0} < 0$ and $\mathrm{d}y/\mathrm{d}t|_{t=0} > 0$ if and only if $H_{xy} < 0$ from Lemma 1. However, we can also show $C_y Z_x = -Z_y$. This leads to

$$\mathbb{E}[Au_Z Z_x Z_y] \leq 0$$

since $C_y > 0$ and Assumption 1. Thus, from (11), even if $Z_{xy} < 0$, $H_{xy} < 0$ does not always hold. That is, we can no longer determine the signs of the comparative static analysis without adding further restrictions.

Acknowledgements

This research was partially supported by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), Grant-in-Aid for 21st Century COE Program.

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