Delay-dependent Stability Analysis and Stabilization of Discrete-time Singular Delay Systems

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Abstract In this paper, delay-dependent stability analysis and stabilization of discrete-time singular delay systems are addressed, respectively. First, a new delay-dependent sufficient condition of admissibility for discrete-time singular delay systems is derived. The proposed method is proved to have some advantages over the existing results. Then, by applying the skill of matrix theory, a state feedback controller is designed to guarantee the closed-loop discrete-time singular delay systems to be admissible. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Key words Discrete-time singular delay systems, delay-dependent stability, stabilization, admissible, state feedback control

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Control of singular systems has been extensively studied in the past years due to the fact that singular systems describe physical systems better than regular ones. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems, or semistate systems whose behaviors are described by differential equations (or difference equations) and algebraic equations. Such systems can preserve the structure of practical systems and have extensive applications in power systems, robotic systems, and networks[1]. However, the control of singular systems cannot be easily treated as that of regular systems. It is well known that study of singular systems is much more complicated than that of regular ones. Recently, more and more attention has been paid to stability and stabilization of singular systems[2−12]. Based on the results, extensive research on H_∞ performance analysis and H_∞ control for singular systems has been carried out[13−18]. Delay is often encountered in various engineering systems, and the existence of delay is frequently a source of instability and poor performance. According to whether dependent on delay or not, the control of singular systems can be respectively classified into two types: delay-dependent[3−8, 16−18] and delay-independent[9−12]. Generally, delay-dependent results are less conservative than the delay-independent ones, especially when the upper bound of the delay is small. If there is no information about the delay, delay-independent results are more useful than delay-dependent ones.

It is noted that stability analysis and stabilization of discrete-time singular delay systems have been focused on in the literature. Many delay-dependent stability criteria for discrete-time singular delay systems have been established and some redundant variables are introduced to decrease conservatism, thus, computation complexity is increased correspondingly. Moreover, it encounters considerable difficulty in designing state feedback controller, such as extremely tedious computation, non-strict linear matrix inequality (LMI), system transformation, etc. Motivated by the above-mentioned factors, in this paper, a novel Lyapunov function is established and a new delay-dependent sufficient condition of admissibility for discrete-time singular delay systems is derived. With respect to the conservatism and computation complexity, the new criterion is proved to have some advantages over the existing results. Then, by utilizing the skill of matrix theory, a simple and efficient approach is proposed to design state feedback controller which ensures the closed-loop discrete-time singular delay systems to be admissible. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Notations. Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{m\times n}$ is the set of all $m\times n$ real matrices. The notation $X>0$ means that X is a positive definite matrix, and $X>Y$ means that $X-Y$ is a positive definite matrix. I denotes an identity matrix with appropriate dimension. The superscript "T" represents transpose of a matrix. The symbol "*" represents the transposed elements in a symmetric matrix.

1 Problem formulation and preliminaries

Consider a discrete-time singular delay system described by

$$\begin{cases} E\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d) + B\boldsymbol{u}(k) \\ \boldsymbol{x}(i) = \boldsymbol{\phi}(i), \quad i=-d, -d+1, \cdots, 0 \end{cases} \tag{1}$$

where $\boldsymbol{x}(k)\in\mathbf{R}^n$ is the state vector, $\boldsymbol{u}(k)\in\mathbf{R}^p$ is the control input. $E\in\mathbf{R}^{n\times n}$ may be singular, rank$E=r\le n$. A, A_d, and B are known constant matrices with appropriate dimensions. The scalar d represents the delay of the system, satisfying $0<d\le d_M$, where d, d_M are positive integers and d_M is the upper bound of delay. $\boldsymbol{\phi}(i)$ is an initial condition.

Consider an unforced discrete-time singular delay system described by

$$E\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d) \tag{2}$$

Definition 1[19]**.** 1) System (2) is said to be regular if $\det(z^{d+1}E - z^dA - A_d)$ is not identically zero; 2) System (2) is said to be causal if it is regular and $\deg(z^{nd}\det(zE-A-z^{-d}A_d)) = nd + \mathrm{rank}(E)$; 3) Let $\rho(E,A,A_d)=\max_{\lambda\in\{z|\det(z^{d+1}E-z^dA-A_d)=0\}} \mid\lambda\mid$ and system (2) is said to be stable if $\rho(E,A,A_d)<1$; 4) System (2) is said to be admissible if it is regular, causal, and stable.

Definition 2[10]**.** The discrete-time singular delay system (2) is regular, causal, and stable if and only if the pair (E,A) is regular, causal, and $\rho(E,A,A_d)<1$.

Lemma 1[20]**.** For any constant matrix $M\ge 0$, $M\in\mathbf{R}^{n\times n}$, $\boldsymbol{\psi}(i)\in\mathbf{R}^n$, positive integers β_1, β_2, and $\beta_2\ge\beta_1\ge 1$, the following inequality holds:

$$-(\beta_2-\beta_1+1)\sum_{i=\beta_1}^{\beta_2}\boldsymbol{\psi}^{\mathrm{T}}(i)M\boldsymbol{\psi}(i)\le$$

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$$-\left(\sum_{i=\beta_1}^{\beta_2}\boldsymbol{\psi}(i)\right)^{\mathrm{T}} M\left(\sum_{i=\beta_1}^{\beta_2}\boldsymbol{\psi}(i)\right) \quad (3)$$

Lemma 2[6]**.** The matrix E in system (2) is nonsingular and system (2) is asymptotically stable for any constant delay d satisfying $0 \le d \le d_M$, if for a given scalar $d_M > 0$, there exist matrices $X > 0$, $Z > 0$, $U > 0$, and N_1, N_2 satisfying the following LMI:

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{21}^{\mathrm{T}} & d_M N_1 \\ \Lambda_{21} & \Lambda_{22} & d_M N_2 \\ d_M N_1^{\mathrm{T}} & d_M N_2^{\mathrm{T}} & -d_M Z \end{bmatrix} < 0 \quad (4)$$

where

$$\begin{aligned}\Lambda_{11} &= A^{\mathrm{T}}XA - E^{\mathrm{T}}XE + U + N_1E + E^{\mathrm{T}}N_1^{\mathrm{T}} + \\ &\quad d_M(A-E)^{\mathrm{T}}Z(A-E)\\ \Lambda_{21} &= A_d^{\mathrm{T}}XA + d_MA_d^{\mathrm{T}}Z(A-E) - E^{\mathrm{T}}N_1^{\mathrm{T}} + N_2E\\ \Lambda_{22} &= A_d^{\mathrm{T}}XA_d + d_MA_d^{\mathrm{T}}ZA_d - U - N_2E - E^{\mathrm{T}}N_2^{\mathrm{T}} \quad (5)\end{aligned}$$

2 Main results

2.1 Delay-dependent admissibility analysis

Theorem 1. Given a scalar $d_M > 0$, for any delay $0 < d \le d_M$, the discrete-time singular delay system (2) is admissible if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $P, Q, R \in \mathbf{R}^{n\times n}$ and a symmetric matrix $\Phi \in \mathbf{R}^{(n-r)\times(n-r)}$ such that

$$\begin{bmatrix} \Psi_{11} & \Psi_{21}^{\mathrm{T}} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} < 0 \quad (6)$$

where

$$\begin{aligned}\Psi_{11} &= A^{\mathrm{T}}XA - E^{\mathrm{T}}PE + Q - \frac{E^{\mathrm{T}}RE}{d_M} + \\ &\quad d_M(A-E)^{\mathrm{T}}R(A-E)\\ \Psi_{21} &= A_d^{\mathrm{T}}XA + d_MA_d^{\mathrm{T}}R(A-E) + \frac{E^{\mathrm{T}}RE}{d_M}\\ \Psi_{22} &= A_d^{\mathrm{T}}XA_d - Q + d_MA_d^{\mathrm{T}}RA_d - \frac{E^{\mathrm{T}}RE}{d_M}\\ X &= P - S^{\mathrm{T}}\Phi S \quad (7)\end{aligned}$$

Matrix $S \in \mathbf{R}^{(n-r)\times n}$ is of full row rank and satisfying $SE = 0$.

Proof. The proof is divided into two parts. The first one deals with the regularity and causality, and the second one treats the stability.

First, we show system (2) is regular and causal. There exist two nonsingular matrices $\hat{M}$ and $\hat{N}$ such that

$$\hat{M}E\hat{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{M}A\hat{N} = \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \\ \hat{A}_3 & \hat{A}_4 \end{bmatrix} \quad (8)$$

Correspondingly,

$$\hat{M}A_d\hat{N} = \begin{bmatrix} \hat{A}_{d1} & \hat{A}_{d2} \\ \hat{A}_{d3} & \hat{A}_{d4} \end{bmatrix} \quad (9)$$

Let

$$\hat{M}^{-\mathrm{T}}P\hat{M}^{-1} = \begin{bmatrix} \hat{P}_1 & \hat{P}_2 \\ \hat{P}_2^{\mathrm{T}} & \hat{P}_3 \end{bmatrix}, \hat{M}^{-\mathrm{T}}R\hat{M}^{-1} = \begin{bmatrix} \hat{R}_1 & \hat{R}_2 \\ \hat{R}_2^{\mathrm{T}} & \hat{R}_3 \end{bmatrix}$$
$$\hat{M}^{-\mathrm{T}}X\hat{M}^{-1} = \begin{bmatrix} \hat{X}_1 & \hat{X}_2 \\ \hat{X}_2^{\mathrm{T}} & \hat{X}_3 \end{bmatrix}, \hat{N}^{\mathrm{T}}Q\hat{N} = \begin{bmatrix} \hat{Q}_1 & \hat{Q}_2 \\ \hat{Q}_2^{\mathrm{T}} & \hat{Q}_3 \end{bmatrix} \quad (10)$$

The partitions of matrix blocks in (9) and (10) are compatible with those in (8). By (6), it is easy to see that

$$\Psi_{11} < 0 \quad (11)$$

Pre- and post-multiplying (11) by $\hat{N}^{\mathrm{T}}$ and $\hat{N}$, respectively, using the expressions in (8) $\sim$ (10), we get

$$\begin{bmatrix} \star & \star \\ \star & \Psi_{11}^{22} \end{bmatrix} < 0 \quad (12)$$

where "$\star$" stands for a matrix that is irrelevant to the following development. Ψ_{11}^{22} represents the $(2,2)$ block of Ψ_{11}.

$$\begin{aligned}\Psi_{11}^{22} &= \hat{A}_2^{\mathrm{T}}\hat{X}_2\hat{A}_4 + \hat{A}_4^{\mathrm{T}}\hat{X}_2^{\mathrm{T}}\hat{A}_2 + \hat{A}_4^{\mathrm{T}}\hat{X}_3\hat{A}_4 + \\ &\quad \hat{A}_2^{\mathrm{T}}\hat{X}_1\hat{A}_2 + \hat{Q}_3 + J\\ J &= d_M\begin{bmatrix} \hat{A}_2^{\mathrm{T}} & \hat{A}_4^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \hat{R}_1 & \hat{R}_2 \\ \hat{R}_2^{\mathrm{T}} & \hat{R}_3 \end{bmatrix}\begin{bmatrix} \hat{A}_2 \\ \hat{A}_4 \end{bmatrix}\end{aligned}$$

By (12), we have

$$\Psi_{11}^{22} < 0 \quad (13)$$

It is noted that

$$E^{\mathrm{T}}XE = E^{\mathrm{T}}(P - S^{\mathrm{T}}\Phi S)E = E^{\mathrm{T}}PE \ge 0 \quad (14)$$

Pre- and post-multiplying (14) by $\hat{N}^{\mathrm{T}}$ and $\hat{N}$, respectively, using the expressions in (8) and (10), we get

$$\begin{bmatrix} \hat{X}_1 & 0 \\ 0 & 0 \end{bmatrix} \ge 0 \quad (15)$$

So $\hat{X}_1 \ge 0$, i.e., $\hat{A}_2^{\mathrm{T}}\hat{X}_1\hat{A}_2 \ge 0$. From $R > 0$, $Q > 0$, using the expressions in (8) and (10), we get $J \ge 0$, $\hat{Q}_3 > 0$, hence,

$$\hat{A}_2^{\mathrm{T}}\hat{X}_1\hat{A}_2 + \hat{Q}_3 + J > 0 \quad (16)$$

By (13), we have

$$\hat{A}_4^{\mathrm{T}}\hat{X}_2^{\mathrm{T}}\hat{A}_2 + \hat{A}_4^{\mathrm{T}}\hat{X}_3\hat{A}_4 + \hat{A}_2^{\mathrm{T}}\hat{X}_2\hat{A}_4 < 0 \quad (17)$$

i.e.,

$$\hat{A}_4^{\mathrm{T}}\left(\hat{X}_2^{\mathrm{T}}\hat{A}_2 + \frac{1}{2}\hat{X}_3\hat{A}_4\right) + \left(\hat{X}_2^{\mathrm{T}}\hat{A}_2 + \frac{1}{2}\hat{X}_3\hat{A}_4\right)^{\mathrm{T}}\hat{A}_4 < 0 \quad (18)$$

From (18), it is seen that $\hat{A}_4$ is nonsingular, thus, (E, A) is regular and causal. By Definition 2, system (2) is regular and causal. Next, we prove the stability.

Choose a Lyapunov functional candidate for system (2):

$$V(k) = V_1(k) + V_2(k) + V_3(k) \quad (19)$$

where

$$V_1(k) = \boldsymbol{x}^{\mathrm{T}}(k)E^{\mathrm{T}}XE\boldsymbol{x}(k) = \boldsymbol{x}^{\mathrm{T}}(k)E^{\mathrm{T}}(P - S^{\mathrm{T}}\Phi S)E\boldsymbol{x}(k) = \boldsymbol{x}^{\mathrm{T}}(k)E^{\mathrm{T}}PE\boldsymbol{x}(k)$$

$$V_2(k) = \sum_{i=k-d}^{k-1} \boldsymbol{x}^{\mathrm{T}}(i)Q\boldsymbol{x}(i)$$

$$V_3(k) = \sum_{i=-d}^{-1}\sum_{j=k+i}^{k-1} \boldsymbol{y}^{\mathrm{T}}(j)E^{\mathrm{T}}RE\boldsymbol{y}(j)$$

where $\boldsymbol{y}(j) = \boldsymbol{x}(j+1) - \boldsymbol{x}(j)$. Define $\Delta V(k) = V(k+1) - V(k)$, and we have

$$\begin{aligned}\Delta V_1(k) = &\boldsymbol{x}^{\mathrm{T}}(k+1)E^{\mathrm{T}}XE\boldsymbol{x}(k+1) - \boldsymbol{x}^{\mathrm{T}}(k)E^{\mathrm{T}}PE\boldsymbol{x}(k) = \\ &(A\boldsymbol{x}(k)+A_d\boldsymbol{x}(k-d))^{\mathrm{T}}X(A\boldsymbol{x}(k)+A_d\boldsymbol{x}(k-d))- \\ &\boldsymbol{x}^{\mathrm{T}}(k)E^{\mathrm{T}}PE\boldsymbol{x}(k) = \boldsymbol{x}^{\mathrm{T}}(k)(A^{\mathrm{T}}XA-E^{\mathrm{T}}PE)\times \\ &\boldsymbol{x}(k) + 2\boldsymbol{x}^{\mathrm{T}}(k)A^{\mathrm{T}}XA_d\boldsymbol{x}(k-d)+ \\ &\boldsymbol{x}^{\mathrm{T}}(k-d)A_d^{\mathrm{T}}XA_d\boldsymbol{x}(k-d)\end{aligned} \tag{20}$$

$$\Delta V_2(k) = \boldsymbol{x}^{\mathrm{T}}(k)Q\boldsymbol{x}(k) - \boldsymbol{x}^{\mathrm{T}}(k-d)Q\boldsymbol{x}(k-d) \tag{21}$$

$$\begin{aligned}\Delta V_3(k) = &\sum_{i=-d}^{-1}\Big(\boldsymbol{y}^{\mathrm{T}}(k)E^{\mathrm{T}}RE\boldsymbol{y}(k)- \\ &\boldsymbol{y}^{\mathrm{T}}(k+i)E^{\mathrm{T}}RE\boldsymbol{y}(k+i)\Big) = \\ &d\boldsymbol{y}^{\mathrm{T}}(k)E^{\mathrm{T}}RE\boldsymbol{y}(k)- \\ &\sum_{i=-d}^{-1}\boldsymbol{y}^{\mathrm{T}}(k+i)E^{\mathrm{T}}RE\boldsymbol{y}(k+i)\end{aligned} \tag{22}$$

$$\begin{aligned}&d\boldsymbol{y}^{\mathrm{T}}(k)E^{\mathrm{T}}RE\boldsymbol{y}(k) \le \\ &\quad d_M\boldsymbol{y}^{\mathrm{T}}(k)E^{\mathrm{T}}RE\boldsymbol{y}(k) = \\ &\quad d_M(E\boldsymbol{x}(k+1) - E\boldsymbol{x}(k))^{\mathrm{T}}R(E\boldsymbol{x}(k+1) - E\boldsymbol{x}(k)) = \\ &\quad d_M((A-E)\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d))^{\mathrm{T}}\times \\ &\quad R((A-E)\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d))\end{aligned} \tag{23}$$

By Lemma 1,

$$\begin{aligned}&-\sum_{i=-d}^{-1}\boldsymbol{y}^{\mathrm{T}}(k+i)E^{\mathrm{T}}RE\boldsymbol{y}(k+i) \le \\ &\quad -\left(\sum_{j=k-d}^{k-1}\boldsymbol{y}(j)\right)^{\mathrm{T}}\frac{E^{\mathrm{T}}RE}{d}\left(\sum_{j=k-d}^{k-1}\boldsymbol{y}(j)\right) \le \\ &\quad -\left(\sum_{j=k-d}^{k-1}\boldsymbol{y}(j)\right)^{\mathrm{T}}\frac{E^{\mathrm{T}}RE}{d_M}\left(\sum_{j=k-d}^{k-1}\boldsymbol{y}(j)\right) = \\ &\quad -(\boldsymbol{x}(k) - \boldsymbol{x}(k-d))^{\mathrm{T}}\frac{E^{\mathrm{T}}RE}{d_M}(\boldsymbol{x}(k) - \boldsymbol{x}(k-d))\end{aligned} \tag{24}$$

From (22) $\sim$ (24), we have

$$\begin{aligned}\Delta V_3(k) \le &\boldsymbol{x}^{\mathrm{T}}(k)\left(d_M(A-E)^{\mathrm{T}}R(A-E)-\frac{E^{\mathrm{T}}RE}{d_M}\right)\boldsymbol{x}(k)+ \\ &2\boldsymbol{x}^{\mathrm{T}}(k)\left(d_M(A-E)^{\mathrm{T}}RA_d+\frac{E^{\mathrm{T}}RE}{d_M}\right)\boldsymbol{x}(k-d)+ \\ &\boldsymbol{x}^{\mathrm{T}}(k-d)\left(d_MA_d^{\mathrm{T}}RA_d - \frac{E^{\mathrm{T}}RE}{d_M}\right)\boldsymbol{x}(k-d)\end{aligned} \tag{25}$$

From (20) $\sim$ (25), we have

$$\Delta V(k) \le \boldsymbol{\zeta}^{\mathrm{T}}(k)\begin{bmatrix}\Psi_{11} & \Psi_{21}^{\mathrm{T}} \\ \Psi_{21} & \Psi_{22}\end{bmatrix}\boldsymbol{\zeta}(k) \tag{26}$$

where $\boldsymbol{\zeta}(k) = \begin{bmatrix}\boldsymbol{x}^{\mathrm{T}}(k) & \boldsymbol{x}^{\mathrm{T}}(k-d)\end{bmatrix}^{\mathrm{T}}$. By (6), we know $\Delta V(k) < 0$, then, system (2) is stable. □

Remark 1. Theorem 1 gives a new delay-dependent sufficient condition of admissibility for system (2). It is noted that inequality (6) is a strict LMI which is convenient to solve the matrix variables. Another important characteristic of Theorem 1 is to introduce a symmetric matrix variable Φ and the matrix S with the property of $SE = 0$, combined into the term $X = P - S^{\mathrm{T}}\Phi S$. Thus, the degree of freedom for matrix variable $P > 0$ is increased, and this would lead to less conservatism. In the section of numerical examples, it is seen that the results of Theorem 1 are less conservative than those of [8].

Remark 2. It is noted that there are no redundant variables to be introduced in Theorem 1, so the numerical complexity is small. The numerical complexity is closely related to the number of decision variables and the number of lines in the LMIs to be solved[21]. Table 1 shows the numbers of decision variables and the number of lines for two results. "Th." is the abbreviation for "Theorem". Similarly to the method adopted in [21], with complexity proportional to $\mathcal{C} = \mathcal{D}^3\mathcal{L}$, where $\mathcal{D}$ represents the number of decision variables and $\mathcal{L}$ represents the number of lines in the LMIs to be solved, we study the quantity $\mathcal{C} = \mathcal{D}^3\mathcal{L}$ for Th. 1 in this paper and Th. 1 in [8]. $(\mathcal{D}^3\mathcal{L})_1$ represents the quantity for Th. 1 in [8] and $(\mathcal{D}^3\mathcal{L})_2$ represents the quantity for Th. 1 in this paper. The ratio between them, denoted by $\mathcal{R} = (\mathcal{D}^3\mathcal{L})_1/(\mathcal{D}^3\mathcal{L})_2$, is depicted in Fig. 1, for various numbers of r. From Fig. 1, it is obvious that in all cases the complexity associated to Th. 1 in [8] is larger than the one associated to Th. 1 in this paper and it demonstrates that the proposed method in this paper is simpler.

Table 1 Numbers of decision variables and lines in the LMIs to be solved in Th. 1 in this paper and Th. 1 in [8]

	Decision variables ($\mathcal{D}$)	Lines ($\mathcal{L}$)
Th. 1 in [8]	$10.5n^2 + (1.5 - 3r)n$	$4n$
Th. 1 in this paper	$2n^2 + (2 - r)n + 0.5(r^2 - r)$	$2n$

If the matrix E in system (2) is nonsingular, then the matrix S is a null matrix in (6). By Theorem 1, we have the following results.

Corollary 1. Assume that the matrix E in system (2) is nonsingular. Given a scalar $d_M > 0$, for any delay $0 < d \le d_M$, the discrete-time delay system (2) is asymptotically stable if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $P, Q, R \in \mathbf{R}^{n\times n}$ such that

$$\begin{bmatrix} \tilde{\Psi}_{11} & \tilde{\Psi}_{21}^{\mathrm{T}} \\ \tilde{\Psi}_{21} & \tilde{\Psi}_{22} \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} \tilde{\Psi}_{11} &= A^{\mathrm{T}}PA - E^{\mathrm{T}}PE + Q - \frac{E^{\mathrm{T}}RE}{d_M} + \\ &\quad d_M(A-E)^{\mathrm{T}}R(A-E) \\ \tilde{\Psi}_{21} &= A_d^{\mathrm{T}}PA + d_M A_d^{\mathrm{T}}R(A-E) + \frac{E^{\mathrm{T}}RE}{d_M} \\ \tilde{\Psi}_{22} &= A_d^{\mathrm{T}}PA_d - Q + d_M A_d^{\mathrm{T}}RA_d - \frac{E^{\mathrm{T}}RE}{d_M} \end{aligned} \tag{28}$$

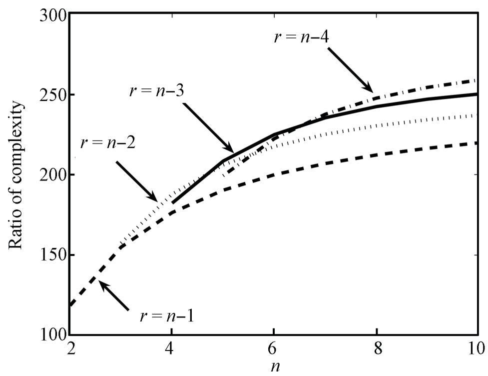

Fig. 1 Evolution of the ratio between the complexity resulting from Th. 1 in [8] and Th. 1 in this paper with respect to n for different r

Remark 3. It is proved that if matrix E in system (2) is nonsingular, the results of Corollary 1 are equivalent to the results of Lemma 2. The proof is given in Appendix. Now, assume that the matrix E is singular, it is proved that Lemma 2 is invalid for discrete-time singular delay system (2). In [6], instead, by introducing augmented state vectors, the troubles caused by the singular matrix E are tackled. It is well known that augmented vectors would increase the dimensions of the system and lead to extremely tedious computation. Compared with the results of [6], Theorem 1 is valid in either case, thus, the proposed method is simple and efficient.

2.2 Controller design

Replacing E, A, A_d with E^{T}, A^{T}, A_d^{T} in (2), we have

$$E^{\mathrm{T}}\boldsymbol{x}(k+1) = A^{\mathrm{T}}\boldsymbol{x}(k) + A_d^{\mathrm{T}}\boldsymbol{x}(k-d) \tag{29}$$

It is obvious that system (29) is admissible if and only if system (2) is admissible. Theorem 1 is rewritten as another form.

Corollary 2. Given a scalar $d_M > 0$, for any delay $0 < d \le d_M$, the discrete-time singular delay system (2) is admissible if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $P, Q, R \in \mathbf{R}^{n\times n}$ and a symmetric matrix $\Phi \in \mathbf{R}^{(n-r)\times(n-r)}$ such that

$$\begin{bmatrix} \Delta_{11} & \Delta_{21}^{\mathrm{T}} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} < 0 \tag{30}$$

where

$$\begin{aligned} \Delta_{11} &= A\mathcal{X}A^{\mathrm{T}} - EPE^{\mathrm{T}} + Q - \frac{ERE^{\mathrm{T}}}{d_M} + \\ &\quad d_M(A-E)R(A-E)^{\mathrm{T}} \\ \Delta_{21} &= A_d\mathcal{X}A^{\mathrm{T}} + d_M A_d R(A-E)^{\mathrm{T}} + \frac{ERE^{\mathrm{T}}}{d_M} \\ \Delta_{22} &= A_d\mathcal{X}A_d^{\mathrm{T}} - Q + d_M A_d R A_d^{\mathrm{T}} - \frac{ERE^{\mathrm{T}}}{d_M} \\ \mathcal{X} &= P - L\Phi L^{\mathrm{T}} \end{aligned} \tag{31}$$

Matrix $L \in \mathbf{R}^{n\times(n-r)}$ is of full column rank and satisfying $EL = 0$.

The following results are now available to design the controller.

Theorem 2. Given a scalar $d_M > 0$, for any delay $0 < d \le d_M$, the discrete-time singular delay system (2) is admissible if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $P, Q, R \in \mathbf{R}^{n\times n}$ and a symmetric matrix $\Phi \in \mathbf{R}^{(n-r)\times(n-r)}$, matrices Y_i, $i = 1, \cdots, 8$, $Y_i \in \mathbf{R}^{n\times n}$, such that

$$\Sigma = \begin{bmatrix} \Sigma_{11} & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} & * \\ \Sigma_{41} & \Sigma_{42} & \Sigma_{43} & \Sigma_{44} \end{bmatrix} < 0 \tag{32}$$

where

$$\begin{aligned} \Sigma_{11} &= -EPE^{\mathrm{T}} + Q + \left(d_M - \frac{1}{d_M}\right) ERE^{\mathrm{T}} + AY_1 + Y_1^{\mathrm{T}}A^{\mathrm{T}} \\ \Sigma_{21} &= \frac{ERE^{\mathrm{T}}}{d_M} + Y_2^{\mathrm{T}}A^{\mathrm{T}} + A_dY_5 \\ \Sigma_{22} &= -Q - \frac{ERE^{\mathrm{T}}}{d_M} + A_dY_6 + Y_6^{\mathrm{T}}A_d^{\mathrm{T}} \\ \Sigma_{31} &= -d_M RE^{\mathrm{T}} + Y_3^{\mathrm{T}}A^{\mathrm{T}} - Y_1 \\ \Sigma_{32} &= Y_7^{\mathrm{T}}A_d^{\mathrm{T}} - Y_2 \\ \Sigma_{33} &= \mathcal{X} + d_M R - Y_3 - Y_3^{\mathrm{T}} \\ \Sigma_{41} &= -d_M RE^{\mathrm{T}} + Y_4^{\mathrm{T}}A^{\mathrm{T}} - Y_5 \\ \Sigma_{42} &= Y_8^{\mathrm{T}}A_d^{\mathrm{T}} - Y_6 \\ \Sigma_{43} &= \mathcal{X} + d_M R - Y_7 - Y_4^{\mathrm{T}} \\ \Sigma_{44} &= \mathcal{X} + d_M R - Y_8 - Y_8^{\mathrm{T}} \\ \mathcal{X} &= P - L\Phi L^{\mathrm{T}} \end{aligned} \tag{33}$$

Proof. By (32) and (33), it is easy to see

$$\Sigma = \Omega + \mathscr{A}\mathscr{Y} + \mathscr{Y}^{\mathrm{T}}\mathscr{A}^{\mathrm{T}} \tag{34}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \dfrac{ERE^{\mathrm{T}}}{d_M} & -d_M ER & -d_M ER \\ \dfrac{ERE^{\mathrm{T}}}{d_M} & -Q - \dfrac{ERE^{\mathrm{T}}}{d_M} & 0 & 0 \\ -d_M RE^{\mathrm{T}} & 0 & \mathcal{X} + d_M R & \mathcal{X} + d_M R \\ -d_M RE^{\mathrm{T}} & 0 & \mathcal{X} + d_M R & \mathcal{X} + d_M R \end{bmatrix}$$

$$\Omega_{11} = \left(d_M - \frac{1}{d_M}\right) ERE^{\mathrm{T}} + Q - EPE^{\mathrm{T}}$$

$$\mathscr{A} = \begin{bmatrix} A^{\mathrm{T}} & 0 & -I_n & 0 \\ 0 & A_d^{\mathrm{T}} & 0 & -I_n \end{bmatrix}^{\mathrm{T}}$$

$$\mathscr{Y} = \begin{bmatrix} Y_1 & Y_2 & Y_3 & Y_4 \\ Y_5 & Y_6 & Y_7 & Y_8 \end{bmatrix}$$

Pre- and post-multiplying (34) by $\mathcal{H}$ and $\mathcal{H}^{\mathrm{T}}$, we have

$$\mathcal{H}\Sigma\mathcal{H}^{\mathrm{T}} = \mathcal{H}\Omega\mathcal{H}^{\mathrm{T}} \tag{35}$$

where $\mathcal{H} = \begin{bmatrix} I_n & 0 & A & 0 \\ 0 & I_n & 0 & A_d \end{bmatrix}$ is a full row rank matrix. By (32), $\Sigma < 0$, so $\mathcal{H}\Sigma\mathcal{H}^{\mathrm{T}} < 0$, i.e., $\mathcal{H}\Omega\mathcal{H}^{\mathrm{T}} < 0$. By computation, we have

$$\mathcal{H}\Omega\mathcal{H}^{\mathrm{T}} = \begin{bmatrix} \Delta_{11} & \Delta_{21}^{\mathrm{T}} \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$$

where Δ_{11}, Δ_{21}, and Δ_{22} are defined as (31). According to Corollary 2, system (2) is admissible. □

Remark 4. Based on Corollary 2 and matrix theory skill, Theorem 2 gives another delay-dependent sufficient condition of admissibility for system (2), which is ready for obtaining the controller.

The state feedback controller has the form of

$$\boldsymbol{u}(k) = G\boldsymbol{x}(k) \tag{36}$$

where $G \in \mathbf{R}^{p\times n}$. Substituting (36) into (1), the closed-loop system is

$$E\boldsymbol{x}(k+1) = (A+BG)\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d) \tag{37}$$

Next, the main aim is to obtain matrix G which guarantees the closed-loop system (37) to be admissible.

Theorem 3. Given a scalar $d_M > 0$, for any delay $0 < d \le d_M$, the closed-loop discrete-time singular delay system (37) is admissible if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $P, Q, R \in \mathbf{R}^{n\times n}$, a symmetric matrix $\Phi \in \mathbf{R}^{(n-r)\times(n-r)}$, matrices $W, Y_i, W \in \mathbf{R}^{p\times n}, Y_i \in \mathbf{R}^{n\times n}$, $i = 5, \cdots, 8$ and a nonsingular matrix Y, $Y \in \mathbf{R}^{n\times n}$, such that

$$\begin{bmatrix} \hat{\Sigma}_{11} & * & * & * \\ \hat{\Sigma}_{21} & \Sigma_{22} & * & * \\ \hat{\Sigma}_{31} & \hat{\Sigma}_{32} & \hat{\Sigma}_{33} & * \\ \hat{\Sigma}_{41} & \Sigma_{42} & \hat{\Sigma}_{43} & \Sigma_{44} \end{bmatrix} < 0 \tag{38}$$

where Σ_{22}, Σ_{42}, Σ_{44} are defined as (33). ρ_i are some given scalars, $i = 1, 2, 3, 4$.

$$\begin{aligned}
\hat{\Sigma}_{11} &= -EPE^{\mathrm{T}} + Q + \left(d_M - \frac{1}{d_M}\right)ERE^{\mathrm{T}} + \\
&\quad \rho_1 AY + \rho_1 Y^{\mathrm{T}}A^{\mathrm{T}} + \rho_1 BW + \rho_1 W^{\mathrm{T}}B^{\mathrm{T}} \\
\hat{\Sigma}_{21} &= \frac{ERE^{\mathrm{T}}}{d_M} + \rho_2 Y^{\mathrm{T}}A^{\mathrm{T}} + A_d Y_5 + \rho_2 W^{\mathrm{T}}B^{\mathrm{T}} \\
\hat{\Sigma}_{31} &= -d_M RE^{\mathrm{T}} + \rho_3 Y^{\mathrm{T}}A^{\mathrm{T}} - \rho_1 Y + \rho_3 W^{\mathrm{T}}B^{\mathrm{T}} \\
\hat{\Sigma}_{32} &= Y_7^{\mathrm{T}}A_d^{\mathrm{T}} - \rho_2 Y \\
\hat{\Sigma}_{33} &= \mathcal{X} + d_M R - \rho_3 Y - \rho_3 Y^{\mathrm{T}} \\
\hat{\Sigma}_{41} &= -d_M RE^{\mathrm{T}} + \rho_4 Y^{\mathrm{T}}A^{\mathrm{T}} - Y_5 + \rho_4 W^{\mathrm{T}}B^{\mathrm{T}} \\
\hat{\Sigma}_{43} &= \mathcal{X} + d_M R - Y_7 - \rho_4 Y^{\mathrm{T}}
\end{aligned} \tag{39}$$

where $\mathcal{X} = P - L\Phi L^{\mathrm{T}}$. The state feedback controller is $\boldsymbol{u}(k) = WY^{-1}\boldsymbol{x}(k)$.

Proof. Based on the results of Theorem 2, replacing A with $A + BG$ in (32), we have

$$\begin{bmatrix} \bar{\Sigma}_{11} & * & * & * \\ \bar{\Sigma}_{21} & \Sigma_{22} & * & * \\ \bar{\Sigma}_{31} & \Sigma_{32} & \Sigma_{33} & * \\ \bar{\Sigma}_{41} & \Sigma_{42} & \Sigma_{43} & \Sigma_{44} \end{bmatrix} < 0 \tag{40}$$

where Σ_{ij}, $i, j = 2, 3, 4$ are defined as (33).

$$\begin{aligned}
\bar{\Sigma}_{11} &= -EPE^{\mathrm{T}} + Q + \left(d_M - \frac{1}{d_M}\right)ERE^{\mathrm{T}} + \\
&\quad AY_1 + Y_1^{\mathrm{T}}A^{\mathrm{T}} + BGY_1 + Y_1^{\mathrm{T}}G^TB^{\mathrm{T}} \\
\bar{\Sigma}_{21} &= \frac{ERE^{\mathrm{T}}}{d_M} + Y_2^{\mathrm{T}}A^{\mathrm{T}} + A_dY_5 + Y_2^{\mathrm{T}}G^{\mathrm{T}}B^{\mathrm{T}} \\
\bar{\Sigma}_{31} &= -d_M RE^{\mathrm{T}} + Y_3^{\mathrm{T}}A^{\mathrm{T}} - Y_1 + Y_3^{\mathrm{T}}G^{\mathrm{T}}B^{\mathrm{T}} \\
\bar{\Sigma}_{41} &= -d_M RE^{\mathrm{T}} + Y_4^{\mathrm{T}}A^{\mathrm{T}} - Y_5 + Y_4^{\mathrm{T}}G^{\mathrm{T}}B^{\mathrm{T}}
\end{aligned} \tag{41}$$

It is noted that (40) is not an LMI, since there exist some quadratic matrices variables, such as, $Y_i^{\mathrm{T}}G^{\mathrm{T}}$, $i = 1, \cdots, 4$. These quadratic matrices variables would bring out difficulty in solving the inequality (40). To effectively tackle the problem, some given scalars ρ_i, $i = 1, \cdots, 4$ and matrix variables Y, W are introduced. Let $Y_i = \rho_i Y$, $W = GY$. Substituting $\rho_i Y$ into Y_i, $i = 1 \cdots, 4$ and replacing GY with W in (41), we have (38). □

Remark 5. Theorem 3 gives the controller design which ensures the closed-loop system (37) to be admissible. Similarly to Remark 2, consider the numerical complexity between the results of Theorem 3 and the ones in [6]. In Theorem 3, the system dimension is not augmented while in [6], the dimension is augmented to twice of the original one, thus, the number of decision variables is increased in [6]. Moreover, there are fewer lines in LMI (38). Table 2 shows the numbers of decision variables and lines in the LMIs to be solved in Th. 4 in [6] and Th. 3 in this paper. $(\mathcal{D}^3\mathcal{L})_3$ represents the quantity for Th. 4 in [6] and $(\mathcal{D}^3\mathcal{L})_4$ represents the quantity for Th. 3 in this paper. The ratio between them, denoted $\mathcal{R} = (\mathcal{D}^3\mathcal{L})_3/(\mathcal{D}^3\mathcal{L})_4$, is depicted in Fig. 2, for $r = n - 1$ and $p = 2$.

Table 2 Numbers of decision variables and lines in the LMIs to be solved in Th. 3 in this paper and Th. 4 in [6]

	Decision variables ($\mathcal{D}$)	Lines ($\mathcal{L}$)
Th. 4 in [6]	$8n^2 + (3 - 2r + p)n + 2r^2$	$12n$
Th. 3 in this paper	$7n^2 + (2 - r + p)n + 0.5(r^2 - r)$	$4n$

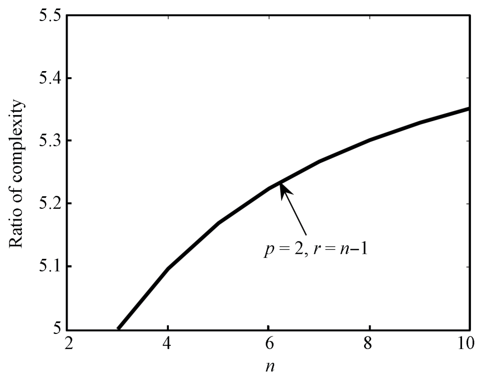

Fig. 2 The ratio between the complexity resulting from Th. 4 in [6] and Th. 3 in this paper with respect to n for $p = 2$ and $r = n - 1$

3 Numerical examples

Example 1. Consider the discrete-time singular delay system[8]:

$$E\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d)$$

where

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.9977+0.1\alpha & 1.1972 \\ 0.1001 & -1.9 \end{bmatrix}$$
$$A_d = \begin{bmatrix} -1.1972 & 1.5772 \\ 0 & 0.9754+0.1\alpha \end{bmatrix}$$

and α is a scalar.

First, let us compare the results of Theorem 1 with the ones of Theorem 2 in [7]. Theorem 2 in [7] gives a delay-dependent stability criterion for system (2) with time-varying delay. If the lower bound of time-varying delay is identical to the upper bound of time-varying delay, then time-varying delay is deduced to be constant time delay. The same case is addressed in this paper.

For given different values of α, using Theorem 2 in [7], the values of d_M are shown in Table 3. It is obvious that the results of [7] are more conservative than the ones of Theorem 1 in this paper. Second, comparisons of d_M with [8] for different α are shown in Table 3. It is seen that the results of Theorem 1 are less conservative than the ones in [8].

Table 3 Comparisons of d_M with different α for Example 1

α	0.5	−0.1	−0.5	−1.0	−1.5
d_M by Th. 2 in [7]	3	4	5	6	10
d_M by Th. 1 in [8]	3	4	5	6	10
d_M by Th. 1 in this paper	3	5	6	8	14

Example 2. Consider the discrete-time singular delay system

$$E\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + A_d\boldsymbol{x}(k-d) + B\boldsymbol{u}(k)$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -0.13 & 0.05 & -0.25 \\ -0.59 & -0.02 & 0.12 \\ 0.12 & -1.12 & 0.18 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.09 & 0.17 & 0.3 \\ -0.02 & -0.07 & -0.04 \\ -0.8 & 0.24 & 0.16 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & -0.5 \\ -0.2 & 0.12 \\ -0.2 & 0.6 \end{bmatrix}$$

The sample time is 0.1 s. First, consider the open-loop system. Assume $d_M = 3$, by Definition 1, it is verified $\rho(E, A, A_d) = 1.5 > 1$ and this demonstrates the open-loop system is unstable. Second, by Theorem 3, choosing $\boldsymbol{L} = [0 \quad 0 \quad 1]^{\mathrm{T}}$, $\rho_1 = -0.1$, $\rho_2 = 0.03$, $\rho_3 = -0.26$, $\rho_4 = -0.34$, and solving LMI (38), we obtain

$$G = \begin{bmatrix} 1.1234 & -4.8451 & -2.3753 \\ -0.9723 & 1.7127 & 0.4890 \end{bmatrix}$$

By computation, $\rho(E, (A+BG), A_d) = 0.89 < 1$, thus, the closed-loop system is stable and is also admissible. Closed-loop system state responses are illustrated in Fig. 3, which clearly show the effectiveness of the proposed method.

In this example, $n = 3$, $r = 2$, $p = 2$, from Table 2, by computation, $\mathcal{R} = 5$.

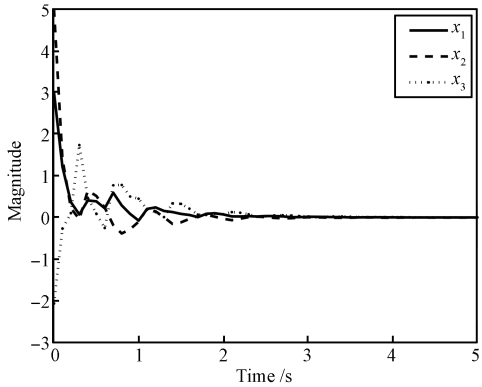

Fig. 3 Closed-loop system state responses

4 Conclusion

A new delay-dependent criterion for admissibility of discrete-time singular delay systems is proposed. The proposed criterion is proved to have some advantages over the existing results. Then, by applying the skill of matrix theory, a state feedback controller is designed to guarantee the closed-loop discrete-time singular delay systems to be admissible. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Appendix

Assume that the matrix E in system (2) is nonsingular. Given a scalar $d_M > 0$, there exist matrices $P > 0$, $Q > 0$, $R > 0$ such that LMI (27) holds if and only if there exist matrices $X > 0$, $U > 0$, $Z > 0$, N_1 and N_2 such that LMI (4) holds.

Proof. (Sufficiency) Pre- and post-multiplying (4) by $\begin{bmatrix} I_n & 0 & -\frac{E^{\mathrm{T}}}{d_M} \\ 0 & I_n & \frac{E^{\mathrm{T}}}{d_M} \\ 0 & 0 & I_n \end{bmatrix}$ and its transpose, respectively, we obtain

$$\begin{bmatrix} \Xi & \Pi^{\mathrm{T}} \\ \Pi & -d_M Z \end{bmatrix} < 0 \tag{A1}$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{21}^{\mathrm{T}} \\ \Xi_{21} & \Xi_{22} \end{bmatrix}$$
$$\Xi_{11} = A^{\mathrm{T}}XA - E^{\mathrm{T}}XE + U - \frac{E^{\mathrm{T}}ZE}{d_M} + d_M(A-E)^{\mathrm{T}}Z(A-E)$$
$$\Xi_{21} = A_d^{\mathrm{T}}XA + d_M A_d^{\mathrm{T}}Z(A-E) + \frac{E^{\mathrm{T}}ZE}{d_M}$$
$$\Xi_{22} = A_d^{\mathrm{T}}XA_d - U + d_M A_d^{\mathrm{T}}ZA_d - \frac{E^{\mathrm{T}}ZE}{d_M}$$
$$\Pi = [\ d_M N_1^{\mathrm{T}} + ZE \quad d_M N_2^{\mathrm{T}} - ZE\ ] \tag{A2}$$

From (A1), we have $\Xi < 0$, that is, (27) holds.

(Necessity) If $\Xi < 0$ holds, then (A1) also holds by taking

$$N_1 = -\frac{E^{\mathrm{T}}Z}{d_M}, \quad N_2 = \frac{E^{\mathrm{T}}Z}{d_M}$$

. □

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