Fermionic K-essence

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Abstract

In the present work, we study the cosmological model with fermionic field and with the non-canonical kinetic term (*fermionic k-essence* or *f-essence*). We also present some important reductions of the model as well as its some generalizations. We also found the exact solution of the model and examine the influence of such gravity-fermion interaction on the observed accelerated expansion of our universe.

1 Introduction

The observational evidence from different sources for the present stage of accelerated expansion of our universe has driven the quest for theoretical explanations of such feature. Assuming the validity of the theory of gravity, one attempt of explanation is the existence of an unregarded, but dominated at present time, ingredient of the energy content of the universe, known as dark energy, with unusual physical properties. The other possibility is modifying the general theory of relativity at large scales. In cosmology, the investigation for the constituents responsible for the accelerated periods in the evolution of the universe is of great interest. The mysterious dark energy has been proposed as a cause for the late time dynamics of the current accelerated phase of the universe.

During last years theories described by the action with non-standard kinetic terms, k-essence, attracted a considerable interest. Such theories were first studied in the context of k-inflation [1], and then the k-essence models were suggested as dynamical dark energy for solving the cosmic coincidence problem [2]-[4]. The action of the k-essence scalar field ϕ minimally coupled to the gravitational field $g_{\mu\nu}$ we write in the form (see e.g. [1]-[4])

$$S = \int d^4x\sqrt{-g}[R + K_1(X, \phi)], \tag{1.1}$$

where

$$X = 0.5g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \tag{1.2}$$

is the canonical kinetic term, ∇_μ is the covariant derivative associated with metric $g_{\mu\nu}$. The important particular reductions of the scalar k-essence (1.1) are:

i) $K_1 = A_1(X)$ (purely kinetic case),
ii) $K_1 = A_1(X)B_1(\phi)$,
iii) $K_1 = A_1(X) + B_1(\phi)$.

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Note that for the FRW metric, the equations of the k-essence (1.1) have the form

$$3H^2 - 0.5\rho_k = 0, \tag{1.3}$$

$$2\dot{H} + 3H^2 + 0.5p_k = 0, \tag{1.4}$$

$$K_{1X}\ddot{\phi} + (\dot{K}_{1X} + 3HK_{1X})\dot{\phi} - K_{1\phi} = 0, \tag{1.5}$$

$$\dot{\rho}_k + 3H(\rho_k + p_k) = 0. \tag{1.6}$$

Here the kinetic term, the energy density and the pressure are given by

$$X = 0.5\dot{\phi}^2, \quad \rho_k = 2K_{1X}X - K_1, \quad p_k = K_1. \tag{1.7}$$

In the recent years several approaches were made to explain the accelerated expansion by choosing fermionic fields as the gravitational sources of energy (see e.g. refs. [5]-[24]). In particular, it was shown that the fermionic field plays very important role in: i) isotropization of initially anisotropic spacetime; ii) formation of singularity free cosmological solutions; iii) explaining late-time acceleration. In the present work, we study the cosmological model with fermionic field, the M_{33} - model, which has the non-canonical kinetic term (*f-essence*). We examine the influence of such gravity-fermionic interaction on the accelerated expansion of the Universe. The formulation of the gravity-fermionic theory has been discussed in detail elsewhere [25]-[28]., so we will only present the result here.

2 Einstein-Dirac equations

In order to have this work self-consistent, in this section we present briefly the techniques that are used to include fermionic sources in the Einstein theory of gravitation and for a more detailed analysis the reader is referred to [25]-[28]. The general covariance principle imposes that the Dirac-Pauli matrices γ^a must be replaced by their generalized counterparts $\Gamma^\mu = e^\mu_a\gamma^a$, whereas the generalized Dirac-Pauli matrices satisfy the extended Clifford algebra, i.e., $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}$. The Einstein-Dirac action reads as

$$S = \int d^4x\sqrt{-g}[R + \epsilon Y - V], \tag{2.1}$$

where $\epsilon = \pm 1$ ($\epsilon = 1$ is the usual case and $\epsilon = -1$ is the phantom case) and

$$Y = 0.5i[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi], \quad V = V(\bar{\psi}, \psi). \tag{2.2}$$

Here ψ and $\bar{\psi} = \psi^+\gamma^0$ denote the fermionic field and its adjoint, respectively and R is the Ricci scalar. The covariant derivatives are given by

$$D_\mu\psi = \partial_\mu\psi - \Omega_\mu\psi, \quad D_\mu\bar{\psi} = \partial_\mu\bar{\psi} + \bar{\psi}\Omega_\mu, \tag{2.3}$$

where the spin connection Ω_μ is given by

$$\Omega_\mu = -0.25g_{\mu\nu}[\Gamma^\nu_{\sigma\lambda} - e^\nu_b(\partial_\sigma e^b_\lambda)]\gamma^\sigma\gamma^\lambda, \tag{2.4}$$

with $\Gamma^\nu_{\sigma\lambda}$ denoting the Christoffel symbols. The closed system of the equations for the model (2.1) looks like

$$R_{\mu\nu} - 0.5Rg_{\mu\nu} + T_{\mu\nu} = 0, \tag{2.5}$$

$$i\Gamma^\mu D_\mu\psi - \frac{dV}{d\bar{\psi}} = 0, \tag{2.6}$$

$$iD_\mu\bar{\psi}\Gamma^\mu + \frac{dV}{d\psi} = 0, \tag{2.7}$$

$$\dot{\rho}_f + 3H(\rho_f + p_f) = 0, \tag{2.8}$$

where the density of energy and pressure are given by

$$\rho_f = V, \quad p_f = \epsilon Y - V. \tag{2.9}$$

3 The M_{33} - model

Let us now we consider the M_{33} - model, which has the action

$$S=\int d^4x\sqrt{-g}[R+K_2(Y,\psi,\bar{\psi})], \tag{3.1}$$

where K_2 is some function of its arguments and the canonical kinetic term has the form

$$Y=0.5i[\bar{\psi}\Gamma^{\mu}D_{\mu}\psi-(D_{\mu}\bar{\psi})\Gamma^{\mu}\psi]. \tag{3.2}$$

We work with a space-time metric of the form

$$ds^2=-dt^2+a^2(dx^2+dy^2+dz^2), \tag{3.3}$$

that is the FRW metric. For this metric,the vierbein is chosen to be

$$(e_a^{\mu})=diag(1,1/a,1/a,1/a),\quad (e_{\mu}^a)=diag(1,a,a,a). \tag{3.4}$$

The Dirac matrices of curved spacetime Γ^{μ} are

$$\Gamma^0=\gamma^0,\quad \Gamma^1=a^{-1}\gamma^1,\quad \Gamma^2=a^{-1}\gamma^2,\quad \Gamma^3=a^{-1}\gamma^3, \tag{3.5}$$

$$\Gamma_0=\gamma^0,\quad \Gamma_1=a\gamma^1,\quad \Gamma_2=a\gamma_2,\quad \Gamma_3=a\gamma_3. \tag{3.6}$$

Hence we get

$$\Omega_0=0,\quad \Omega_1=0.5\dot{a}\gamma^1\gamma^0,\quad \Omega_2=0.5\dot{a}\gamma^2\gamma^0,\quad \Omega_3=0.5\dot{a}\gamma^3\gamma^0. \tag{3.7}$$

We now ready to write the equations of the M_{33} - model (3.1). We have

$$3H^2+0.5[K_2+0.5(K_{2\psi}\psi+K_{2\bar{\psi}}\bar{\psi})] = 0, \tag{3.8}$$

$$2\dot{H}+3H^2+0.5K_2 = 0, \tag{3.9}$$

$$K_{2Y}\dot{\psi}+0.5(3HK_{2Y}+\dot{K}_{2Y})\psi-i\gamma^0K_{2\bar{\psi}} = 0, \tag{3.10}$$

$$K_{2Y}\dot{\bar{\psi}}+0.5(3HK_{2Y}+\dot{K}_{2Y})\bar{\psi}+iK_{2\psi}\gamma^0 = 0, \tag{3.11}$$

$$\dot{\rho}_f+3H(\rho_f+p_f) = 0, \tag{3.12}$$

where $Y=0.5i(\bar{\psi}\gamma^0\dot{\psi}-\dot{\bar{\psi}}\gamma^0\psi)$ and

$$\rho_f=K_{2Y}Y-K_2=-[K_2+0.5(K_{2\psi}\psi+K_{2\bar{\psi}}\bar{\psi})],\quad p_f=K_2 \tag{3.13}$$

are the energy density and pressure of the fermionic field. If $K_2=Y-V$, then from the system (3.8)-(3.12) we get the corresponding equations of the Einstein-Dirac model.

3.1 Submodels

3.1.1 The M_{33A} - model

The Langrangian of the M_{33A} - model has the form

$$K_2=A_2(Y). \tag{3.14}$$

So the M_{33A} - model is the purely kinetic fermionic k-essence.

3.1.2 The M_{33B} - model

The Langrangian of the M_{33B} - model reads as

$$K_2=A_2(Y)B_2(\psi,\bar{\psi}). \tag{3.15}$$

3.1.3 The M_{33C} - model

The Langrangian of the M_{33A} - model has the form

$$K_2 = A_2(Y) + B_2(\psi, \bar{\psi}). \tag{3.16}$$

3.2 Solution

In this subsection we want to construct a solution of the M_{33}-model. Let $K_2 = K_2(Y, u)$, where $u = \bar{\psi}\psi$. Then the system (3.8)-(3.12) becomes

$$\begin{aligned}
3H^2 + 0.5[K_2 + K_2^{'}u] &= 0, & (3.17)\\
2\dot{H} + 3H^2 + 0.5K_2 &= 0, & (3.18)\\
K_{2Y}\dot{\psi} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\psi - i\gamma^0 K_2^{'}\psi &= 0, & (3.19)\\
K_{2Y}\dot{\bar{\psi}} + 0.5(3HK_{2Y} + \dot{K}_{2Y})\bar{\psi} + iK_2^{'}\bar{\psi}\gamma^0 &= 0, & (3.20)\\
\dot{\rho}_f + 3H(\rho_f + p_f) &= 0, & (3.21)
\end{aligned}$$

where $K_2^{'} = dK_2/du$ and

$$\rho_f = -[K_2 + K_2^{'}u]. \quad p_f = K_2. \tag{3.22}$$

We now consider the submodel (3.16) that is the M_{33C}-model, where we assume that $A_2 = \alpha Y^n, \quad B_2 = \beta u^m$ that is the case $K_2 = \alpha Y^n + \beta u^m$. Let $a = a_0 t^{\lambda}$. Then we have the following solution

$$Y = \left\{\left[-\frac{6m\lambda^2 - 4(m+1)\lambda}{\alpha m}\right] t^{-2}\right\}^{\frac{1}{n}}, \quad u = \left\{\left[\frac{-4\lambda}{\beta m}\right] t^{-2}\right\}^{\frac{1}{m}}, \tag{3.23}$$

where

$$\lambda = \frac{2n - 2m + 2mn}{3mn}, \quad a_0 = \sqrt[3]{\frac{c}{\alpha n}\left[-\frac{6m\lambda^2 - 4(m+1)\lambda}{\alpha m}\right]^{\frac{1-n}{n}}\left[-\frac{4\lambda}{\beta m}\right]^{-\frac{1}{m}}}. \tag{3.24}$$

Finally we present the following formulas

$$u = ca^{-3}K_{2Y}^{-1}, \quad \psi_j = c_j a^{-1.5} K_{2Y}^{-0.5} e^{i\gamma^0 \int K_2^{'} K_{2Y}^{-1} dt}, \tag{3.25}$$

where $c = |c|_1^2 + |c|_2^2 - |c|_3^2 - |c|_4^2, \quad c_j = consts.$

4 The M_{34} - model

We now would like to present the M_{34} - model (the generalized k-essence or *g-essence* for short) which has the following action [29]

$$S = \int d^4x\sqrt{-g}[R + K(X, Y, \phi, \psi, \bar{\psi})]. \tag{4.1}$$

For the FRW metric (3.3), the equations of *g-essence* (4.1) have the form

$$\begin{aligned}
3H^2 - 0.5\rho &= 0, & (4.2)\\
2\dot{H} + 3H^2 + 0.5p &= 0, & (4.3)\\
K_X\ddot{\phi} + (\dot{K}_X + 3HK_X)\dot{\phi} - K_{\phi} &= 0, & (4.4)\\
K_Y\dot{\psi} + 0.5(3HK_Y + \dot{K}_Y)\psi - i\gamma^0 K_{\bar{\psi}} &= 0, & (4.5)\\
K_Y\dot{\bar{\psi}} + 0.5(3HK_Y + \dot{K}_Y)\bar{\psi} + iK_{\psi}\gamma^0 &= 0, & (4.6)\\
\dot{\rho} + 3H(\rho + p) &= 0. & (4.7)
\end{aligned}$$

Here the kinetic terms, the energy density and the pressure are given by

$$X = 0.5\dot{\phi}^2, \quad Y = 0.5i(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) \tag{4.8}$$

and

$$\rho = 2K_X X + K_Y Y - K, \quad p = K. \tag{4.9}$$

Note that the model (4.1) contents some important particular submodels. For example:

i) the scalar k-essence (1.1) as $K = K_1(X, \phi)$;

ii) the M_{33} - model (3.1) as $K = K_2(Y, \psi, \bar{\psi})$;

iii) the M_{34A} - model as $K = K_1(X, \phi)K_2(Y, \psi, \bar{\psi})$;

vi) the M_{34B} - model as $K = K_1(X, \phi) + K_2(Y, \psi, \bar{\psi})$.

Some properties of the M_{34} - model (4.1) were studied in [29]. In particular it is shown that it can describes the late-time acceleration of the universe.

5 Conclusion

We briefly summarize the present work. We first derived the equations of the M_{33} - model for the FRW space-time. Then we found their exact solution for the *f-essence* $K_2 = \alpha Y^n + \beta u^m$. Finally, let us we present the expressions for the equation of state and deceleration parameters. For the our particular solution (3.19) they take the form

$$w_f = -1 + \frac{mn}{n - m + mn}, \quad q = -1 + \frac{3mn}{2(n - m + mn)}. \tag{5.1}$$

These formulas tell us that the M_{33} - model can describes the observed accelerated expansion of our universe.

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