Agnesi Weighting for the Measure Problem of Cosmology *

Don N. Page †
Theoretical Physics Institute
Department of Physics, University of Alberta
Room 238 CEB, 11322 – 89 Avenue
Edmonton, Alberta, Canada T6G 2G7

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Abstract

The measure problem of cosmology is how to assign normalized probabilities to observations in a universe so large that it may have many observations occurring at many different spacetime locations. I have previously shown how the Boltzmann brain problem (that observations arising from thermal or quantum fluctuations may dominate over ordinary observations if the universe expands sufficiently and/or lasts long enough) may be ameliorated by volume averaging, but that still leaves problems if the universe lasts too long. Here a solution is proposed for that residual problem by a simple weighting factor $1/(1+t^2)$ to make the time integral convergent.

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†Internet address: profdonpage@gmail.com

Introduction

The high degree of spatial flatness observed for the constant-time hypersurfaces of our universe leads to the idea that our universe is much larger than what we can presently observe. The leading explanation for this flatness, cosmological inflation in the early universe, suggests that in fact the universe is enormously larger than what we can see, perhaps arbitrarily large if an indefinitely long period of eternal inflation has occurred in the past. Furthermore, the recent observations of the acceleration of the universe suggest that our universe may expand exponentially yet more into a very distant future. As a result, spacetime may already be, or may become, so large that a vast number of different observations (by which I mean observational results, what it is that is actually observed) will recur a huge number of times throughout the universe.

If we restrict to theories that only predict whether or not a particular observation (e.g., ours) occurs, there are likely to be many such theories predicting that our observation almost certainly occurs, so that we would have very little observational evidence to distinguish between such theories. This would seem to imply that observational science would come to an end for such theories.

However, even for a very large universe there can be theories that are much more testable by predicting not just whether a particular observation occurs, but also the probability that this particular observation is made rather than any of the other possible observations. Then one can use the probability the theory assigns to our actual observation as the likelihood of the theory, given the observation (actually the conditional probability of the observation, given the theory). One can then draw statistical conclusions about alternative theories from their likelihoods. For example, in a Bayesian analysis in which one assigns prior probabilities to the theories, one can multiply these priors by the likelihoods and divide by the sum of the products over all theories to get the normalized posterior probabilities of the theories.

Therefore, it would be desirable to have theories that each predict normalized probabilities for all possible observations. (These probabilities can be normalized measures for a set of observations that in a global sense all actually occur, as in Everettian versions of quantum theory in which quantum probabilities are not propensities for a wide class of potentialities to be converted to a narrower class of actualities. All observations with positive measure could actually occur in such a completely deterministic version of quantum theory, but with different measures, which, if normalized, can be used as likelihoods in a statistical analysis.)

However, in a very large universe in which many observations recur many times, it can become problematic what rule to use to calculate the normalized measure (or probability) for each one. If one had a definite classical universe in which each observation occurs a fixed finite number of times, and if the total number of all observations is also a finite number, one simple rule would be to take the normalized measure for each observation to be the fraction of its occurrence, the number of times it occurs divided by the total number of all observations. But in a quantum universe in which there are amplitudes for various situations, it is less obvious what to do.

I have shown that Born's rule, taking the normalized measure of each observation to be the expectation value of a corresponding projection operator, does not work in a sufficiently large universe [1, 2, 3, 4]. The simplest class of replacement rules would seem to be to use instead the expectation values of other positive operators, but then the question arises as to what these operators are.

For a universe that is a quantum superposition of eigenstates that each have definite finite numbers of each observation, one simple choice for the normalized measures would be to take the expectation values of the frequencies of each observation (say frequency averaging), and a different simple choice would be to take the expected numbers of each observation divided by the expected total number of all observations (say number averaging). For example, suppose that the quantum state giving only two possible observations (say of a loon or of a bear, to use the animals on the one- and two-dollar Canadian coins) is

$$|\psi\rangle = \cos\theta |mn\rangle + \sin\theta |MN\rangle, \tag{1}$$

where the first eigenstate $|mn\rangle$ corresponds to m loon observations and n bear observations and the second eigenstate $|MN\rangle$ corresponds to M loon observations and N bear observations. (For simplicity I am assuming all of the loon observations are precisely identical but different from all of the bear observations that are themselves precisely identical.) Then the first choice above, frequency averaging, would give

$$\begin{aligned} P_f(\text{loon}) &= \frac{m}{m+n}\cos^2\theta + \frac{M}{M+N}\sin^2\theta, \\ P_f(\text{bear}) &= \frac{n}{m+n}\cos^2\theta + \frac{N}{M+N}\sin^2\theta, \end{aligned} \tag{2}$$

whereas the second choice above, number averaging, would give

$$\begin{aligned} P_n(\text{loon}) &= \frac{m\cos^2\theta + M\sin^2\theta}{(m+n)\cos^2\theta + (M+N)\sin^2\theta}, \\ P_n(\text{bear}) &= \frac{n\cos^2\theta + N\sin^2\theta}{(m+n)\cos^2\theta + (M+N)\sin^2\theta}. \end{aligned} \tag{3}$$

Therefore, even in this very simple case, there is no uniquely-preferred rule for converting from the quantum state to the observational probabilities. One would want $P(\text{loon})$ to be between the two loon-observation frequencies for the two eigenstates, between $m/(m+n)$ and $M/(M+N)$, as indeed both rules above give, but unless one believes in the collapse of the wavefunction (which would favor frequency averaging), there does not seem to be any clear choice between the two. (One can easily see that in this example, there is no state-independent projection operator whose expectation value is always between $m/(m+n)$ and $M/(M+N)$ for arbitrary m, n, M, N and θ, so Born's rule fails.)

The problem becomes even more difficult when each quantum component may have an infinite number of observations. Then it may not be clear how to get definite values for the frequencies of the observations in each eigenstate, or how to get definite values for the ratios of the infinite expectation values for the numbers of each different observation. Most of the work on the measure problem in cosmology has focused on regularizing these infinite numbers of observations [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71]. However, I have emphasized [1, 2, 3, 4, 51] that there is also the ambiguity described above even for finite numbers of occurrence of identical observations.

One challenge is that many simple ways to extract observational probabilities from the quantum state appear to make them dominated by Boltzmann brain observations, observations produced by thermal or vacuum fluctuations rather than by a long-lived observer [72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100]. But if Boltzmann brains dominate, we would expect that our observations would have much less order than they are observed to have, so we have strong statistical evidence against our observations' being Boltzmann brains. We would therefore like theories that does not have the measures for observations dominated by Boltzmann brains.

The main way in which theories tend to predict domination by Boltzmann brains is by having the universe last so long that after ordinary observers die out, a much larger number (say per comoving volume) of Boltzmann brains eventually appear. A big part of the problem is that the volume of space seems to be beginning to grow exponentially as the universe enters into an era in which a cosmological constant (or something very much like it) dominates the dynamics. Therefore, the expected number of Boltzmann brain observations per unit time would grow exponentially with the expansion of the universe and would eventually become larger than the current number of observations per time by ordinary observers, leading to Boltzmann brain domination in number averaging (which I have previously called volume weighting because the number per time is proportional to the spatial volume for observations at a fixed rate per four-volume).

To avoid this part of the problem that occurs for what I have called volume weighting (or what I now prefer to call number averaging, setting the measure for a observation proportional to the expectation value or quantum average of the number of occurrences of the observation), I have proposed using instead volume averaging (or what I now prefer to call spatial density averaging), setting the measure for each observation on a particular hypersurface to be proportional to the expectation value of the spatial density of the occurrences of that observation, the expected number divided by the spatial volume. This would lead to the contribution per time for hypersurfaces at late times being the very low asymptotically constant spacetime density of Boltzmann brains. This density is presumably enormously lower than the spacetime density of ordinary observers today, so per time, observations today dominate greatly over Boltzmann brains at any time in the distant future.

However, if the universe lasts for a sufficiently long time (exponentially longer than the time it would have to last for Boltzmann brains to dominate in number averaging), then integrating over time would cause even the contribution from the very tiny spatial density of Boltzmann brains eventually to dominate over the contributions of ordinary observers that presumably exist only during a finite time. (For simplicity I am ignoring the contributions from tunnelings to new vacua.) Therefore, it appears that we need not only a shift from number averaging (volume weighting) to spatial density averaging (volume averaging), but that we also need something else to suppress the divergence in the Boltzmann brain contributions at infinite times.

In the scale-factor measure [49, 52, 53, 63], one puts a cutoff where the volume of cross sections of certain sets of timelike geodesics reaches some upper limit. If the universe is dominated by a cosmological constant at late times, geodesics other than those that stay within bound matter configurations expand indefinitely at an asymptotically exponential rate, so that all such geodesics eventually reach the cutoff. Then if the contributions within the bound matter configurations, where the geodesics are not cut off, do not dominate, then one only gets a finite set of observations of each type, and one can apply either frequency averaging or number averaging. (One might expect the matter in bound matter configurations eventually to decay away, so that one does not have to worry about the timelike geodesics there that would never expand to the cutoff if the matter configuration persisted.)

The usual answer that one gets is that if the universe tends to a quasi-stationary eternal inflation picture in which new bubbles are forming and decaying at an asymptotically fixed rate, and if the cutoff is applied at a sufficiently great volume, then the precise value at which it is applied does not matter [49, 52, 53, 63]. Furthermore, if the tunneling rate to new bubbles that lead to more ordinary observers is greater than the rate for Boltzmann brains to form, ordinary observers can dominate over Boltzmann brains. However, it is important in this approach that there be a cutoff, which is crucial for defining the ensemble of observations. It has even been noted [69] that using this cutoff, "Eternal inflation predicts that time will end."

In the scale-factor measure, it is not specified precisely where the cutoff is to be imposed (at what volume, relative to some initial volume of each set of timelike geodesics), but it is just pointed out that the resulting observational probabilities appear to be insensitive to the value of the cutoff so long as it is sufficiently late (or large). However, for a precise theory with a cutoff, one would like a precise cutoff. It then seems a bit *ad hoc* to have the cutoff at some particular very late (or large) value, as seems to be necessary with the scale-factor cutoff. (What simple explanation could be given for the very large value of the cutoff?)

Here I am proposing to replace the scale-factor cutoff that has an unspecified late value with a particular simple explicit weighting factor to suppress the measures for late-time observations, such as Boltzmann brains, in a precisely specified way. The idea is to supplement the spatial density averaging (volume averaging), which greatly ameliorates the Boltzmann brain problem, with a measure over time that integrates to a finite value over infinite time. The measure over time is chosen to be $dt/(1+t^2)$, where t is the proper time in Planck units, which is the simplest analytic weighting I could think of that gives a convergent integral over time. Since the curve $y = 1/(1+x^2)$ is named the witch of Agnesi, I shall call this Agnesi weighting. (The witch of Agnesi was named after the Italian linguist, mathematician, and philosopher Maria Gaetana Agnesi, 1718-1799, after a misidentification of the Italian word for "curve" with the word for "woman contrary to God," so that it was mistranslated "witch").

Probabilities of observations with Agnesi weighting

In this paper I shall use a semiclassical approximation for gravity, since I do not know how to do Agnesi weighting in full quantum gravity. Assume that the quantum state corresponds to a superposition of semiclassical spacetime geometries. Further assume that the postulated operators whose expectation values give the measures for observations commute approximately with the projection operators to the semiclassical geometries, so that for the measures one can regard the quantum state as if it were an ensemble of 4-geometries with probabilities $p({}^4g)$ given by the absolute squares of the amplitudes for each geometry. There is no guarantee that this approximation is good, but here I shall make it for simplicity. Perhaps later one can go back and look at refinements, though it may be hard to do that without knowing more about the postulated operators.

I shall assume that each semiclassical 4-geometry has a preferred hypersurface. In a standard big-bang model, this could be the singular surface at the big bang. In my Symmetric-Bounce model for the universe [101], which I have argued is more predictive, the preferred hypersurface would be the hypersurface in which the semiclassical geometry has zero trace of the extrinsic curvature. (In this model, to semiclassical accuracy, the entire extrinsic curvature would vanish on this hypersurface of time symmetry.) If one had a different semiclassical model in which there is a bounce rather than a singular big bang, the preferred hypersurface could be the hypersurface of zero trace of the extrinsic curvature, the one that minimizes the spatial volume if there are more than one such extremal hypersurfaces.

Then for each point of the spacetime, I shall choose the simplest choice of a time function t, the proper time of the longest timelike curve from that point to the preferred hypersurface. This will be a timelike geodesic intersecting the hypersurface perpendicularly. If there are two sides to the hypersurface, as in my Symmetric-Bounce model, arbitrarily take t positive on one side of it (its future) and negative on the other side of it (its past). Take the preferred foliation of the spacetime given by the hypersurfaces of constant t. These will be spatial hypersurfaces, though they may have kinks where one goes from one region of the spacetime with one smooth congruence of timelike geodesics that maximize the proper time to the preferred hypersurface to another region with a discontinuously different smooth congruence of geodesics. Let $V(t)$ be the spatial 3-volume of each such hypersurface. I shall assume that $V(0)$ at $t = 0$ is a local minimum of the spatial volume, the preferred hypersurface which can have $V(0) > 0$ in a bounce model or $V(0) = 0$ in a big bang model. The proper 4-volume between infinitesimally nearly hypersurfaces of the foliation is $dV_4 = V(t)dt$.

In a WKB approximation to the Wheeler-DeWitt equation for canonical quantum gravity, the absolute square of the wavefunctional for the hypersurfaces integrated over an infinitesimal sequence of hypersurfaces in a foliation is proportional to the conserved WKB flux multiplied by the infinitesimal proper time between hypersurfaces [102], so here I shall take the quantum probability for the hypersurface to be one of the foliation hypersurfaces between t and $t+dt$ to be $p(^4g)dt$. Note that for semiclassical 4-geometries that have t running to infinity, the Wheeler-DeWitt inner product or quantum probability diverges when integrated over the hypersurfaces corresponding to all t. This fact also highlights the need to put in a weighting factor or do something else to get finite observational probabilities out from a quantum state of canonical quantum gravity.

Let us assume that the semiclassical spacetime ^4g gives a spacetime density expectation value $n_j(t, x^i)$ for the observation O_j to occur at the time t and spatial location x^i. Let $\bar{n}_j(t)$ be the spatial average of $n_j(t, x^i)$ over the spatial hypersurface. Then the expected number of occurrences of the observation O_j between t and $t+dt$ is $dN_j = \bar{n}_j(t)V(t)dt$. If we were doing number averaging (volume weighting), we would seek to integrate dN_j over t to get a measure for the observation O_j contributed by the semiclassical geometry ^4g if it were the only 4-geometry. However, if t can go to infinity, this integral would diverge. If $V(t)$ grows exponentially with t, it would still diverge even if we included the weighting factor $1/(1+t^2)$. One would need an exponentially decreasing weight factor (with a coefficient of t in the exponential that is greater than the Hubble constant of the fastest expanding vacuum in the landscape) to give a convergent integral if one just used a function of t with number averaging. Such a rapidly decaying weight factor would lead to the youngness problem [37].

Things are much better if we use spatial density averaging (volume averaging), which divides dN_j by $V(t)$ to get $\bar{n}_j(t)dt$, the spatial average of the density of the observation O_j multiplied by the proper time dt. If we then combine this spatial density averaging over the spatial hypersurfaces with Agnesi weighting for the time, we get that the semiclassical 4-geometry ^4g contributes $\int \bar{n}_j(t)dt/(1+t^2)$ to the measure for O_j. Next, we sum this over the quantum probabilities of the 4-geometries ^4g to get the relative probability of the observation O_j as

$$p_j = \sum_{^4g} p(^4g) \int \bar{n}_j(t) \frac{dt}{1+t^2} = \sum_{^4g} p(^4g) \int \frac{dN_j}{dt} \frac{1}{V(t)} \frac{dt}{1+t^2}. \tag{4}$$

Here, of course, the expectation value of the spatially averaged density $\bar{n}_j(t)$ of the observations O_j, and thus also the expectation value of the rate of observations per time dN_j/dt, depend implicitly on the 4-geometry ${}^4g$, and by a semiclassical 4-geometry I am including the quantum state of the matter fields on that 4-geometry, on which the expectation value $n_j(t, x^i)$ and hence $\bar{n}_j(t)$ and dN_j/dt are likely to depend, as well as on the 4-geometry itself.

Finally, we get the normalized probabilities for the observations O_j by dividing by the sum of the unnormalized relative probabilities p_j:

$$P_j = \frac{p_j}{\sum_k p_k}. \tag{5}$$

Consequences of Agnesi weighting for our universe

We see that with the combination of spatial density averaging (volume averaging) and Agnesi weighting, the expectation value dN_j/dt of the number of observations O_j per proper time is divided by both the 3-volume $V(t)$ of the hypersurface and by the Agnesi factor $1+t^2$. This tends to favor observations early in the universe, so let us see how great a youngness effect it gives, say between the origin of the solar system and a time equally far in the future, near its expected demise.

Let us use what I call the Mnemonic University Model (MUM, which itself might be considered a British term of endearment for Mother Nature) for the universe, a spatially flat universe dominated by dust and a cosmological constant, with present age $t_0 = H_0^{-1} = 10^8$ years$/\alpha$, where $\alpha \approx 1/137036000$ [103] is the electromagnetic fine structure constant, and with the solar age $t_0/3$. The present observations give a universe age of 13.69 ± 0.13 Gyr [103] that is 0.999 ± 0.009 times the MUM value of 13.7036 Gyr, a Hubble constant of 72 ± 3 km s^{-1} Mpc^{-1} [103] that is 1.009 ± 0.042 times the MUM value of 71.3517 km s^{-1} Mpc^{-1}, and a solar system age of 4.5681 ± 0.0003 Gyr [104] that is 1.00005 ± 0.00007 times the MUM value of 4.56787 Gyr. Thus the MUM values are all within the present observational uncertainties for the universe age, Hubble constant, and solar system age.

The metric for the MUM model is

$$ds^2 = -dt^2 + \sinh^{4/3}(1.5 H_\Lambda t)(dx^2 + dy^2 + dz^2), \tag{6}$$

where $H_\Lambda = \sqrt{\Lambda/3}$ is the asymptotic value of the Hubble expansion rate

$$H = \frac{\dot{a}}{a} = H_\Lambda \coth(1.5 H_\Lambda t). \tag{7}$$

For $t_0 = H_0^{-1}$, we need $H_\Lambda t_0 = \tanh(1.5 H_\Lambda t_0)$ or $H_\Lambda t_0 \approx 0.858560$, and then $t_0 = 10^8$years$/\alpha$ gives $H_\Lambda \approx (15.96115\text{Gyr})^{-1} \approx 61.2597$ km s^{-1} Mpc^{-1}. One can also calculate that the MUM predicts that at present the dark energy corresponding to the cosmological constant gives a fraction of the total (closure) energy density that is $\Omega_\Lambda = \tanh^2(1.5 H_\Lambda t_0) = (H_\Lambda t_0)^2 \approx 0.737125$, in good agreement with the observational value of 0.74 ± 0.03 [103] that is 1.004 ± 0.041 times the MUM value.

Some features of the MUM are that with the conformal time that is given by $\eta = \int_0^t dt'/\sinh^{2/3}(1.5 H_\Lambda t')$, the total conformal time is $\eta_\infty \approx 44.76088$ Gyr, and the present value of the conformal time is $\eta_0 \approx 33.8825$ Gyr $\approx 0.756967\eta_\infty$. (This is using the normalization above that $a(t) = \sinh^{2/3}(1.5 H_\Lambda t)$, which gives $a_0 \equiv a(t_0) \approx 1.41014$; if one had instead set $a_0 = 1$ so $a(t) = \sinh^{2/3}(1.5 H_\Lambda t)/\sinh^{2/3}(1.5 H_\Lambda t_0)$, one would have $\eta = \int_0^t dt'/a(t')$ giving $\eta_\infty \approx 63.1193$ Gyr and $\eta_0 \approx 47.7792$ Gyr.) Thus we see that although there is only a finite proper time in the past and an infinite proper time in the future, over three-quarters of the total finite conformal time of the MUM has already passed.

The cosmological event horizon for the comoving observer at $r = \sqrt{x^2 + y^2 + z^2} = 0$ (which we shall take to be our worldline) is at $r = \eta_\infty - \eta$, so on the constant-time hypersurface $t = t_0$ (and hence $\eta = \eta_0$), it is at $r = r_1 = \eta_\infty - \eta_0 \approx 10.8784$ Gyr, at a distance along this hypersurface of $a_0 r_1 \approx 15.3401$ Gyr (times the speed of light c, which I am setting to unity; e.g., this distance is 15.3401 billion light years). The actual spacetime geodesic distance from us to the point on the comoving worldline at $r = r_1$ that is crossing our cosmological event horizon when its proper time from the big bang is the same as ours is 16.2282 Gyr, greater than the distance along a geodesic of the constant-time hypersurface, because geodesics of that hypersurface are not geodesics of spacetime but instead bend in the timelike direction, shortening their length. The actual geodesic of spacetime joining the two events goes forward in the time t from t_0 to $t \approx 1.17686 t_0 \approx 16.1272$ Gyr, to a point with $a \approx 1.18725 a_0$, before bending back in t to get back to t_0 at the cosmological event horizon.

Like de Sitter spacetime with the same value of the cosmological constant, the MUM has a maximal separation of two events connected by a spatial geodesic, which is $\pi/H_\Lambda \approx 50.1434$ Gyr. All events with $r \geq 2\eta_\infty - \eta_0 - \eta$ cannot be reached by any geodesics from our location in spacetime. The events on this boundary at $t = t_0$ are at $r = r_2 = 2(\eta_\infty - \eta) \approx 21.7567$ Gyr, which is at a distance of 30.6802 Gyr along the $t = t_0$ hypersurface, though the geodesic distance is the maximal value of 50.1434 Gyr. (Actually, there is no geodesic to this boundary itself, but this maximal value is the limit of the geodesic distance as r approaches the boundary.)

A third preferred distance on the $t = t_0$ hypersurface of homogeneity is at $r = r_3 = \eta_0 \approx 33.8825$ Gyr, which is where a comoving worldline that started at the big bang on our past light cone reaches after the same proper time t_0 from the big bang as we are. That is, this is the present location of a worldline which started at our particle horizon. This value of r corresponds to a physical distance along this hypersurface of 47.7792 Gyr. There are no geodesics from us to that point, so even if we had a tachyon gun, we could not hit that worldline at a point on it after its proper time passed our value of t_0.

The MUM also allows on to calculate the geodesic distance from us to each of these three worldlines along a geodesic that is orthogonal to our worldline at its intersection here and now. This distance to $r = r_1$ (the comoving worldline that crosses our cosmological event horizon at a proper time of t_0) is 11.3244 Gyr, to $r = r_2$ (the worldline that after proper time t_0 reaches the boundary of where geodesics from us can reach) is 14.3274 Gyr, and to $r = r_3$ (the worldline that starts at the big bang on our past light cone) is 14.6863 Gyr. This spacelike geodesic never reaches our cosmological event horizon but instead ends at the big bang at a distance of 14.6889 Gyr from us, where $r = r_4 \approx 41.0459$ Gyr (or $a_0 r_4 \approx 57.8806$ Gyr for the distance along the $t = t_0$ hypersurface to the comoving worldline with $r = r_4$), which is less than the value $r = r_5 \approx 44.7609$ Gyr where our cosmological event horizon intersects the big bang, whose comoving worldline is at a distance $a_0 r_5 \approx 63.1193$ Gyr from us along the $t = t_0$ hypersurface. That is, if we define simultaneity by spacelike geodesics orthogonal to our worldline, the big bang is still going on right now [105], at a distance of 14.7 billion light years from us in the Mnemonic Universe Model.

Yet another comoving worldline that one may define is the one that emerges from the big bang from the boundary of the region that can be reached from us by spacetime geodesics. This is at $r = r_6 = 2\eta_\infty - \eta_0 \approx 55.6393$ Gyr, which as measured along the $t = t_0$ hypersurface is at the distance $a_0 r_6 \approx 78.4594$ billion light years from us. This is the upper limit to the current distance (over a constant-time hypersurface, not along a spatial geodesic of spacetime that has a maximum length of 50.1434 billion light years in the MUM) of any comoving worldline that can be reached by any geodesics from our current location in spacetime. The limit of the spatial geodesics that reach from us to comoving worldlines as $r \to r_6$ is a null geodesic that goes from us to the spacelike future boundary at $\eta = \eta_\infty$ and then returns to the big bang along another null geodesic; the spacelike geodesics approaching this limit approach the maximum spacelike geodesic length of 50.1434 billion light years, this length occurring in the de Sitter region in the arbitrarily distant future where the spatial geodesic turns around from going toward the future in t to going back toward the past in t.

Now let us use the MUM to calculate the youngness effect from the formation of the solar system, at a time $t_0/3$ before the present, or at $t = 2t_0/3$ after the big bang, to a time equally equally far in the future, at $t = 4t_0/3$, which we shall use as a very crude approximation for the mnemonic demise of the solar system. Since both of these times are enormously longer than the Planck time (with $t_0 = 8.021 \times 10^{60}$ in Planck units), we can drop the 1 that is included in the Agnesi weighting to avoid a divergence at $t = 0$. Then we see that on a per-time basis, the Agnesi weighting factor of $1/(1+t^2)$ is four times smaller at the demise of the solar system than at its formation. However, the spatial volume of the universe also goes up by a factor of 7.75 during this 'lifetime' of the solar system, so if we had a fixed comoving density of observers, the combination of the Agnesi and spatial density averaging (volume averaging) factors would give about 31 times the weight for observations at the formation of the solar system than at its end.

This would imply that if the same number of observations occurred per proper time and per comoving volume at the formation and at the demise of the solar system, the ones at the demise would have only about 3% of the measure of the ones at the formation. Half of the measure would occur within the first 18% of the solar system lifetime. This effect would tend to favor observations early in the history of the solar system.

However, it seems highly plausible that a factor of only about 31 would be negligible in comparison with the factors that determine the numbers of observations. Presumably if one sampled a huge number of solar systems, only a very tiny fraction of the observations would occur very close to the formation, because of the time needed for evolution. If the probability for evolution to intelligent life to have occurred rises sufficiently rapidly with the time after the formation time (e.g., significantly faster than the linear rise one would expect if evolution were a single event that occurred statistically at a constant rate per time per solar system), then it would not be at all surprising that we exist at a time when 85% of the measure would have passed *if* the number of observations were instead uniform in time.

The shift of the measure (say calibrated for a fixed comoving density of observers making a constant number of observations per time) from being uniform in the time to having the weighting factors of the inverse three-volume (from the spatial density averaging) and of very nearly the inverse square of the time (from the Agnesi weighting) would have an effect on the number of hard steps n Brandon Carter estimated for the evolution of intelligent life on earth [106, 107]. A hard step (or 'critical' step in the first of these papers) is one whose corresponding timescale is at least a significant fraction of the available time for it to occur (e.g., the lifetime of the sun). Carter emphasized [106] that unless there is an unexplained (and therefore *a priori* improbable) coincidence, the timescale of a step is not likely to be close to the available time, so generically a hard step has a timescale much longer than the available time. Therefore, a hard step is unlikely to occur within the available time on a random suitable planet in which the previous steps have occurred.

In the first of these papers [106], Carter assumed that since we are about halfway through the predicted lifetime of the sun, we arose about halfway through the life-permitting period on earth and about halfway through the measure if the measure were uniform in time. He then concluded that the number of hard steps n would likely be 1 or 2. In the second paper [107], Carter used more recent information [108] that the sun may become too luminous for life to continue on earth just one billion years in the future rather than five. Then we would be a fraction $f \sim 5/6$ of the way through the available period for life, and this would lead to an estimate for the number of hard steps to be $n \sim f/(1-f) \sim 5$. (Carter suggested $4 \lesssim n \lesssim 8$ and favored $n = 6$ if the first hard step occurred on Mars.)

Now let us see how these estimates for the number of hard steps to us would be modified with the spatial density averaging and Agnesi weighting. If we take the assumptions of Carter's original paper, that the available time is the entire solar lifetime and that we are halfway through it, without any measure factors f would be 0.5, but with my measure factors this fraction would be changed to $f = 0.85$, which would then give $n \sim 6$ even without the natural global warming effects of rising solar flux. On the alternative assumption that there is only one gigayear left for life on earth, my measure factors change Carter's $f = 5/6$ to $f = 0.94$ and hence give the number of hard steps as $n \sim f/(1-f) \sim 16$.

Therefore, if we could really learn what the number of hard steps were for the evolution of intelligent life here on earth, we could in principle test between different proposals for the measure, such as my proposed spatial density averaging and Agnesi weighting. However, this currently seems like a very hard problem. (Would it be another hard step for intelligent life to solve it?) All I can say at present is that it does not seem obviously in contradiction with observations that the number of hard steps might be higher than Carter's estimates, so our present knowledge does not appear to provide strong evidence against the proposed spatial density averaging and Agnesi weighting.

Conclusions

Agnesi weighting gives a precise weighting factor that may be an improvement over the indefinite cutoff proposed by other proposals, such as the scale-factor measure. Unlike what occurs in the latter, in which time comes to a sharp end at an unspecified time, in Agnesi weighting old universes never die, they just fade away.

When combined with number density averaging (which I previously called volume averaging [51]) and with a suitable quantum state for the universe (such as the Symmetric-Bounce state [101]), Agnesi weighting gives a finite measure for observations in the universe and appears to avoid the Boltzmann brain problem and other potential problems of cosmological measures. It leads to a very mild youngness effect, but one which is well within the current uncertainties of how rapidly intelligent life is likely to have evolved on earth.

Phenomenologically, Agnesi weighting appears to work well. However, it is surely not the last word on the subject. For one thing, although it is quite simple, it is rather *ad hoc* (like all other solutions to the measure problem proposed so far), so one would like to learn some principle that would justify it or an improvement to it. Second, it is presently formulated only in the semiclassical approximation to quantum cosmology, so one would want a fully quantum version. These challenges will be left for future work.

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