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Supersymmetry restoration in lattice formulations of 2D $\mathcal{N} = (2, 2)$ WZ model based on the Nicolai map

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Abstract

For lattice formulations of the two-dimensional $\mathcal{N} = (2, 2)$ Wess–Zumino (2D $\mathcal{N} = (2, 2)$ WZ) model on the basis of the Nicolai map, we show that supersymmetry (SUSY) and other symmetries are restored in the continuum limit without fine tuning, to all orders in perturbation theory. This provides a theoretical basis for use of these lattice formulations for computation of correlation functions.

Keywords: Supersymmetry, lattice field theory, Nicolai map, continuum limit

1. Introduction

It is believed that at long distance, 2D $\mathcal{N} = (2, 2)$ WZ model with a quasi-homogeneous superpotential[1] provides a Landau–Ginzburg description of $\mathcal{N} = (2, 2)$ superconformal field theories (SCFT) [1, 2, 3, 4, 5, 6, 7, 8, 9]. See §14.4 of Ref. [10] for a review. Although this expectation has been tested in various ways, it is very difficult to confirm this WZ/SCFT correspondence directly by comparing general correlation functions in both theories; 2D WZ model is strongly coupled in low energies and for such a comparison, one needs a certain powerful tool which enables nonperturbative calculation.

In a recent paper [11], Kawai and Kikukawa reconsidered this problem and they computed some correlation functions in 2D WZ model by numerical

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[1] A polynomial $W(\phi)$ of variables ϕ_I ($I = 1, 2, \ldots, N$) is called quasi-homogeneous, when there exist some weights ω_I such that $W(\phi_I \to \Lambda^{\omega_I}\phi_I) = \Lambda W(\phi)$.

simulation of a lattice formulation developed in Ref. [12]. They considered the WZ model with a cubic superpotential $W(\phi) = \lambda\phi^3/3$, which, according to the conjectured correspondence, should provide a Landau–Ginzburg description of the A_2 model. The central charge of the A_2 model is $c = 1$ (the gaussian model) and a (unique) chiral primary field in the NS sector, $\Phi_{0,0}$, which should be given by the scalar field of the WZ model in the infrared, has conformal dimensions $(h, \overline{h}) = (1/6, 1/6)$. Finite-size scalings of scalar two-point functions observed in Ref. [11] are remarkably consistent with the above expectation. Ref. [11] thus certainly demonstrated a use for lattice formulations in studying nonperturbative dynamics of supersymmetric field theory (there exist preceding numerical simulations of the 2D $\mathcal{N} = (2,2)$ WZ model with a massive cubic superpotential $W(\phi) = m\phi^2/2 + \lambda\phi^3/3$ [13, 14, 15, 16, 17]).

Having observed the success of Ref. [11], one is naturally lead to consider the 2D $\mathcal{N} = (2,2)$ WZ model with more general (quasi-homogeneous) superpotentials. It would be interesting to generalize the study of Ref. [11] to $W(\phi) = \lambda\phi^n/n$ with $n > 3$, for example, which is thought to correspond to the A_{n-1} model, or to $W(\phi) = \lambda\phi^n/n + \lambda'\phi\phi'^2/2$ with $n \geq 3$, where ϕ and ϕ' are independent scalar fields, which should correspond to the D_{n+1} model.

Before going into such study of physical questions, however, one has to be sure at least within perturbation theory[2] that symmetries which are broken by lattice regularization (including SUSY) are restored in the continuum limit without tuning lattice parameters. Somewhat surprisingly, such an argument for symmetry restoration in lattice formulations of the 2D $\mathcal{N} = (2,2)$ WZ model is not found in the literature, except those for the cubic superpotential with a single supermultiplet: Ref. [19] for a lattice formulation of Ref. [20] and Ref. [11] for a formulation of Ref. [12]. In fact, at first glance, it appears that rather complicated enumeration of possible symmetry breaking operators is required for an argument for general superpotentials. The purpose of the present article is to point out that there actually exists a very simple way to see the symmetry restoration in the continuum limit for lattice formulations [20, 12, 16] based on the Nicolai map [21, 22, 23, 24, 25] for general superpotentials. We can show that SUSY and other symmetries are restored

[2] In the context of the Landau–Ginzburg description of nontrivial SCFT, one is interested in the WZ model without mass term for which, strictly speaking, 2D perturbation theory is a formal one due to severe infrared divergences. Thus, it is eventually desirable to confirm the symmetry restoration in a non-perturbative manner, as had been done in Ref. [18] for the 2D $\mathcal{N} = (2,2)$ supersymmetric Yang–Mills theory.

in the continuum limit without fine tuning to all orders of perturbation theory.[3]

2. Lattice formulations based on the Nicolai map

Lattice formulations of 2D $\mathcal{N}=(2,2)$ WZ model based on the Nicolai map [20, 12, 16] can be succinctly expressed in the following form (a denotes the lattice spacing):

$$
\begin{aligned}
S_{\text{2DWZ}}^{\text{LAT}} &= Qa^2 \sum_x \left[-\psi_{I-}G_I - \psi_{I+}\eta_I(\phi,\phi^*) - \psi_{I-}\eta_I^*(\phi,\phi^*)\right] \\
&= a^2 \sum_x \Bigg[-G_I^*G_I - G_I\eta_I(\phi,\phi^*) - G_I^*\eta_I^*(\phi,\phi^*) \\
&\qquad\qquad - (\psi_{I+},\psi_{I-}) \begin{pmatrix} \frac{\partial\eta_I}{\partial\phi_J} & \frac{\partial\eta_I}{\partial\phi_J^*} \\ \frac{\partial\eta_I^*}{\partial\phi_J} & \frac{\partial\eta_I^*}{\partial\phi_J^*} \end{pmatrix} \begin{pmatrix} \overline{\psi}_{J-} \\ \overline{\psi}_{J+} \end{pmatrix} \Bigg],
\end{aligned} \tag{1}
$$

where $(\phi_I^{(*)}, \psi_{\pm I}, \overline{\psi}_{\mp I}, G_I^{(*)})$ $(I=1,\,2,\,\ldots,\,N)$ denotes a supermultiplet and the summation over repeated "flavor" indices I, J, $\ldots$ is understood; the superscript in the form $x^{(*)}$ implies either x or x^* throughout this article. Q is one particular spinor component of the $\mathcal{N}=(2,2)$ super transformation[4] and its explicit form is given by

$$
\begin{aligned}
&Q\phi_I = -\overline{\psi}_{I-}, && Q\overline{\psi}_{I-} = 0, \\
&Q\phi_I^* = -\overline{\psi}_{I+}, && Q\overline{\psi}_{I+} = 0, \\
&Q\psi_{I+} = G_I, && QG_I = 0, \\
&Q\psi_{I-} = G_I^*, && QG_I^* = 0.
\end{aligned} \tag{2}
$$

Since this fermionic transformation is nilpotent, $Q^2=0$, the lattice action (1) is manifestly invariant under this transformation, $QS_{\text{2DWZ}}^{\text{LAT}}=0$, for any choice of the functions $\eta_I(\phi,\phi^*)$. Actually, lattice actions in Refs. [20, 12, 16] are actions obtained after integrating over the auxiliary fields G_I (G_I is a "shifted"

[3]There also exists a valid lattice formulation of the 2D $\mathcal{N}=(2,2)$ WZ model on the basis of the SLAC derivative in which SUSY and other symmetries are manifest [26, 27].

[4]The explicit form of the $\mathcal{N}=(2,2)$ super transformation can be found, for example, in Appendix A of Ref. [28]. Spinor components in the present article and those in Ref. [28] are related by: $\psi_+=\psi_R$, $\psi_-=\overline{\psi}_L$, $\overline{\psi}_-=\psi_L$ and $\overline{\psi}_+=\overline{\psi}_R$.

auxiliary field and in the continuum theory, it is defined from the conventional auxiliary field F_I by $G_I \equiv F_I + (\partial_0 + i\partial_1)\phi_I)$. In this article, we instead use representation (1) because with explicit auxiliary fields, the Q transformation is nilpotent even without using the equation of motion. The action (1) is also invariant under the $U(1)_V$ transformation,[5]

$$\psi_I \equiv \begin{pmatrix} \psi_{I+} \\ \psi_{I-} \end{pmatrix} \to e^{-i\alpha}\psi_I, \qquad \overline{\psi}_I \equiv (\overline{\psi}_{I-}, \overline{\psi}_{I+}) \to e^{i\alpha}\overline{\psi}_I. \tag{3}$$

Although the Q-invariance of Eq. (1) holds for any choice of $\eta_I(\phi, \phi^*)$, for the lattice action to have a correct classical continuum limit, $\eta_I(\phi, \phi^*)$ should become in the classical continuum limit a combination that specifies the Nicolai map in 2D $\mathcal{N} = (2, 2)$ WZ model, $\eta_I(\phi, \phi^*) \xrightarrow{a \to 0} \partial W(\phi)/\partial\phi_I - (\partial_0 - i\partial_1)\,\phi_I^*$. (The Nicolai map in 2D $\mathcal{N} = (2, 2)$ WZ model is the field transformation from (ϕ, ϕ^*) to the combination in the right-hand side and its complex conjugate.) Here, $W(\phi)$ is the superpotential, a holomorphic polynomial of scalar fields ϕ_I,

$$W(\phi) = \sum_{\{m\}} \frac{\lambda_{\{m\}}}{\prod_{m_I \neq 0} m_I} \phi_1^{m_1} \phi_2^{m_2} \cdots \phi_N^{m_N}, \tag{4}$$

and $\{m\} \equiv \{m_1, m_2, \ldots, m_N\}$ is a collection of non-negative integers. In what follows, we assume that field variables are chosen so that $W(\phi)$ and thus the scalar potential in the WZ model, $V(\phi, \phi^*) = \sum_I |\partial_I W(\phi)|^2$, do not have any linear tadpole terms. Note that mass dimensions of the scalar fields ϕ_I, the spinor fields ψ_I and the auxiliary fields G_I are 0, 1/2 and 1, respectively. As a consequence, all the coupling constants $\lambda_{\{m\}}$ in Eq. (4) have the mass dimension 1. Also, as an additional requirement, the functions $\eta_I(\phi, \phi^*)$ should be chosen such that the resulting lattice Dirac operator does not have the species doublers.

In the present lattice system (1), the partition function can (almost) be trivialised as in the continuum theory [21, 22, 23, 24, 25], by changing bosonic integration variables from (ϕ, ϕ^*) to (η, η^*). The Jacobian associated with this change of variables precisely cancels the absolute value of the fermion determinant and then the functional integral becomes (after integrating over

[5] The *continuum* action of the 2D $\mathcal{N} = (2, 2)$ WZ model possesses another R-symmetry, a $\mathbb{Z}_2$ symmetry, that is defined by $\phi_I \leftrightarrow \phi_I^*$, $\psi_I \leftrightarrow i\sigma_2\overline{\psi}_I^T$ and $F_I \leftrightarrow F_I^*$.

the auxiliary fields) gaussian one up to a sign factor associated with the fermion determinant. This "almost trivialized" representation provides a remarkable simulation algorithm that is completely free from the critical slowing down and a usual difficulty of massless fermions. See Refs. [13, 11].

So far, three different choices of $\eta_I(\phi,\phi^*)$ (lattice Nicolai map function) have been studied. In Ref. [20], the authors adopted (see Refs. [29, 25, 30] for corresponding Hamiltonian formulations)

$$\eta_I(\phi,\phi^*) = \frac{\partial W(\phi)}{\partial \phi_I} - \left(\partial_0^S - i\partial_1^S\right)\phi_I^* - \frac{a}{2}\sum_\mu \partial_\mu^*\partial_\mu\phi_I, \tag{5}$$

where $\partial_\mu^S \equiv (\partial_\mu^* + \partial_\mu)/2$ and ∂_μ and ∂_μ^* are the forward and backward lattice difference operators, respectively. This choice of the lattice Nicolai map function leads to (we set $\gamma_0 \equiv \sigma_1$, $\gamma_1 \equiv -\sigma_2$ and $\gamma_5 \equiv i\gamma_0\gamma_1 = \sigma_3$),

$$\begin{aligned} S_{\mathrm{2DWZ}}^{\mathrm{LAT}} = a^2 \sum_x \Bigg[& -G_I^* G_I - G_I \eta_I(\phi,\phi^*) - G_I^*\eta_I^*(\phi,\phi^*) \\ & + \overline{\psi}_I \left(D_{\mathrm{w}} + \frac{\partial^2 W(\phi)}{\partial\phi_I\partial\phi_J}\frac{1+\gamma_5}{2} + \frac{\partial^2 W(\phi^*)}{\partial\phi_I^*\partial\phi_J^*}\frac{1-\gamma_5}{2}\right)\psi_J \Bigg], \end{aligned} \tag{6}$$

where D_{w} is the Wilson-Dirac operator,

$$D_{\mathrm{w}} \equiv \frac{1}{2}\sum_\mu \left\{\gamma_\mu(\partial_\mu^* + \partial_\mu) - a\partial_\mu^*\partial_\mu\right\}. \tag{7}$$

In Ref. [16], the authors consider

$$\eta_I(\phi,\phi^*) = \frac{\partial W(\phi)}{\partial \phi_I} - \left(\partial_0^S - i\partial_1^S\right)\phi_I^* + i\frac{a}{2}\sum_\mu \partial_\mu^*\partial_\mu\phi_I. \tag{8}$$

The resulting lattice action is given by Eq. (6) with $D_{\mathrm{w}} \to \widetilde{D}_{\mathrm{w}}$, where the "twisted" Wilson-Dirac operator $\widetilde{D}_{\mathrm{w}}$ is defined by

$$\widetilde{D}_{\mathrm{w}} \equiv \frac{1}{2}\sum_\mu \left\{\gamma_\mu(\partial_\mu^* + \partial_\mu) + ia\gamma_5\partial_\mu^*\partial_\mu\right\}. \tag{9}$$

Finally, in Ref. [12] (see also Ref. [11]),

$$\begin{aligned} & \eta_I(\phi,\phi^*) \\ & = \frac{\partial W(\phi)}{\partial \phi_I} + \left(\phi_I^* - \frac{a}{2}\frac{\partial W(\phi^*)}{\partial\phi_I^*}\right)(S_0 - iS_1) + \left(\phi_I - \frac{a}{2}\frac{\partial W(\phi)}{\partial\phi_I}\right)T, \end{aligned} \tag{10}$$

where S_μ and T denote the matrix elements of

$$
\begin{aligned}
S_\mu &= \frac{1}{2}(\partial_\mu^* + \partial_\mu)(A^\dagger A)^{-1/2}, \\
T &= \frac{1}{a}\left\{1 - \left(1 + \frac{1}{2}a^2 \sum_\mu \partial_\mu^* \partial_\mu\right)(A^\dagger A)^{-1/2}\right\},
\end{aligned} \tag{11}
$$

and the combination $A \equiv 1 - aD_{\mathrm{w}}$ is defined from Wilson-Dirac operator (7). The resulting lattice action is

$$
\begin{aligned}
S_{\text{2DWZ}}^{\text{LAT}} = a^2 \sum_x \Bigg[&-G_I^* G_I - G_I \eta_I(\phi,\phi^*) - G_I^* \eta_I^*(\phi,\phi^*) \\
&+ \overline{\psi}_I \left(D + \frac{1+\gamma_5}{2}\frac{\partial^2 W(\phi)}{\partial\phi_I \partial\phi_J}\frac{1+\hat{\gamma}_5}{2} + \frac{1-\gamma_5}{2}\frac{\partial^2 W(\phi^*)}{\partial\phi_I^* \partial\phi_J^*}\frac{1-\hat{\gamma}_5}{2}\right)\psi_J\Bigg],
\end{aligned} \tag{12}
$$

where D is the overlap-Dirac operator [31, 32]

$$
D = \begin{pmatrix} T & S_0 + iS_1 \\ S_0 - iS_1 & T \end{pmatrix}, \tag{13}
$$

which fulfills the Ginsparg-Wilson relation [33],

$$
\gamma_5 D + D\hat{\gamma}_5 = 0, \qquad \hat{\gamma}_5 \equiv \gamma_5(1 - aD). \tag{14}
$$

As a result of this relation [34], when the superpotential is quasi-homogeneous, the lattice action possesses an invariance under the discrete subgroup $\mathbb{Z}_n$ of $U(1)_A$ [12] (see below).

3. Perturbative proof of symmetry restoration in the continuum limit

The basic idea of the perturbative proof of symmetry restoration is common to that of Refs. [35, 36, 37, 38]: assuming that symmetries under consideration do not suffer from the anomaly, in the continuum limit, symmetry breaking owing to lattice regularization appears only in local terms in the effective action, which correspond to 1PI diagrams with non-negative superficial degree of divergence. Thus, we enumerate all local (bosonic) polynomials of fields whose mass dimension is less than or equal to two, because terms with the mass dimension higher than two correspond to diagrams with negative superficial degree of divergence. The spacetime integral of these local polynomials must be invariant under Q, Eq. (2), and under $U(1)_V$, Eq. (3), because these are manifest symmetries of the present lattice formulations. From mass dimensions of fields and transformation law (2), we see that the mass dimension of Q is 1/2. Also, under $U(1)_V$, Eq. (3), Q transforms as

$$Q \to e^{i\alpha} Q, \tag{15}$$

as again can be seen from transformation law (2).

A key observation which allows for a quick enumeration of relevant local terms is the triviality of the (local) Q-cohomology. From transformation law (2), it is easy to see that the Q-cohomology is trivial. That is,

$$QX([\varphi]) = 0 \iff X([\varphi]) = QY([\varphi]) + \text{const.}, \tag{16}$$

where $[\varphi]$ collectively denotes all fields and X and Y are local polynomials of fields at point x, for example. Moreover, by combining Eq. (16) with techniques of Refs. [39, 40, 41, 42] (especially the algebraic Poincaré lemma [40]), it is straightforward to show that the local Q-cohomology is also trivial; this means,

$$Q \int d^2x\, X([\varphi]) = 0 \iff X([\varphi]) = QY([\varphi]) + \partial_\mu Z_\mu([\varphi]) + \text{const.}, \tag{17}$$

where all X, Y and Z_μ are local polynomials of fields. This shows that in enumerating Q-invariant local terms in the effective action, we can restrict ourselves to local polynomials of fields of the form QY. (Another possibility, a constant being independent of all fields, has no physical consequence and can be neglected.) Here, the combination Y must contain an odd number of ψ_I (or $\overline{\psi}_I$) for QY to be bosonic. Also, it should be proportional to at least one coupling constant $\lambda_{\{m\}}^{(*)}$, because we are interested in terms induced by radiative corrections (the classical continuum limit reproduces the target theory by construction). Therefore, from the limitation of the mass dimension 2, allowed local terms are at most linear in $\lambda_{\{m\}}^{(*)}$, Q and ψ_I (or $\overline{\psi}_I$). Taking also the $U(1)_V$ symmery into account, only ψ_I is possible. Thus, possible terms are given by a linear combination of

$$\begin{aligned}
&\lambda_{\{m\}}^{(*)} Q\left(f(\phi^*,\phi)\psi_{I\pm}\right)\\
&= \lambda_{\{m\}}^{(*)}\left(-\frac{\partial f(\phi^*,\phi)}{\partial\phi_J^*}\overline{\psi}_{J+}\psi_{I\pm} - \frac{\partial f(\phi^*,\phi)}{\partial\phi_J}\overline{\psi}_{J-}\psi_{I\pm} + f(\phi^*,\phi)G_I^{(*)}\right),
\end{aligned} \tag{18}$$

where $f(\phi^*, \phi)$ is a local monomial of scalar fields. We can see, however, that this combination cannot be induced by perturbative radiative corrections in the above lattice formulations.

Let us first consider the lattice action, Eq. (6) with Eq. (5). For example, the only way to have the last term of Eq. (18) that is linear in $\lambda_{\{m\}}^{(*)}$ and $G_I^{(*)}$, is to connect scalar lines in the vertex

$$a^2 \sum_x G_I^{(*)} \frac{\partial W(\phi^{(*)})}{\partial \phi_I^{(*)}}, \tag{19}$$

to make a 1PI tadpole diagram. However, to have such a diagram, we need a free propagator between ϕ_J and ϕ_K, $\langle \phi_J(x)\phi_K(y)\rangle_0$ (or between ϕ_J^* and ϕ_K^*, $\langle \phi_J^*(x)\phi_K^*(y)\rangle_0$). As can easily be verified from Eqs. (6) and (5), free propagators of these types identically vanish. Note that the lattice action possesses the invariance under $\phi_I \to e^{-i\alpha}\phi_I$ and $\phi_I^* \to e^{i\alpha}\phi_I^*$ in the free theory. Thus the last term of Eq. (18) cannot be induced by radiative corrections.

The situation is similar for other terms in Eq. (18). To have the term containing $\overline{\psi}_{J-}\psi_{I+}$, for example, we have to connect scalar lines in the Yukawa interaction

$$a^2 \sum_x \overline{\psi}_J \frac{\partial^2 W(\phi)}{\partial \phi_J \partial \phi_I} \frac{1+\gamma_5}{2} \psi_I, \tag{20}$$

to make a tadpole. This is again impossible, because we do not have a free propagator of the type $\langle \phi_K(x)\phi_L(y)\rangle_0$.

From these considerations, we observe that *no* local term that corresponds to a 1PI diagram with non-negative superficial degree of divergence is induced by perturbative radiative corrections to the effective action. From this, we infer that all symmetries broken by the lattice regularization are restored in the continuum limit to all orders in perturbation theory.[6] Note that the fact that $\lambda_{\{m\}}^{(*)}$ are dimensionful and the present 2D system is super-renormalizable is crucial for the above proof.

The argument goes almost identically for other lattice actions, because they have common features: Q and $U(1)_V$ invariance[7] and no free propagators

[6] In this regard, one of us (H.S.) would like to apologize the authors of Ref. [20] for his wrong statement made in Ref. [43] that a discrete lattice symmetry $\mathbb{Z}_n$ (see below), which the lattice formulation of Ref. [20] does not have, is crucial for the SUSY restoration. In reality, as shown above, the discrete lattice symmetry is not indispensable for the SUSY restoration.

[7] One can easily modify the above proof so that it does not require the $U(1)_V$ invariance.

of the types $\langle\phi_I(x)\phi_J(y)\rangle_0$ and $\langle\phi_I^*(x)\phi_J^*(y)\rangle_0$, as can easily be verified.

4. Conclusion

In this article, we have shown to all orders in perturbation theory that for lattice formulations of 2D $\mathcal{N}=(2,2)$ WZ model on the basis of the lattice Nicolai map, Eqs. (5), (8) and (10), SUSY and other symmetries broken by lattice regularization are restored in the continuum limit without fine tuning. Our this result provides a theoretical basis for using these lattice formulations for computation of correlation functions.

All the above lattice formulations are thus equivalent in the sense that they all require no fine tuning to reach a SUSY point in the continuum limit. The way of approaching the continuum theory can, however, be different. Generally speaking, a lattice formulation might be regarded superior if higher symmetries are preserved with it. In this respect, the formulation with Eq. (10) is superior, because it possesses a higher symmetry when the superpotential is quasi-homogeneous [12]. When the superpotential is quasi-homogeneous (see footnote 1),

$$W(\phi_I \to e^{i\omega_I\alpha}\phi_I) = e^{i\alpha}W(\phi), \tag{21}$$

and thus the continuum action (after integrating over the auxiliary fields) possesses an invariance under a $U(1)_A$ transformation that is given by,

$$\begin{aligned} &\phi_I \to e^{i\omega_I\alpha}\phi_I, && \phi_I^* \to e^{-i\omega_I\alpha}\phi_I^*, \\ &\psi_I \to e^{i(\omega_I-1/2)\alpha\gamma_5}\psi_I, && \overline{\psi}_I \to \overline{\psi}_I e^{i(\omega_I-1/2)\alpha\gamma_5}. \end{aligned} \tag{22}$$

This symmetry cannot be promoted to a lattice symmetry in the cases of Eq. (5) and Eq. (8), because the resulting (twisted) Wilson term cannot be compatible with the chiral γ_5 rotation. With the choice (10), on the other hand, thanks to the Ginsparg–Wilson relation (14), the part of the action quadratic in the spinor field possesses a lattice $U(1)_A$ symmetry corresponding to Eq. (22):

$$\begin{aligned} &\phi_I \to e^{i\omega_I\alpha}\phi_I, && \phi_I^* \to e^{-i\omega_I\alpha}\phi_I^*, \\ &\psi_I \to e^{i(\omega_I-1/2)\alpha\hat{\gamma}_5}\psi_I, && \overline{\psi}_I \to \overline{\psi}_I e^{i(\omega_I-1/2)\alpha\gamma_5}. \end{aligned} \tag{23}$$

Although this $U(1)_A$ invariance for arbitrary α is broken by a term in the lattice action (after integrating over the auxiliary fields),

$$-\frac{\partial W(\phi)}{\partial\phi_I}(S_0+iS_1)\phi_I, \tag{24}$$

the so-called would-be surface term [12] (and its complex conjugate), not all is lost. Since the above would-be surface term is also quasi-homogeneous with same weights ω_I as $W(\phi)$, a discrete subgroup $\mathbb{Z}_n$ of $U(1)_A$, which is given by Eq. (23) with the angles $\alpha = 2\pi k$, $k = 0, 1, 2, \ldots, n-1$ (where the integer n is determined by weights ω_I), remains an exact lattice symmetry. This exact lattice symmetry could imply a faster approach to the continuum theory; this point deserves further study.

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