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Magnetohydrostatic equilibrium in starspots: dependences on color (T_{eff}) and surface gravity (g)

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Abstract. Temperature contrasts and magnetic field strengths of sunspot umbrae broadly follow the thermal-magnetic relationship obtained from magnetohydrostatic equilibrium. Using a compilation of recent observations, especially in molecular bands, of temperature contrasts of starspots in cool stars, and a grid of Kurucz stellar model atmospheres constructed to cover layers of sub-surface convection zone, we examine how the above relationship scales with effective temperature (T_{eff}), surface gravity g and the associated changes in opacity of stellar photospheric gas. We calculate expected field strengths in starpots and find that a given relative reduction in temperatures (or the same darkness contrasts) yield increasing field strengths against decreasing T_{eff} due to a combination of pressure and opacity variations against T_{eff}.

Keywords. Sun: magnetic fields, sunspots, stars: magnetic fields, stars: spots, stars: activity

1. Introduction

Despite a lack of deductive magnetohydrodynamic explanation for the formation and the equilibrium of a sunspot, extensive observations in combination with magnetohydrostatic models have provided reasonable understanding of the thermal-magnetic structure of sunspots in the observable layers. Reduced temperature and gas pressure inside sunspots dictate dominantly the thermal-magnetic relationship derived from the magneto-hydrostatic balance. Such a relationship causes the Wilson depression – geometrical depression of the observable optical depth unity level within a sunspot –, which provides an explanation for the field strengths observed in sunspots and also relates the intensity (or the brightness) contrasts to field strengths. Here we examine how such a thermal-magnetic relationship scales with the stellar parameters, viz. the effective temperature T_{eff} and surface gravity g as well as the associated changes in the opacity of the stellar photospheric gas. We then discuss the implications of such scalings for the interpretations of observed field strengths. We also discuss how such scalings could be crucial players in the activity related photospheric brightness variations and their correlation with other activity measures.

2. Method of Calculation

Thermal-magnetic relationship for starspots is obtained from a simplified magnetohydrostatic (MHS) condition, that relates the magnetic field strength to the temperature at the axis of a vertical column of magnetic field (Solanki *et al.* (1993)). The radial component of the magneto-hydrostatic force-balance equation (Maltby 1977, Solanki et al. 1993), after neglecting the magnetic curvature force and with the use of the equation of state $P = R\rho T/\mu$, yields the thermal-magnetic relation,

$$\frac{T(r,z)}{T_e(z)} = \frac{\mu(r,z)\rho_e(z)}{\mu_e(z)\rho(r,z)}\left[1 - \frac{B_z^2(r,z)}{8\pi P_e(z)}\right], \tag{2.1}$$

where P, T, ρ, μ and R denote the gas pressure, temperature, density, mean molecular weight and the gas constant respectively. The subscript e stands for the external atmosphere. B_z is the vertical component of magnetic field and r is the radial distance from the center of the spot and z is the depth measured from continuum optical depth $\tau_{c,e}$=1. We further neglect the r-dependence of quantities in the above equation, thus the calculated quantities refer to the axis of the spot, and we refer B_z simply as B hereafter. Eqn.2.1 is valid for each level z. The variation of external atmospheric quantities with depth z are determined by g and T_{eff} of the parent star and are taken from the grid of Kurucz stellar models constructed to cover deeper regions of the convection zone using the ATLAS9 stellar atmosphere code (Kurucz 2001). We prescribe temperature contrasts for starspots under two cases, case (i): use the empirical relation $\Delta T = (590. * logg) - 680K$ (O'Neal *et al.* (1996)) to determine the effective temperature of the spot $T_{eff,spt}$ in a parent star characterised by g and T_{eff}: $T_{spt}(\tau_c = 2/3) = T_{eff,spt} = T_{eff} - \Delta T$, case (ii): the temperature contrasts are independent of g and are of a constant ratio of T_{eff}: $\Delta T = 0.3T_{eff}$ (Berdyugina (2005)). Hydrostatic equilibrium inside the spot yields, to a very good approximation (Cox and Giuli 1968),

$$P(\tau_c = 2/3) = \frac{2}{3}\frac{g}{\kappa_R(\rho, T_{eff,spt})}. \tag{2.2}$$

The density inside the spot at $\tau_c = 2/3$ is determined by solving the above equation with the use of Rosseland mean opacities κ_R from the tables of Kurucz (1993) and Alexander and Ferguson (1994). Saha's equation is used to determine the mean molecular weight. The density difference between the $\tau_c = 2/3$ and $\tau_c = 1$ levels within the spot is assumed to be negligible, i.e., $\rho(\tau_c = 1) \approx \rho(\tau_c = 2/3)$. Using the assumption that $\rho(\tau_c = 1) = \rho_e(z = Z_w)$, i.e., the densities inside and outside the spot are equal at the level of Wilson depression Z_w (Maltby (1977)), we find the depth z at which the above

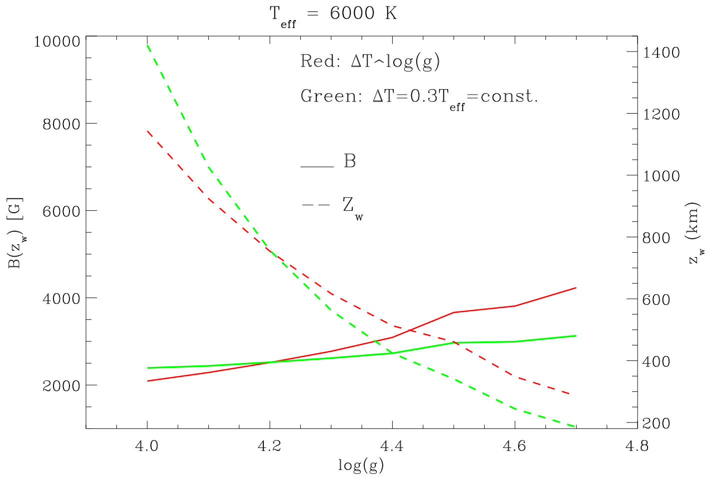

Figure 1. Variation of field strength B (solid lines) and the Wilson depression Z_w (dashed lines) against $log(g)$ for $T_{eff} = 6000K$

equality is satisfied in the parent stellar model. This gives the value of Z_w, and now we determine the only unknown quantity B at this level using Eqn.2.1.

3. Results and Discussions

Figures 1 and 2 summarise the main results of our study. Each figure shows the variation of observable field strengths $B(Z_w)$ on the spot axis as a function of $log(g)$ for a particular T_{eff} for the two distinct kinds of variation of spot temperature given as case (i) (*red curves*) and (ii) (*green curves*) in the previous Section. The Wilson depressions, Z_w, are shown as dashed curves and the B values as solid curves. Z_w are scaled in terms of pressure scale heights H_p at the τ_c=1 level in the quiet photospheres. Fig.1, for $T_{eff} = 6000K$, shows that the B values are not very different for the two cases above, and moreover the Wilson depressions Z_w are of the order of scale heights. In contrast, Figure 2, for $T_{eff} = 5000K$, shows that the results for the above two cases are very different: the gravity dependent temperature contrasts (as implied by the observed relation of O'Neal et al.(1996)) requires very strong magnetic fields for starspot equilibrium, which stems from the large values of Z_w. On the other hand, a constant value of temperature

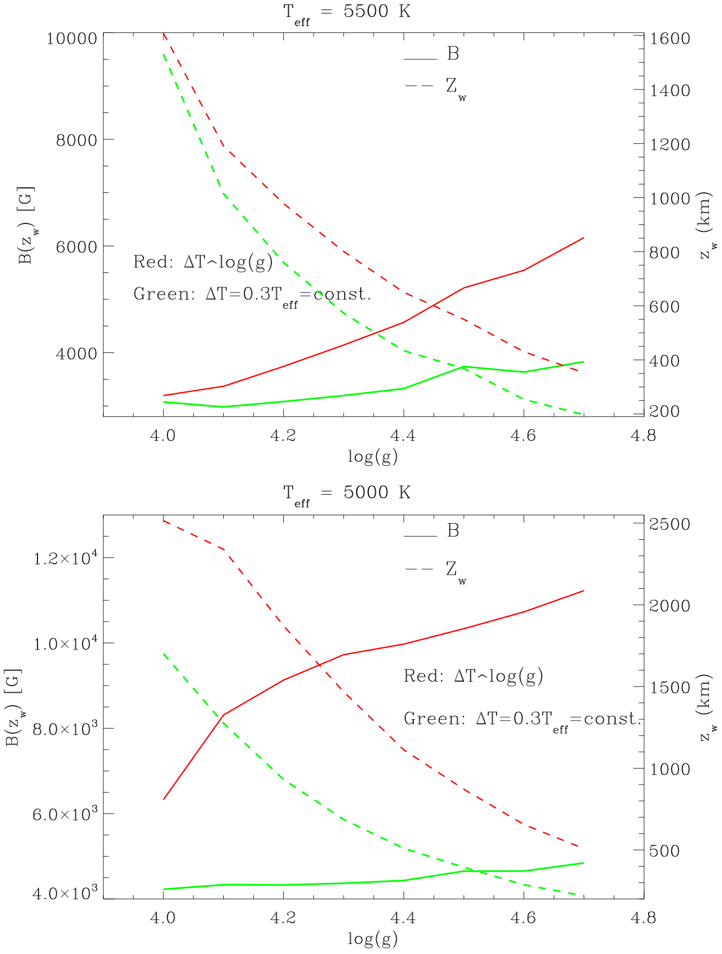

Figure 2. Same as Fig. 1 but for $T_{eff} = 5500K$ (top) and $5000K$ (bottom)

reduction (case ii above) over the range of surface gravity values used yields B values within the observed ranges (see Berdyugina (2005) and references therein). Consequently, a relatively less temperature reduction and hence a less amount of gas evacuation is sufficient to attain a given value of field strength in cooler stars than that in stars hotter than about $5500K$. In other words, spots of a given field strength would appear much darker in a star of $T_{eff} = 6000K$ than those in $T_{eff} = 5000K$.

According to Lockwood et al.(1992), younger and faster rotating stars are 'spot dominated', i.e. more flux in spots than small-scale fields and faculae, and hence grow darker as activity increases. If such a 'spot domination' is purely dependent on age (rotation) but not on color (spectral type), then it could be thought of as a phenomenon not contradictory with the g dependent temperature contrasts for spots derived by O'Neal et al. (1996). However, our results in Figure 2 imply unrealistically large B and Z_w values for spots in such cases. On the other hand, consistency between Lockwood et al.'s results requiring 'spot domination' for younger stars and the situation of case (ii) of our results requires that there be a color T_{eff} dependence of spot properties in addition to the age dependence.

Alternatively, results of Lockwood et al.(1992), but without the requirement of 'spot domination' in younger stars, could be made consistent with our case (ii) of less gas evacuated spots if the small-scale fields forming the 'faculae' too are of such less evacuated state. This would imply that faculae in younger and cooler stars are less bright and therefore these stars grow darker as activity increases. Interestingly, the superadiabaticity that drives the convective collapse of small-scale flux tubes indeed linearly decrease with T_{eff} and is found not to intensify such fields to a high degree of evacuation as in stars hotter than about 5000 K (Rajaguru *et al.* (2002)). Hence, it would appear that the near-surface thermal structure of cool stars crucially determine the key properties of magnetic structures small and large. We conclude that the scaling of thermal and magnetic properties of starspots with both the gravity g (age) and T_{eff} (color) are crucial for a consistent interpretation of observed correlations between activity measures that sample the different heights in the outer atmospheres.

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