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On messengers and metastability in gauge mediation

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Abstract

One notoriously difficult problem in perturbative gauge mediation of supersymmetry breaking via messenger fields is the generic presence of a phenomenologically unacceptable vacuum with messenger vevs, with a lower energy than the desired ("MSSM") vacuum. We investigate the possibility that quantum corrections promote the latter to the ground state of the theory, and find that this is indeed feasible. For this to happen, the couplings of the messengers to the goldstino superfield must be small, and this implies an additional suppression of the MSSM soft terms with respect to the supersymmetry breaking scale. This in turns sets a lower limit on the masses of the messengers and of the supersymmetry breaking fields, which makes both sectors inaccessible at colliders. Contrary to other scenarios like direct gauge mediation, gaugino masses are unsuppressed with respect to scalar masses.

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1 Introduction and conclusions

Gauge mediation of supersymmetry breaking [1] is an attractive way of solving the flavour problem of supersymmetric theories. In its minimal version, it leads to a highly predictive spectrum which has been extensively studied[4] from a phenomenological viewpoint [6, 7]. On the other hand, the construction of an explicit supersymmetry breaking sector[5] coupled to messenger fields [9] responsible for the generation of the MSSM

[4]More recently, there has been an intense activity aiming at providing generalized gauge mediation models [2, 3, 4] and at studying their phenomenology [5].

[5]For a review of the recent progress on the subject, see e.g. Ref. [8].

soft terms leads to instabilities of the scalar potential in the messenger direction, and therefore to dangerous vacua breaking the electric charge and colour. While the desired MSSM vacuum can be locally stable with a lifetime exceeding the age of the Universe [10], it would clearly be more satisfactory to avoid the messenger instabilities. Recently progress was made in this direction in scenarios in which messengers are part of the supersymmetry breaking sector, dubbed direct gauge mediation models [11]. However these generally have difficulties in generating large enough gaugino masses, and more work is needed in order to construct fully realistic models.

The purpose of the present letter is to investigate whether it is possible at all to avoid messenger instabilities in explicit, perturbative supersymmetry breaking models coupled to messenger fields. Based on the analysis of a specific class of models of the O'Raifeartaigh type, we find evidence that this is indeed possible, provided that the coupling of the messengers to the goldstino superfield is sufficiently suppressed with respect to their couplings to other fields from the supersymmetry breaking sector. We shall consider the following class of models, written below in the canonical form of Refs. [12, 13]:

$$W \;=\; f\,X + \frac{1}{2}\left(h_a^{(X)} X + h_a^{(\chi_i)} \chi_i\right) \varphi_a^2 \,+\, m_a \varphi_a Y_a \,+\, \phi\,(\lambda_X X + \lambda_i \chi_i + M)\,\tilde{\phi}\ , \qquad (1)$$

where X is the goldstino superfield, χ_i, φ_a and Y_a are the other fields needed to break supersymmetry, and $(\phi,\, \tilde{\phi})$ are the messenger superfields. Here and in the following, summation over repeated indices is understood. Notice that the R-symmetry of the O'Raifeartaigh sector [14] is broken by the messenger couplings. As we are going to show in Section 3, a necessary condition for avoiding messenger instabilities in the one-loop effective potential reads (written for simplicity in the case of equal O'Raifeartaigh masses $m_a = m$ and with all couplings evaluated at the scale $\mu = m$):

$$|\lambda_X| \;<\; \frac{1}{8\pi^2}\;|\sum_a h_a^{(X)} (\lambda\cdot \bar{h}_a)|\ , \qquad (2)$$

where $(\lambda\cdot\bar{h}_a) \equiv \sum_i \lambda_i \bar{h}_a^{(\chi_i)}$. This result is valid when the masses of the O'Raifeartaigh fields are small compared with the messenger mass M.

We emphasize that, once the condition (2) is imposed, gaugino and scalar masses of the same order of magnitude are generated by loops of messenger fields. In particular, there is no contradiction between the one-loop stability of the MSSM vacuum and non-vanishing gaugino masses. This is to be contrasted with the tree-level supersymmetry breaking models discussed in Ref. [13], in which gaugino masses are not generated at the one-loop level and at leading order in supersymmetry breaking. The class of models we consider evade the conclusions of Ref. [13] because the pseudo-modulus space is not stable at $\lambda_X X + \lambda_i \chi_i + M = 0$.

As we are going to see in Section 3, the one-loop stability of the MSSM vacuum requires heavy messenger and O'Raifeartaigh fields, which are therefore out of reach of the LHC.

2 Generic O'Raifeartaigh models coupled to messenger fields

In this section, we review the tree-level vacuum structure of generic O'Raifeartaigh models coupled to messenger fields, and point out the instability of the scalar potential in the messenger direction. We adopt the parametrization of Refs. [13, 15]:

$$W \;=\; X_i\, f_i(\varphi_a) \,+\, g(\varphi_a) \,+\, \phi\, (\lambda\!\cdot\! X + M)\, \tilde{\phi} \;, \tag{3}$$

where X_i, $i = 1 \cdots N$ and φ_a, $a = 1 \cdots P$ are O'Raifeartaigh fields, $(\phi, \tilde{\phi})$ are messenger fields, and we have defined $\lambda\!\cdot\! X \equiv \sum_i \lambda_i X_i$.

2.1 Tree-level vacuum structure and messenger instability

The F-term equations of motion are given by:

$$\begin{aligned} -\bar{F}_i = f_i(\varphi_a) + \lambda_i \phi\tilde{\phi} \;, \quad & -\bar{F}_a = X_i\; \partial_a f_i(\varphi_b) + \partial_a g(\varphi_b) \;, \\ -\bar{F}_\phi \;=\; (\lambda\!\cdot\! X + M)\, \tilde{\phi} \;, \quad & -\bar{F}_{\tilde{\phi}} \;=\; \phi\, (\lambda\!\cdot\! X + M) \;, \end{aligned} \tag{4}$$

and the tree-level scalar potential reads:

$$\begin{aligned} V \;=\; & \sum_i \left| f_i(\varphi_a) \;+\; \lambda_i \phi\tilde{\phi} \right|^2 \;+\; \sum_a |X_i \partial_a f_i(\varphi_b) + \partial_a g(\varphi_b)|^2 \\ & + |\lambda\!\cdot\! X + M|^2 \,(|\phi|^2 + |\tilde{\phi}|^2) \;. \end{aligned} \tag{5}$$

In the following, we will assume that the MSSM D-term vanish at the minimum of the scalar potential, such that $\langle\phi\rangle = \langle\tilde{\phi}\rangle$. Let us now review the conditions for tree-level supersymmetry breaking. Supersymmetry is broken for $N > P$ in the absence of messenger fields, and for $N > P + 1$ when they are present. With this condition, the equations $F_a = F_\phi = F_{\tilde{\phi}} = 0$ can always be satisfied, leaving $N - P$ tree-level flat directions, which are linear combinations of the fields X_i. The vevs of the fields φ_a, on the contrary, are completely determined and the functions $f_i(\varphi_a)$ can be chosen such that $\langle\varphi_a\rangle = 0$ (this will be the case of the models we will specialize to in Section 3).

The models defined above possess two tree-level vacua (or more precisely two local minima extending to pseudo-moduli spaces):

- a vacuum with vanishing messenger vevs, $\phi = \tilde{\phi} = 0$, and energy

$$V_1 \;=\; f^2 \,, \tag{6}$$

  where we have defined $f^2 \equiv \sum_i \bar{f}_i f_i$. This is the phenomenologically desired vacuum, and we shall refer to it as the MSSM vacuum.

- a vacuum with non-vanishing messenger vevs

$$\phi\tilde{\phi} \;=\; -\frac{1}{|\lambda|^2}\,\bar{\lambda}\cdot f \;, \tag{7}$$

  located at $\lambda\cdot X + M = 0$, and energy

$$V_2 \;=\; \sum_i \left| f_i - \frac{\lambda_i}{|\lambda|^2}\,\bar{\lambda}\cdot f \right|^2 \;=\; f^2 \;-\; \frac{|\bar{\lambda}\cdot f|^2}{|\lambda|^2} \;, \tag{8}$$

  where we have defined $|\lambda|^2 \equiv \sum_i \bar{\lambda}_i \lambda_i$ and $\bar{\lambda}\cdot f \equiv \sum_i \bar{\lambda}_i f_i$. We shall refer to this vacuum as the messenger vacuum.

Comparing Eq. (6) with Eq. (8), we can see that the unwanted messenger vacuum is the ground state of the model. Moreover, the pseudo-moduli space extending the MSSM vacuum is not stable everywhere: at $\lambda\cdot X + M = 0$, $\phi = \tilde{\phi} = 0$ becomes a local maximum and one is driven to the messenger vacuum. This is the vacuum stability problem mentioned in the introduction. The purpose of the present letter is to find appropriate conditions ensuring that the MSSM vacuum is the global minimum of the one-loop effective potential.

2.2 Flat directions and their lifting

In order to remove the instabilities of the tree-level vacuum along the pseudo-moduli space, quantum corrections should stabilize all flat directions of the O'Raifeartaigh sector[6]. A necessary condition for this to happen is that the $N - P$ flat directions

[6]Notice that if we accept to live in a metastable vacuum, this condition is actually sufficient for phenomenological viability.

appear in the fermionic and/or scalar mass matrix, since these matrices determine the one-loop effective potential through the Coleman-Weinberg formula [16]. In the absence of messenger fields, the fermionic mass matrix takes the form:

$$M_F \;=\; \begin{pmatrix} 0 & \partial_a f_i(\varphi) \\ \partial_b f_j(\varphi) & X_i \partial_a \partial_b f_i(\varphi) + \partial_a \partial_b g(\varphi) \end{pmatrix} \,, \tag{9}$$

in which the tree-level flat directions appear through the $P(P+1)/2$ combinations:

$$\chi_{ab} \;\equiv\; X_i \, \partial_a \partial_b f_i(\varphi) \,, \tag{10}$$

out of which $\min\{N, P(P+1)/2\}$ are independent. It is easy to check that the scalar mass matrix depends on the same combinations of fields. Taking into account the condition for supersymmetry breaking, we arrive at the following necessary conditions:

$$P+1 \;<\; N \;\leq\; \frac{P(P+3)}{2} \,. \tag{11}$$

2.3 Comparison of the tree-level vacuum energies

The purpose of this letter is to show that quantum corrections can promote the MSSM vacuum to the ground state of the theory at the price of suppressing the coupling of the messengers to the low-energy goldstino superfield. For this to be possible, the difference between the tree-level MSSM and messenger vacuum energies,

$$\Delta V \;\equiv\; V_1 - V_2 \;=\; -\,\frac{|\bar{\lambda} \cdot f|^2}{|\lambda|^2} \,, \tag{12}$$

should be small compared with V_1 and V_2. This requires:

$$|\bar{\lambda} \cdot f|^2 \;\ll\; |\lambda|^2 f^2 \,. \tag{13}$$

The condition (13) has a simple interpretation in terms of goldstino couplings. The low-energy goldstino superfield is defined by:

$$X \;\equiv\; \frac{1}{f} \sum_i f_i X_i \,, \tag{14}$$

such that, in the MSSM vacuum, $F_X = f$ while the orthogonal combinations χ_i ($i = 1 \cdots N-1$) have vanishing F-terms. Denoting by $\lambda_X \equiv (\lambda \cdot \bar{f})/f$ the coupling of the messengers to the goldstino superfield, we can rewrite the condition (13) in the simpler form:

$$|\lambda_X| \ \ll \ |\lambda| \ = \ \left(|\lambda_X|^2 + \sum_{i=1}^{N-1} |\lambda_{X_i}|^2\right)^{1/2} . \tag{15}$$

When Eq. (15), or equivalently Eq. (14), is satisfied, the tree-level MSSM and messenger vacua are sufficiently close in energy for quantum corrections to significantly affect the vacuum structure of the theory.

3 One-loop corrections to the vacuum energy

We now turn to the explicit computation of the one-loop effective potential in the subclass of models defined by the following superpotential:

$$W \ = \ X_i \, (f_i + \frac{1}{2} h_a^{(i)} \varphi_a^2) \ + \ m_a \, \varphi_a Y_a \ + \ \phi \, (\lambda \cdot X + M) \, \tilde{\phi} \ , \tag{16}$$

where $i = 1 \cdots Q$ and $a = 1 \cdots P$. The F-term equations of motions read:

$$\begin{aligned} -\bar{F}_{X_i} &= f_i + \frac{1}{2} h_a^{(i)} \varphi_a^2 + \lambda_i \phi \tilde{\phi} \ , \quad -\bar{F}_{Y_a} = m_a \varphi_a \ , \\ -\bar{F}_{\varphi_a} &= X_i h_a^{(i)} \varphi_a + m_a Y_a \ , \\ -\bar{F}_{\phi} &= (\lambda \cdot X + M) \tilde{\phi} \ , \quad -\bar{F}_{\tilde{\phi}} = (\lambda \cdot X + M) \phi \ , \end{aligned} \tag{17}$$

and the tree-level scalar potential is given by:

$$\begin{aligned} V \ = \ & \sum_i \left| f_i + \frac{1}{2} h_a^{(i)} \varphi_a^2 + \lambda_i \phi \tilde{\phi} \right|^2 + \sum_a |m_a \varphi_a|^2 + \sum_a \left| X_i h_a^{(i)} \varphi_a + m_a Y_a \right|^2 \\ & + \ |\lambda \cdot X + M|^2 \ (|\tilde{\phi}|^2 + |\phi|^2) \ . \end{aligned} \tag{18}$$

At tree level, $\langle \varphi_a \rangle = \langle Y_a \rangle = 0$ is realized for large enough values of m_a (for instance, in the MSSM vacuum the condition reads $m_a^2 > |\bar{h}_a \cdot f|$). Supersymmetry is broken for any $Q > 1$. In the MSSM vacuum, X_i fields are tree-level flat directions, whereas in the messenger vacuum there are $Q - 1$ flat directions if one imposes the D-term constraint $\phi = \tilde{\phi}$. The fermionic mass matrix has the general form:

$$M_F \ = \ \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix} , \tag{19}$$

where

$$\mathcal{M}_1 \;=\; \begin{pmatrix} h_a^{(i)} X_i & m_a \\ m_a & 0 \end{pmatrix} \tag{20}$$

and

$$\mathcal{M}_2 \;=\; \begin{pmatrix} 0 & \lambda{\cdot}X+M & \lambda_j\tilde{\phi} \\ \lambda{\cdot}X+M & 0 & \lambda_j\phi \\ \lambda_i\tilde{\phi} & \lambda_i\phi & 0 \end{pmatrix} . \tag{21}$$

In order to compute the one-loop vacuum energies, we shall perform the approximate calculation of the effective potential using the one-loop Kähler potential [17]:

$$K^{(1)} \;=\; -\,\frac{1}{32\pi^2}\,\mathrm{Tr}\left(M_F M_F^\dagger \,\ln \frac{M_F M_F^\dagger}{\Lambda^2}\right) . \tag{22}$$

Then the one-loop scalar potential is given by

$$V \;=\; (K^{-1})_{ij}\; F_i \bar{F}_j \;\equiv\; V_0 \;+\; V^{(1)} \;, \tag{23}$$

where at the linearized order in the corrections to the Kahler metric we find:

$$V^{(1)} \;=\; \frac{1}{32\pi^2}\sum_\alpha \left[\frac{\partial^2 \mu_\alpha^2}{\partial X_i \partial \bar{X}_j}\left(\ln\frac{\mu_\alpha^2}{\Lambda^2}+1\right) + \frac{1}{\mu_\alpha^2}\frac{\partial \mu_\alpha^2}{\partial X_i}\frac{\partial \mu_\alpha^2}{\partial \bar{X}_j}\right] F_i \bar{F}_j \;. \tag{24}$$

In Eqs. (23) and (24), $(K^{-1})_{ij}$ is the inverse of the Kähler metric $K_{ij} = \frac{\partial^2 K}{\partial X_i \partial \bar{X}_j}$, and μ_α^2 are the eigenvalues of $M_F M_F^\dagger$.

The eigenvalues of the mass matrix of the O'Raifeartaigh fields φ_a and Y_a, $\mathcal{M}_1$, are easily found. Writing:

$$\mathcal{M}_1^a \mathcal{M}_1^{a\,\dagger} \;=\; \begin{pmatrix} |h_a^{(i)} X_i|^2 + m_a^2 & m_a h_a^{(i)} X_i \\ m_a \bar{h}_a^{(i)} X_i^\dagger & m_a^2 \end{pmatrix} , \tag{25}$$

one obtains the eigenvalues ($a = 1 \cdots P$):

$$\mu_{a,\pm}^2 \;=\; \frac{1}{2}\left(2m_a^2 + |h_a^{(i)} X_i|^2 \pm |h_a^{(i)} X_i| \sqrt{|h_a^{(i)} X_i|^2 + 4m_a^2}\right) , \tag{26}$$

where, without loss of generality, the m_a have been assumed to be real parameters. The contribution of the φ_a, Y_a fields to the effective Kähler potential is then:

$$\begin{aligned} \mathrm{Tr}\left(\mathcal{M}_1 \mathcal{M}_1^\dagger \,\ln\frac{\mathcal{M}_1\mathcal{M}_1^\dagger}{\Lambda^2}\right) \;=\; \sum_a \Bigg\{ & \left(|h_a^{(i)} X_i|^2 + 2m_a^2\right) \ln \frac{m_a^2}{\Lambda^2} \\ & + 2\,|h_a^{(i)} X_i| \sqrt{|h_a^{(i)} X_i|^2 + 4m_a^2}\;\ln \frac{|h_a^{(i)} X_i| + \sqrt{|h_a^{(i)} X_i|^2 + 4m_a^2}}{2m_a} \Bigg\} . \end{aligned} \tag{27}$$

In the absence of messenger fields, the one-loop effective potential can be easily analyzed in the Kähler approximation (in the small supersymmetry breaking limit). In this case the fermion mass matrix reduces to $\mathcal{M}_1$, and the Kähler metric is given by:

$$K_{ij} \;=\; \delta_{ij} + Z_a h_a^{(i)} \bar{h}_a^{(j)} \;, \tag{28}$$

where

$$\begin{aligned} Z_a \;=\; & -\frac{1}{32\pi^2}\left\{ \ln\frac{m_a^2}{\Lambda^2} + 2 - \frac{2m_a^2}{|h_a^{(i)}X_i|^2 + 4m_a^2} \right. \\ & \left. + \frac{2}{|h_a^{(i)}X_i|}\frac{|h_a^{(i)}X_i|^4 + 6m_a^2|h_a^{(i)}X_i|^2 + 4m_a^4}{(|h_a^{(i)}X_i|^2 + 4m^2)^{3/2}} \ln\frac{|h_a^{(i)}X_i| + \sqrt{|h_a^{(i)}X_i|^2 + 4m_a^2}}{2m_a} \right\} . \end{aligned} \tag{29}$$

Let us define $\chi_a \equiv h_a^{(i)} X_i$. The functions Z_a are monotonically decreasing functions of $|\chi_a|$, whose limiting values are given by:

$$\begin{aligned} Z_a\left(|\chi_a| \ll m_a\right) \;&\simeq\; -\frac{1}{32\pi^2}\left(2 + \ln\frac{m_a^2}{\Lambda^2} + \frac{2|\chi_a|^2}{3m_a^2}\right) , \\ Z_a\left(|\chi_a| \gg m_a\right) \;&\simeq\; -\frac{1}{32\pi^2}\ln\frac{|\chi_a|^2}{\Lambda^2} \;. \end{aligned} \tag{30}$$

Since $Z_a \ll 1$, the inverse Kähler metric is simply:

$$K^{-1}_{ij} \;=\; \delta_{ij} \;-\; Z_a\, h_a^{(i)} \bar{h}_a^{(j)} \,, \tag{31}$$

and the one-loop effective potential is:

$$V^{(1)} \;=\; -\, Z_a(|\chi_a|)\; |\bar{h}_a\!\cdot\! f|^2 \,. \tag{32}$$

The effect of the one-loop corrections is to lift the tree-level flat directions and to stabilize the pseudo-moduli fields X_i at the origin. More precisely, all X_i's are stabilized at the origin for $P \geq Q$ if the couplings $h_a^{(i)}$ are generic, while some flat directions are still present for $P < Q$. This can easily be seen by expanding the effective potential (32) for small X_i values:

$$V^{(1)} \;\simeq\; \text{const} \;+\; \frac{1}{32\pi^2}\left(2 + \ln\frac{m_a^2}{\Lambda^2} + \frac{2}{3m_a^2}\,\bar{h}_a^{(i)} h_a^{(j)} \bar{X}_i X_j\right) |\bar{h}_a\!\cdot\! f|^2 \,. \tag{33}$$

All pseudo-moduli fields are stabilized at $X_i = 0$ if the positive matrix

$$M^2_{ij} \;\equiv\; \sum_a \frac{|\bar{h}_a\!\cdot\! f|^2}{m_a^2}\,\bar{h}_a^{(i)} h_a^{(j)} \tag{34}$$

has rank Q. For generic $h_a^{(i)}$ couplings, this is the case for $P \geq Q$ (notice that in the case $P > Q$ the fields χ_a are not independent of each other). One should also keep in mind that the constraint $m_a^2 > |h_a \cdot \bar{f}|$ has to be imposed in order to stabilize the fields φ_a and Y_a at the origin.

Let us now reintroduce the messenger fields. The second part of the fermion mass matrix, coming from the messenger fields, gives:

$$\mathcal{M}_2 \mathcal{M}_2^\dagger = \begin{pmatrix} |\lambda \cdot X + M|^2 + |\lambda|^2 |\tilde{\phi}|^2 & |\lambda|^2 \tilde{\phi} \phi^\dagger & \bar{\lambda}_j \phi^\dagger (\lambda \cdot X + M) \\ |\lambda|^2 \phi \tilde{\phi}^\dagger & |\lambda \cdot X + M|^2 + |\lambda|^2 |\phi|^2 & \bar{\lambda}_j \tilde{\phi}^\dagger (\lambda \cdot X + M) \\ \lambda_i \phi \, (\lambda \cdot X + M)^\dagger & \lambda_i \tilde{\phi} \, (\lambda \cdot X + M)^\dagger & \lambda_i \bar{\lambda}_j (|\phi|^2 + |\tilde{\phi}|^2) \end{pmatrix} . \quad (35)$$

It can be shown that this matrix has $Q - 1$ zero eigenvalues, corresponding to $Q - 1$ tree-level flat directions present both in the MSSM and in the messenger vacuum. The remaining eigenvalues are the solutions of the following equation:

$$\mu^2 \left(\mu^2 - |\lambda|^2 (|\phi|^2 + |\tilde{\phi}|^2) - |\lambda \cdot X + M|^2 \right)^2 = 4 \, |\lambda|^4 \, |\phi \tilde{\phi}|^2 \, |\lambda \cdot X + M|^2 . \quad (36)$$

In the MSSM vacuum, $\phi = \tilde{\phi} = 0$ and another zero eigenvalue is found, corresponding to the Q^{th} flat direction of the tree-level scalar potential.

3.1 One-loop corrections to the MSSM vacuum energy

Due to the vanishing messenger vevs in the MSSM vacuum, the matrix $\mathcal{M}_2 \mathcal{M}_2^\dagger$ has only two equal nonzero eigenvalues $\mu^2 = |\lambda \cdot X + M|^2$. Hence:

$$\mathrm{Tr} \left(\mathcal{M}_2 \mathcal{M}_2^\dagger \ln \frac{\mathcal{M}_2 \mathcal{M}_2^\dagger}{\Lambda^2} \right) = 2 \, |\lambda \cdot X + M|^2 \ln \frac{|\lambda \cdot X + M|^2}{\Lambda^2} . \quad (37)$$

Putting all contributions together, the Kähler metric is given by:

$$K_{ij} = \delta_{ij} + Z_a h_a^{(i)} \bar{h}_a^{(j)} + Z' \lambda_i \bar{\lambda}_j , \quad (38)$$

where the functions $Z_a(|\chi_a|)$ are given by Eq. (29) as before, and

$$Z' = -\frac{1}{16\pi^2} \left(\ln |\lambda \cdot X + M|^2 + 2 \right) . \quad (39)$$

In order to be able to write some analytic minimization conditions, let us assume that the pseudo-moduli fields X_i are stabilized close to the origin, namely that $|X_i| \ll m_a, M$

(later on we will derive condition for this to be the case). We can then expand the one-loop effective potential

$$V^{(1)} \;\simeq\; -Z_a\,|\bar{h}_a\cdot f|^2 \;-\; Z'\,|\bar{\lambda}\cdot f|^2\,, \tag{40}$$

and, for $P \geq Q$, we find a minimum at:

$$M^2_{ij}X_j \;=\; -\,\frac{3\bar{\lambda}_i}{M}\;|\bar{\lambda}\cdot f|^2\,, \tag{41}$$

where the matrix M^2_{ij} has been defined in Eq. (34). The pseudo-moduli fields X_i are therefore stabilized at small values $|X_i| \ll m_a, M$ as soon as $m_a \ll M$ (or even $m_a < M$ if the couplings $h_a^{(i)}$ are of order 1), implying that the messengers cannot be too light. Setting $X_i = 0$ in the effective potential (40) then gives a very good approximation of the one-loop MSSM vacuum energy:

$$V_1 \;=\; f^2 \;+\; \frac{1}{32\pi^2}\left[\sum_a |\bar{h}_a\cdot f|^2\left(\ln\frac{m_a^2}{\Lambda^2}+2\right) + \,2\,|\bar{\lambda}\cdot f|^2\left(\ln\frac{M^2}{\Lambda^2}+2\right)\right]. \tag{42}$$

Using the renormalization group equations of Appendix A, it is easy to show that the $\ln\Lambda$-dependent terms in V_1 are precisely renormalizing the tree-level vacuum energy f^2. One can therefore write:

$$V_1 \;=\; f^2(\mu) \;+\; \frac{1}{32\pi^2}\left[\sum_a |\bar{h}_a\cdot f|^2\left(\ln\frac{m_a^2}{\mu^2}+2\right) + \,2\,|\bar{\lambda}\cdot f|^2\left(\ln\frac{M^2}{\mu^2}+2\right)\right], \tag{43}$$

where the couplings in Eq. (43) are evaluated at the renormalization group scale μ.

3.2 One-loop corrections to the messenger vacuum energy

In the messenger vacuum, one has:

$$\lambda\cdot X + M \;=\; 0\;, \qquad \phi\tilde{\phi} \;=\; -\frac{1}{|\lambda|^2}\,\bar{\lambda}\cdot f\;(1+\epsilon_\phi)\;, \tag{44}$$

where ϵ_ϕ is a one-loop correction to the tree-level messenger vevs. In Eq. (44) we anticipated the fact that $\lambda\cdot X + M = 0$ is also valid at the one-loop level, since there are no anomalous dimensions mixing the messenger fields with the O'Raifeartaigh fields ($\gamma^{\phi}_{X_i} = \gamma^{\tilde{\phi}}_{X_i} = 0$). Thus, $F_\phi = F_{\tilde{\phi}} = 0$ still holds at the one-loop level, and since $\bar{F}_\phi = -(\lambda\cdot X + M)\,\tilde{\phi}$ this also implies $\lambda\cdot X + M = 0$. However, in order to keep the full X_i-dependence of the one-loop effective potential, one must solve Eq. (36) for $\lambda\!\cdot\! X + M \neq 0$. This can be done in the limit $|\lambda\!\cdot\! X + M|^2 \ll |\lambda|^2(|\phi|^2 + |\tilde{\phi}|^2)$, in which:

$$\mu_1^2 \simeq \frac{4|\phi\tilde{\phi}|^2}{(|\phi|^2 + |\tilde{\phi}|^2)^2}\, |\lambda\!\cdot\! X + M|^2 \,, \tag{45}$$

$$\mu_{2,3}^2 \simeq |\lambda|^2(|\phi|^2 + |\tilde{\phi}|^2) \pm \frac{2\,|\lambda\phi\tilde{\phi}|}{(|\phi|^2 + |\tilde{\phi}|^2)^{1/2}}\, |\lambda\!\cdot\! X + M| + \frac{|\phi|^4 + |\tilde{\phi}|^4}{(|\phi|^2 + |\tilde{\phi}|^2)^2}\, |\lambda\!\cdot\! X + M|^2 \,.$$

The Kähler metric then reads:

$$K_{ij} = \delta_{ij} + Z_a h_a^{(i)} \bar{h}_a^{(j)} + Z' \lambda_i \bar{\lambda}_j \;, \tag{46}$$

where

$$Z' = -\frac{1}{16\pi^2} - \frac{1}{16\pi^2(|\phi|^2 + |\tilde{\phi}|^2)^2} \left\{ 2\,|\phi\tilde{\phi}|^2 \left(\ln \frac{4\,|\phi\tilde{\phi}|^2\,|\lambda\!\cdot\! X + M|^2}{(|\phi|^2 + |\tilde{\phi}|^2)^2\,\Lambda^2} + 2 \right) \right.$$
$$\left. + \,(|\phi|^4 + |\tilde{\phi}|^4)\, \ln \frac{|\lambda|^2\,(|\phi|^2 + |\tilde{\phi}|^2)}{\Lambda^2} \right\} . \tag{47}$$

Since $\lambda_i F_i = 0$ in the tree-level messenger vacuum, the term proportional to Z' contributes to the effective potential only at higher loop level. Hence, the one-loop effective potential reduces to:

$$V^{(1)} \simeq -\sum_a Z_a\, |\bar{h}_a\!\cdot\! f + (\bar{h}_a\!\cdot\!\lambda)\, \phi\tilde{\phi}|^2 \,. \tag{48}$$

Minimization of Eq. (48) with respect to ϕ, $\tilde{\phi}$ confirms that the messenger vevs are of the expected form (44) and yields the one-loop vacuum energy[7]:

$$V_2\,(\chi_a) = \sum_i \left| f_i - \frac{\lambda_i}{|\lambda|^2}\, \bar{\lambda}\!\cdot\! f \right|^2 - \sum_a Z_a \left| \bar{h}_a\!\cdot\! f - (\bar{h}_a\!\cdot\!\lambda)\, \frac{\bar{\lambda}\!\cdot\! f}{|\lambda|^2} \right|^2 , \tag{49}$$

where the minimization with respect to the X_i fields remains to be done. Each function $Z_a(|\chi_a|)$ is separately minimized for $\chi_a = 0$. However, for $P > Q$ the χ_a's are not independent variables, so that it is not possible to set all of them to zero. Hence the one-loop messenger vacuum energy will in general be larger than $V_2(\chi_a = 0)$:

$$V_2 > \sum_i \left| f_i - \frac{\lambda_i}{|\lambda|^2}\, \bar{\lambda}\!\cdot\! f \right|^2 + \frac{1}{32\pi^2} \sum_a \left(\ln \frac{m_a^2}{\Lambda^2} + 2 \right) \left| \bar{h}_a\!\cdot\! f - (\bar{h}_a\!\cdot\!\lambda)\, \frac{\bar{\lambda}\!\cdot\! f}{|\lambda|^2} \right|^2 . \tag{50}$$

[7]Since V_2 corresponds to a stationary point of the scalar potential, it does not depend linearly on the one-loop correction ϵ_ϕ. Terms quadratic in ϵ_ϕ would be formally two-loop and have been omitted.

Using the renormalization group equations of Appendix A, one can show that the $\ln \Lambda$-dependent term in V_2, which has exactly the same form as the one in the RHS of Eq. (50), renormalizes the tree-level vacuum energy. One can therefore write:

$$V_2 \; > \; \sum_i \left| f_i - \frac{\lambda_i}{|\lambda|^2}\, \bar{\lambda}\cdot f \right|^2 (\mu) \; + \; \frac{1}{32\pi^2} \sum_a \left(\ln \frac{m_a^2}{\mu^2} + 2 \right) \left| \bar{h}_a \cdot f - (\bar{h}_a \cdot \lambda)\, \frac{\bar{\lambda}\cdot f}{|\lambda|^2} \right|^2 \; . \quad (51)$$

where the couplings in Eq. (51) are evaluated at the renormalization group scale μ.

3.3 Comparison of the one-loop energies of the two vacua

Let us now write the condition for the one-loop energy of the MSSM vacuum to be lower than the one of the messenger vacuum. Using Eqs. (43) and (51), we obtain the following upper bound on $\Delta V \equiv V_1 - V_2$:

$$\begin{aligned} \Delta V \; < \; & |\bar{\lambda}\cdot f|^2 \left\{ \frac{1}{|\lambda|^2} + \frac{1}{32\pi^2} \left[2 \left(\ln \frac{M^2}{\mu^2} + 2 \right) - \frac{1}{|\lambda|^4} \sum_a \left(\ln \frac{m_a^2}{\mu^2} + 2 \right) |\bar{h}_a \cdot \lambda|^2 \right] \right\} \\ & + \frac{1}{32\pi^2 |\lambda|^2} \sum_a \left(\ln \frac{m_a^2}{\mu^2} + 2 \right) \left[\, (h_a \cdot \bar{f})(\bar{h}_a \cdot \lambda)(\bar{\lambda}\cdot f) + \text{c.c.} \,\right] . \end{aligned} \quad (52)$$

The first line in Eq. (52) is dominated by the tree-level term and is therefore positive, while the second line does not have a definite sign. Since the latter is proportional to $\bar{\lambda}\cdot f$, it can overcome the former, which is proportional to $|\bar{\lambda}\cdot f|^2$ (remember that we have required $|\bar{\lambda}\cdot f|^2 \ll |\lambda|^2 f^2$), and promote the MSSM vacuum to the ground state of the theory. For this to happen, a sufficient condition is that the superpotential parameters be such that the RHS of Eq. (52) is negative. This condition simplifies in the case of equal O'Raifeartaigh masses $m_a = m$ to:

$$\begin{aligned} & |\bar{\lambda}\cdot f|^2 \left\{ 1 + \frac{1}{16\pi^2} \left[|\lambda|^2 \left(\ln \frac{M^2}{m^2} + 2 \right) - \sum_a \frac{|\bar{h}_a \cdot \lambda|^2}{|\lambda|^2} \right] \right\} \\ & \qquad\qquad < \; - \frac{1}{8\pi^2}\, \text{Re} \left[(\bar{\lambda}\cdot f) \sum_a (h_a \cdot \bar{f})(\bar{h}_a \cdot \lambda) \right] , \end{aligned} \quad (53)$$

with all couplings evaluated at the renormalization group scale $\mu = m$. Neglecting the terms suppressed by a loop factor in the LHS of Eq. (53), one arrives at the simpler, approximate condition (to be supplemented with an appropriate choice of the coupling phases):

$$|\bar{\lambda}\cdot f| \; < \; \frac{1}{8\pi^2} \left| \sum_a (\lambda \cdot \bar{h}_a)(h_a \cdot \bar{f}) \right| , \quad (54)$$

which is the main result of this paper. In terms of the couplings of the low-energy goldstino superfield, the same condition is expressed by Eq. (2).

Let us now summarize all the requirements we imposed on the superpotential parameters in order to arrive at Eq. (54):

$$|\bar{\lambda}\cdot f| \ll |\lambda| f \ , \qquad m_a \ll M \ , \qquad |\bar{h}_a\cdot f| < m_a^2 \ , \qquad |\bar{\lambda}\cdot f| < M^2 \ . \tag{55}$$

The first inequality is the condition imposed on the difference of the tree-level MSSM and messenger vacuum energies, and is no longer relevant once Eq. (54) is satisfied[8]. The second inequality ensures that the pseudo-moduli fields X_i are stabilized close to the origin in the MSSM vacuum, a fact that was taken into accound in the computation of the MSSM vacuum energy. The last two inequalities are required to avoid the presence of tachyons in the O'Raifeartaigh and messenger sectors, respectively (the fourth one is actually an automatic consequence of the other constraints, which even imply $|\bar{\lambda}\cdot f| \ll M^2$).

4 Final comments

Let us review the assumptions made in the derivation of the condition (54). First it was obtained in a specific class of perturbative supersymmetry breaking models coupled to messenger fields. The computation of the vacuum energies was limited to the one-loop level and made in the Kähler approximation (this is however legitimate in the limit of small supersymmetry breaking, $|\bar{h}_a\cdot f| \ll m_a^2$ and $|\bar{\lambda}\cdot f| \ll M^2$). Finally, the validity of our one-loop computation is strictly speaking limited to the vicinity of the tree-level vacua, and we cannot exclude the presence of other minima in the one-loop scalar potential, although we view this as a rather unlikely possibility. All in all we believe that, while they do not constitue a rigourous proof, our computations and arguments provide strong evidence that quantum corrections can make the MSSM vacuum absolutely stable, even though instabilities in the direction of the messenger fields are present at tree level. An important point is that gaugino masses are not suppressed relative to soft scalar masses, in contrat to the tree-level supersymmetry breaking models discussed in Ref. [13].

Finally, we would like to comment on the constraints set by Eqs. (54) and (55) on the mass scales involved in the class of models we have considered. Imposing a perturbative

[8]For $h_a^{(i)}$ couplings at most of order one, as required by perturbativity, Eq. (54) actually implies $|\bar{\lambda}\cdot f| \ll |\lambda| f$, or a weaker form of it.

upper bound of order 1 on dimensionless parameters, we obtain $|\bar{\lambda}\cdot f| \lesssim m_a^2/(8\pi^2)$ and $|\bar{\lambda}\cdot f| \ll M^2/(8\pi^2)$. Since MSSM soft terms in the few 100 GeV – 1 TeV range require $|\bar{\lambda}\cdot f|/M \sim 100\,\text{TeV}$ in perturbative gauge mediation, heavy O'Raifeartaigh and messenger fields are required:

$$M \gg m_a \gtrsim \sqrt{(10^4 \text{ TeV})M} \,. \tag{56}$$

The minimal allowed values for the various mass scales involved are:

$$m_a \sim 10^5\,\text{TeV} \,, \quad M \sim 10^6\,\text{TeV} \,, \quad f \sim \lambda_X^{-1}\,(10^4\,\text{TeV})^2 \,, \tag{57}$$

where $\lambda_X = (\lambda\cdot\bar{f})/f$ is the goldstino-messenger coupling. As for the masses of the pseudo-moduli fields X_i, they are given by the eigenvalues of the matrix M^2_{ij} defined in Eq. (34) and do not possess a model-independent lower bound; for $h_a^{(i)} \sim 1$ and $\lambda_X \sim 10^{-2}$, they are of order 10^5 TeV. All these states are well beyond the reach of high-energy colliders. Notice that the lowest achievable gravitino mass is of order $m_{3/2} \sim 10^{-2}$ GeV (corresponding to $\lambda_X \sim 10^{-2}$), which allows to evade the most severe BBN problems associated with NLSP decays. Such a gravitino mass is also consistent with gravitino as cold dark matter.

We conclude that heavy messenger and supersymmetry breaking fields seem to be required in order for one-loop corrections to ensure the stability of the MSSM vacuum.

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A Renormalization group equations

The renormalization group equations for the superpotential couplings (16) are:

$$\frac{d}{dt}\,h_a^{(i)} = \frac{1}{16\pi^2}\left[2|h|^2 h_a^{(i)} + \frac{1}{2}\,h_b^{(i)}\bar{h}_b^{(j)}h_a^{(j)} + (h_a\cdot\bar{\lambda})\lambda_i\right] , \tag{A.1}$$

$$\frac{d}{dt}\,\lambda_i = \frac{1}{16\pi^2}\left[3|\lambda|^2\lambda_i + \frac{1}{2}\,(\bar{h}_b\cdot\lambda)h_b^{(i)}\right] , \tag{A.2}$$

$$\frac{d}{dt}\,f_i = \frac{1}{16\pi^2}\left[\frac{1}{2}\,h_a^{(i)}\bar{h}_a^{(j)} + \lambda_i\bar{\lambda}_j\right] f_j \,, \tag{A.3}$$

where $|h|^2 \equiv \sum_a \sum_i h_a^{(i)} \bar{h}_a^{(i)}$.

B More about the messenger vacuum

In Section 3.2, we argued that the term proportional to Z' contributes to the effective potential only at higher loop level, since $\lambda_i F_i = 0$ in the tree-level messenger vacuum. However, since the Z' function (47) diverges for $\lambda{\cdot}X + M \to 0$ one can wonder whether it is legitimate to do so. In this appendix, we propose an alternative computation of the one-loop effective potential in the messenger vacuum, based on the exact inversion of the Kähler metric, which supports the result of Section 3.2.

Starting from the Kähler metric:

$$K_{ij} \;=\; \delta_{ij} + Z_a h_a^{(i)} \bar{h}_a^{(j)} + Z' \lambda_i \bar{\lambda}_j \;, \tag{B.1}$$

we can formally invert it exactly into:

$$\begin{aligned}(K^{-1})_{jk} \;=\;& \delta_{jk} - Z_b\, h_a^{(j)} N_{ab}^{-1} \bar{h}_b^{k} - \frac{Z' Z_c Z_d}{1+|\lambda|^2 Z'}\, (\lambda{\cdot}\bar{h}_c)(\bar{\lambda}{\cdot}h_d)\, h_a^{(j)} N_{ab}^{-1} (M^{-1})^\dagger_{db} \bar{h}_b^{(k)} \\ &+ \frac{Z' Z_b}{1+|\lambda|^2 Z'} \left[h_a^{(j)} M_{ab}^{-1} (\bar{h}_b{\cdot}\lambda) \bar{\lambda}_k + \lambda_j (\bar{\lambda}{\cdot}h_b) (M^{-1})^\dagger_{ba} h_a^{(k)} \right] \\ &- \frac{Z'}{1+|\lambda|^2 Z'} \left[1 + \frac{Z' Z_b}{1+|\lambda|^2 Z'}\, (\bar{\lambda}{\cdot}h_a)(\bar{h}_b{\cdot}\lambda) M_{ab}^{-1} \right] \lambda_j \bar{\lambda}_k \;,\end{aligned} \tag{B.2}$$

where the matrices M and N are defined by:

$$\begin{aligned} M_{ab} \;&=\; \delta_{ab} \,+\, Z_a\, \bar{h}_a{\cdot}h_b \,-\, \frac{Z_a Z'}{1+|\lambda|^2 Z'}\, (\bar{h}_a{\cdot}\lambda)(\bar{\lambda}{\cdot}h_b) \;, \\ N_{ab} \;&=\; \delta_{ab} \,+\, Z_a\, \bar{h}_a{\cdot}h_b \;. \end{aligned} \tag{B.3}$$

In the limit $Z' \gg 1$, $Z_a \ll 1$, we obtain:

$$\begin{aligned}(K^{-1})_{jk} \;=\;& \delta_{jk} - \frac{\lambda_j \bar{\lambda}_k}{|\lambda|^2} - Z_a \left[h_a^{(j)} \bar{h}_a^{(k)} - \frac{\bar{h}_a{\cdot}\lambda}{|\lambda|^2}\, h_a^{(j)} \bar{\lambda}_k - \frac{h_a{\cdot}\bar{\lambda}}{|\lambda|^2}\, \bar{h}_a^{(k)} \lambda_j \right. \\ &\left. + \frac{(\bar{\lambda}{\cdot}h_a)(\bar{h}_a{\cdot}\lambda)}{|\lambda|^4}\, \lambda_j \bar{\lambda}_k \right] + \mathcal{O}\left(\frac{1}{Z'}, Z_a \right) .\end{aligned} \tag{B.4}$$

Thus, even though Z' diverges, the inverse Kähler metric remains finite. It is interesting to note that taking the limit $Z' \to \infty$ leaves a term in Eq. (B.4) which is not suppressed by a loop factor. Now the one-loop vacuum energy reads:

$$V_2(\chi_a) = (K^{-1})_{ij} \, F_i \bar{F}_j = f^2 - \frac{1}{|\lambda|^2} |\bar{\lambda}\cdot f|^2 - Z_a \left| \bar{h}_a \cdot f - (\bar{h}_a \cdot \lambda) \frac{\bar{\lambda}\cdot f}{|\lambda|^2} \right|^2 , \qquad (B.5)$$

(where we have inserted the tree-level messenger vevs $\phi\tilde{\phi} = -\bar{\lambda}\cdot f/|\lambda|^2$), in agreement with Eq. (49).

References

[1] G. F. Giudice and R. Rattazzi, Phys. Rept. **322** (1999) 419 [arXiv:hep-ph/9801271].

[2] C. Cheung, A. L. Fitzpatrick and D. Shih, JHEP **0807** (2008) 054 [arXiv:0710.3585 [hep-ph]]; P. Meade, N. Seiberg and D. Shih, Prog. Theor. Phys. Suppl. **177**, 143 (2009) [arXiv:0801.3278 [hep-ph]]; L. M. Carpenter, M. Dine, G. Festuccia and J. D. Mason, Phys. Rev. D **79**, 035002 (2009) [arXiv:0805.2944 [hep-ph]]; H. Ooguri, Y. Ookouchi, C. S. Park and J. Song, Nucl. Phys. B **808**, 121 (2009) [arXiv:0806.4733 [hep-th]]; J. Distler and D. Robbins, arXiv:0807.2006 [hep-ph]; K. A. Intriligator and M. Sudano, JHEP **0811**, 008 (2008) [arXiv:0807.3942 [hep-ph]]; K. Benakli and M. D. Goodsell, Nucl. Phys. B **816**, 185 (2009) [arXiv:0811.4409 [hep-ph]]; M. Buican, P. Meade, N. Seiberg and D. Shih, JHEP **0903**, 016 (2009) [arXiv:0812.3668 [hep-ph]]; M. Buican and Z. Komargodski, JHEP **1002**, 005 (2010) [arXiv:0909.4824 [hep-ph]].

[3] E. Dudas, S. Lavignac and J. Parmentier, Nucl. Phys. B **808** (2009) 237 [arXiv:0808.0562 [hep-ph]]; N. J. Craig and D. R. Green, Phys. Rev. D **79** (2009) 065030 [arXiv:0808.1097 [hep-ph]]; M. Ibe and T. T. Yanagida, Phys. Rev. D **81** (2010) 035017 [arXiv:0912.4221 [hep-ph]].

[4] D. Green, A. Katz and Z. Komargodski, arXiv:1008.2215 [hep-th]; M. McGarrie and R. Russo, Phys. Rev. D **82** (2010) 035001 [arXiv:1004.3305 [hep-ph]]; M. McGarrie, arXiv:1009.0012 [hep-ph]; M. Sudano, arXiv:1009.2086 [hep-ph].

[5] S. Abel, M. J. Dolan, J. Jaeckel and V. V. Khoze, JHEP **0912** (2009) 001 [arXiv:0910.2674 [hep-ph]]; S. Shirai, M. Yamazaki and K. Yonekura, JHEP **1006** (2010) 056 [arXiv:1003.3155 [hep-ph]]; S. Abel, M. J. Dolan, J. Jaeckel, V. V. Khoze, M. J. Dolan, J. Jaeckel and V. V. Khoze, arXiv:1009.1164 [hep-ph].

[6] S. Dimopoulos, S. D. Thomas and J. D. Wells, Nucl. Phys. B **488** (1997) 39 [arXiv:hep-ph/9609434]; J. A. Bagger, K. T. Matchev, D. M. Pierce and R. j. Zhang, Phys. Rev. D **55** (1997) 3188 [arXiv:hep-ph/9609444].

[7] M. Drees, R. Godbole and P. Roy, "Theory and phenomenology of sparticles: An account of four-dimensional N=1 supersymmetry in high energy physics", *Hackensack, USA: World Scientific (2004) 555 p*; P. Binetruy, "Supersymmetry: Theory, experiment and cosmology," *Oxford, UK: Oxford Univ. Pr. (2006) 520 p.*

[8] K. A. Intriligator and N. Seiberg, Class. Quant. Grav. **24** (2007) S741 [arXiv:hep-ph/0702069].

[9] M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D **51** (1995) 1362 [arXiv:hep-ph/9408384]; M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53** (1996) 2658 [arXiv:hep-ph/9507378].

[10] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [arXiv:hep-th/0602239].

[11] R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75** (2007) 045022 [arXiv:hep-ph/0612139]; C. Csaki, Y. Shirman and J. Terning, JHEP **0705** (2007) 099 [arXiv:hep-ph/0612241]; S. Abel, C. Durnford, J. Jaeckel and V. V. Khoze, Phys. Lett. B **661** (2008) 201 [arXiv:0707.2958 [hep-ph]]. N. Haba and N. Maru, Phys. Rev. D **76** (2007) 115019 [arXiv:0709.2945 [hep-ph]]; A. Giveon, A. Katz and Z. Komargodski, JHEP **0907** (2009) 099 [arXiv:0905.3387 [hep-th]].

[12] S. Ray, Phys. Lett. B **642** (2006) 137 [arXiv:hep-th/0607172].

[13] Z. Komargodski and D. Shih, JHEP **0904** (2009) 093 [arXiv:0902.0030 [hep-th]].

[14] A. E. Nelson and N. Seiberg, Nucl. Phys. B **416** (1994) 46 [arXiv:hep-ph/9309299].

[15] K. A. Intriligator, N. Seiberg and D. Shih, JHEP **0707** (2007) 017 [arXiv:hep-th/0703281].

[16] S. R. Coleman and E. J. Weinberg, Phys. Rev. D **7** (1973) 1888.

[17] M. T. Grisaru, M. Rocek and R. von Unge, Phys. Lett. B **383** (1996) 415 [arXiv:hep-th/9605149].