

Shock Wave Collisions and Thermalization in AdS₅

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We study heavy ion collisions at strong 't Hooft coupling using AdS/CFT correspondence. According to the AdS/CFT dictionary heavy ion collisions correspond to gravitational shock wave collisions in AdS₅. We construct the metric in the forward light cone after the collision perturbatively through expansion of Einstein equations in graviton exchanges. We obtain an analytic expression for the metric including all-order graviton exchanges with one shock wave, while keeping the exchanges with another shock wave at the lowest order. We read off the corresponding energy-momentum tensor of the produced medium. Unfortunately this energy-momentum tensor does not correspond to ideal hydrodynamics, indicating that higher order graviton exchanges are needed to construct the full solution of the problem. We also show that shock waves must completely stop almost immediately after the collision in AdS₅, which, on the field theory side, corresponds to complete nuclear stopping due to strong coupling effects, likely leading to Landau hydrodynamics. Finally, we perform trapped surface analysis of the shock wave collisions demonstrating that a bulk black hole, corresponding to ideal hydrodynamics on the boundary, has to be created in such collisions, thus constructing a proof of thermalization in heavy ion collisions at strong coupling.

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§1. General Setup: Expansion in Graviton Exchanges

AdS/CFT correspondence conjectures that the dynamics of $\mathcal{N} = 4$ $SU(N_c)$ SYM theory in four space-time dimensions is dual to the type IIB superstring theory on $AdS_5 \times S_5$.¹⁾ In the limit of large number of colors N_c and large 't Hooft coupling $\lambda = g^2 N_c$ (with g the gauge coupling constant) such that $N_c \gg \lambda \gg 1$, AdS/CFT correspondence reduced to the gauge-gravity duality: $\mathcal{N} = 4$ $SU(N_c)$ SYM theory at $N_c \gg \lambda \gg 1$ is dual to (weakly coupled) classical supergravity in AdS₅. Hence the gauge theory dynamics at strong coupling, which includes all-orders quantum effects, is equivalent to the classical dynamics of supergravity. Instead of summing infinite classes of Feynman diagrams in the gauge theory or using other non-perturbative methods, one can simply study classical supergravity in 5 dimensions. For a review of AdS/CFT correspondence see.²⁾

Our goal is to describe the isotropization (and thermalization) of the medium created in heavy ion collisions assuming that the medium is strongly coupled and using AdS/CFT correspondence to study its dynamics. We want to construct a metric in AdS₅ which is dual to an ultrarelativistic heavy ion collision as pictured in Fig. 1. Throughout the discussion we will use Bjorken approximation of the nuclei having an infinite transverse extent and being homogeneous (on the average) in the transverse direction, such that nothing in our problem would depend on the transverse coordinates x^1, x^2 .³⁾

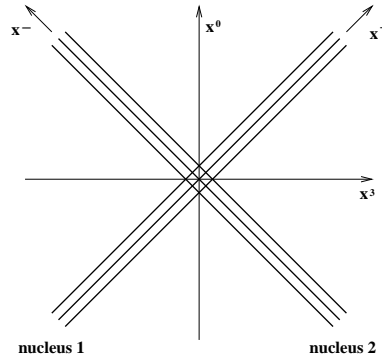


Fig. 1. The space-time picture of the ultrarelativistic heavy ion collision in the center-of-mass frame. The collision axis is labeled x^3 , the time is x^0 .

We start with a metric for a single shock wave moving along a light cone:⁴⁾

$$ds^2 = \frac{L^2}{z^2} \left\{ -2 dx^+ dx^- + \frac{2\pi^2}{N_c^2} \langle T_{--}(x^-) \rangle z^4 dx^{-2} + dx_\perp^2 + dz^2 \right\}. \quad (1.1)$$

Here $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$, z is the coordinate describing the 5th dimension such that the boundary of the AdS space is at $z = 0$, and L is the radius of S_5 . According to holographic renormalization,⁵⁾ $\langle T_{--}(x^-) \rangle$ is the expectation value of the energy-momentum tensor for a single ultrarelativistic nucleus moving along the light-cone in the x^+ -direction in the gauge theory. We assume that the nucleus is made out of nucleons consisting of N_c^2 “valence gluons” each, such that $\langle T_{--}(x^-) \rangle \propto N_c^2$, and the metric (1.1) has no N_c^2 -suppressed terms in it.

The metric in Eq. (1.1) is an exact solution of Einstein equations in AdS₅:

$$R_{\mu\nu} + \frac{4}{L^2} g_{\mu\nu} = 0. \quad (1.2)$$

It can also be represented perturbatively as a single graviton exchange between the source nucleus at the AdS boundary and the location in the bulk where we measure the metric/graviton field. This is shown in Fig. 2, where the solid line represents the nucleus and the wavy line is the graviton propagator. Incidentally a single graviton exchange, while being a first-order perturbation of the empty AdS space, is also an exact solution of Einstein equations. This means higher order tree-level graviton diagrams are zero (cf. classical gluon field of a single nucleus in covariant gauge in the Color Glass Condensate (CGC) formalism⁶⁾).

Now let us try to find the geometry dual to a collision of two shock waves with the metrics like that in Eq. (1.1). Defining $t_1(x^-) \equiv \frac{2\pi^2}{N_c^2} \langle T_{1--}(x^-) \rangle$ and $t_2(x^+) \equiv \frac{2\pi^2}{N_c^2} \langle T_{2++}(x^+) \rangle$ for the energy-momentum tensors of the two shock waves in the boundary theory, we write the metric resulting from such a collision as

$$ds^2 = \frac{L^2}{z^2} \left\{ -2 dx^+ dx^- + dx_\perp^2 + dz^2 + t_1(x^-) z^4 dx^{-2} + t_2(x^+) z^4 dx^{+2} \right. \\ \left. + \text{higher order graviton exchanges} \right\} \quad (1.3)$$

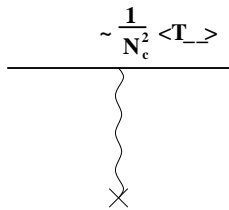


Fig. 2. A representation of the metric (1.1) as a graviton (wavy line) exchange between the nucleus at the boundary of AdS space (the solid line) and the point in the bulk where the metric is measured (denoted by a cross).

The metric of Eq. (1.3) is illustrated in Fig. 3. The first two terms in Fig. 3 (diagrams A and B) correspond to one-graviton exchanges which constitute the individual metrics of each of the nuclei, as shown in Eq. (1.1). Our goal below is to calculate higher-order corrections to these terms, which are illustrated by the diagram C in Fig. 3 and by the ellipsis following it. Fig. 3 illustrates that construction of dual geometry to a shock wave collision in AdS₅ consists of summing up all tree-level graviton exchange diagrams, similar diagrammatically to the classical gluon field formed by heavy ion collisions in CGC.^{7),8)} While classical gluon fields lead to free-streaming final state,⁹⁾ as we will argue below, their AdS graviton “dual” will lead to an ideal hydrodynamic final state for the gauge theory similar to the one found in,⁴⁾ though at the same time different from⁴⁾ due to non-trivial rapidity dependence in the case at hand.

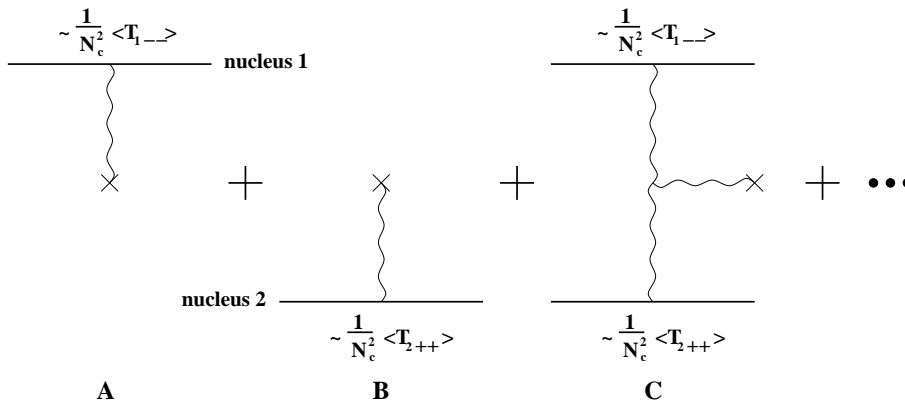


Fig. 3. Diagrammatic representation of the metric in Eq. (1.3). Wavy lines are graviton propagators between the boundary of the AdS space and the bulk. Graphs A and B correspond to the metrics of the first and the second nucleus correspondingly. Diagram C is an example of the higher order graviton exchange corrections.

§2. Perturbative Solution of Einstein Equations

We begin by parametrizing the metric as

$$ds^2 = \frac{L^2}{z^2} \left\{ - [2 + g(x^+, x^-, z)] dx^+ dx^- + [t_1(x^-) z^4 + f(x^+, x^-, z)] dx^{-2} \right.$$

$$+ \left[t_2(x^+) z^4 + \tilde{f}(x^+, x^-, z) \right] dx^{+2} + [1 + h(x^+, x^-, z)] dx_{\perp}^2 + dz^2 \Big\}, \quad (2.1)$$

where the functions f , \tilde{f} , g , and h are zero before the collision, i.e., for either $x^+ < 0$ and/or $x^- < 0$. The exact Einstein equations (1.2) for f , \tilde{f} , g , and h are rather complicated and are not going to be presented here. Instead we are going to solve Einstein equations perturbatively.

To be more specific let us consider in the boundary theory a collision of two ultrarelativistic nuclei with large light-cone momenta per nucleon p_1^+ , p_2^- , and atomic numbers A_1 and A_2 . Writing

$$t_1(x^-) = \mu_1 \delta(x^-), \quad t_2(x^+) = \mu_2 \delta(x^+) \quad (2.2)$$

we want to expand Einstein equations in the powers of μ_1 and μ_2 .^{10),11)} The two scales μ_1 and μ_2 can be expressed terms of physical parameters in the problem^{11),12)}

$$\mu_1 \sim p_1^+ A_1^2 A_1^{1/3}, \quad \mu_2 \sim p_2^- A_2^2 A_2^{1/3}. \quad (2.3)$$

A_1 and A_2 are the typical transverse momentum scales describing the two nuclei,¹¹⁾ similar to the saturation scales. Note that μ_1 and μ_2 are independent of N_c . As follows from a simple dimensional analysis, combined with Lorentz-transformation properties of the relevant quantities, the expansion parameters in four dimensions would be

$$\mu_1 (x^-)^2 x^+, \quad \text{and} \quad \mu_2 (x^+)^2 x^-, \quad (2.4)$$

such that the expansion is valid only at early times, when these parameters are small.

Linearizing Einstein equations (1.2) in f , \tilde{f} , g , and h we solve the obtained system of differential equation to obtain¹¹⁾

$$h(x^+, x^-, z) = h_0(x^+, x^-) z^4 + h_1(x^+, x^-) z^6 \quad (2.5)$$

where h_0 and h_1 are determined by the causal solutions of the following equations

$$(\partial_+ \partial_-)^2 h_0(x^+, x^-) = 8 t_1(x^-) t_2(x^+), \quad (2.6)$$

$$\partial_+ \partial_- h_1(x^+, x^-) = \frac{4}{3} t_1(x^-) t_2(x^+). \quad (2.7)$$

f , \tilde{f} , and g are easily expressed in terms of $h(x^+, x^-, z)$ from Eq. (2.5) (see¹¹⁾).

This lowest-order perturbative solution leads to the energy density of the produced medium (in the center-of-mass frame)^{10),11)}

$$\epsilon(\tau) = \frac{N_c^2}{\pi^2} \mu_1 \mu_2 \tau^2 \quad (2.8)$$

where $\tau = \sqrt{2x^+ x^-}$. The corresponding center-of-mass energy-momentum tensor is

$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & 2\epsilon(\tau) & 0 & 0 \\ 0 & 0 & 2\epsilon(\tau) & 0 \\ 0 & 0 & 0 & -3\epsilon(\tau) \end{pmatrix} \quad (2.9)$$

in terms of the x^0, x^1, x^2, x^3 components. One can see that the longitudinal pressure component in Eq. (2.9) is large and negative. Under boosts the T^{00} and T^{33} components of the energy-momentum tensor mix. This implies that there is a frame in which the energy density is negative $T'_{00} < 0$. At this point it is not clear whether this result presents a problem, as there may be nothing wrong with energy density becoming negative for a short period of time.¹⁰⁾ As we will see below, further evolution of the energy density with time leads to disappearance of this negative energy-density problem.

Higher-order perturbative corrections to the energy-momentum tensor (2.9) in powers of μ_1 and μ_2 have been found in^{10),11)} up to the fourth order in μ_i (i.e., up to $O(\mu_1^3\mu_2), O(\mu_1^2\mu_2^2), O(\mu_1\mu_2^3)$).

§3. All-Order Resummation in μ_2

The exact solution of Einstein equations for the collision of two shock waves involves resummation of both parameters in Eq. (2.4) to all orders: such calculation appears to be very hard to do. Instead one can resum all orders of $\mu_2 (x^+)^2 x^-$ while keeping at the lowest order in $\mu_1 (x^-)^2 x^+$. The corresponding diagram is shown in Fig. 4: in it one resums multiple graviton exchanges with one shock wave (t_2), while exchanging only one graviton with the other shock wave (t_1). By analogy with perturbative QCD calculations we will refer to these class of diagrams as to the “proton-nucleus” scattering, with the shock wave t_1 being the proton and the shock wave t_2 the nucleus. The applicability region of such approximation is defined by

$$\mu_1 (x^-)^2 x^+ \ll 1, \quad \mu_2 (x^+)^2 x^- \sim 1, \quad (3.1)$$

which shows that the resummation is applicable to nucleus-nucleus collisions, only in the part of the forward light-cone defined by Eq. (3.1).

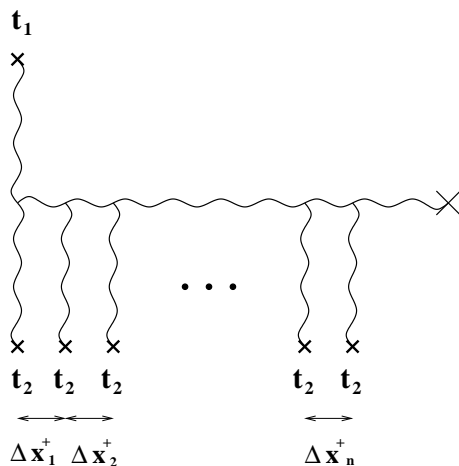


Fig. 4. A diagram contributing to the metric of an asymmetric collision of two shock waves.

Such resummation was performed in¹³⁾ using the eikonal approximation. The

result for the expectation value of the energy-momentum tensor reads¹³⁾

$$\langle T^{++} \rangle = -\frac{N_c^2}{2\pi^2} \frac{4\mu_1\mu_2(x^+)^2\theta(x^+)\theta(x^-)}{[1+8\mu_2(x^+)^2x^-]^{3/2}}, \quad (3.2a)$$

$$\begin{aligned} \langle T^{--} \rangle &= \frac{N_c^2}{2\pi^2} \theta(x^+)\theta(x^-) \frac{\mu_1}{2\mu_2(x^+)^4} \frac{1}{[1+8\mu_2(x^+)^2x^-]^{3/2}} \\ &\times \left[3 - 3\sqrt{1+8\mu_2(x^+)^2x^-} + 4\mu_2(x^+)^2x^- \right. \\ &\left. \times \left(9 + 16\mu_2(x^+)^2x^- - 6\sqrt{1+8\mu_2(x^+)^2x^-} \right) \right], \end{aligned} \quad (3.2b)$$

$$\langle T^{+-} \rangle = \frac{N_c^2}{2\pi^2} \frac{8\mu_1\mu_2x^+x^-\theta(x^+)\theta(x^-)}{[1+8\mu_2(x^+)^2x^-]^{3/2}}, \quad (3.2c)$$

$$\langle T^{ij} \rangle = \delta^{ij} \frac{N_c^2}{2\pi^2} \frac{8\mu_1\mu_2x^+x^-\theta(x^+)\theta(x^-)}{[1+8\mu_2(x^+)^2x^-]^{3/2}}. \quad (3.2d)$$

Provided the complexity of the problem at hand, the resulting formulas (3.2) for the energy-momentum tensor are remarkably simple!

Now we can ask a question: what kind of medium is produced in these strongly coupled proton-nucleus collisions? Is it described by ideal hydrodynamics, just like Bjorken hydrodynamics was obtained in⁴⁾? In our case the produced matter distribution is obviously rapidity-dependent, so it is slightly more tricky to check whether Eqs. (3.2) constitute an ideal hydrodynamics, i.e., whether it can be written as

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu} \quad (3.3)$$

with the positive energy density ϵ and pressure p . $\eta^{\mu\nu}$ is the metric of the four-dimensional Minkowski space-time and u^μ is the fluid 4-velocity.

For the particular case at hand it is easy to see that the energy-momentum tensor in Eq. (3.2) can not be cast in the ideal hydrodynamics form of (3.3). In the case of ideal hydrodynamics one has

$$T^{++} = (\epsilon + p)(u^+)^2 > 0. \quad (3.4)$$

At the same time $\langle T^{++} \rangle$ in Eq. (3.2a) is *negative definite*. Therefore the ideal hydrodynamic description is not achieved in the proton-nucleus collisions. We believe this result is due to limitations of this proton-nucleus approximation. As we will show below, thermalization (black hole production) is inevitable in the collision of the two shock waves considered here. Our conclusion is then that thermalization/isotropization of the medium does not happen in the space-time region defined by the bounds in Eq. (3.1). What we found in Eq. (3.2) is a medium at some intermediate stage, on its way to thermalization at a later time. It appears that one needs to solve the full nucleus-nucleus scattering problem to all orders in both μ_1 and μ_2 , to obtain a medium described by ideal hydrodynamics.

§4. Stopping of Nuclei After Collision

To better understand dynamics of the shock wave collisions let us follow one of the shock waves after the interaction. First we “smear” the delta-function profile of that shock wave:

$$t_1(x^-) = \frac{\mu_1}{a_1} \theta(x^-) \theta(a_1 - x^-). \quad (4.1)$$

Here $a_1 \propto R_1 \frac{A_1}{p_1^+} \propto \frac{A_1^{1/3}}{p_1^+}$, where the nucleus of radius R_1 has A_1 nucleons in it. The “++” component of the energy-momentum tensor of a shock wave after the collision at $x^- = a_1/2$ is¹¹⁾

$$\langle T^{++}(x^+, x^- = a_1/2) \rangle = \frac{N_c^2}{2\pi^2} \frac{\mu_1}{a_1} [1 - 2\mu_2 x^{+2} a_1]. \quad (4.2)$$

The first term on the right of Eq. (4.2) is due to the original shock wave while the second term describes energy loss due to graviton emission. Eq. (4.2) shows that $\langle T^{++} \rangle$ of a nucleus becomes zero at light-cone times (as $p_1^+ \approx p_2^-$ in the center-of-mass frame)

$$x_{stop}^+ \sim \frac{1}{\sqrt{\mu_2 a_1}} \sim \frac{1}{A_2 A_1^{1/6} A_2^{1/6}}. \quad (4.3)$$

Zero $\langle T^{++} \rangle$ would mean *stopping* of the shock wave and the corresponding nucleus. The result can be better understood by doing all-order resummation of graviton exchanges with one shock wave performed above for “proton-nucleus collisions”.¹³⁾ The full result for the proton’s “++” component of the energy-momentum tensor is

$$\langle T^{++} \rangle = \frac{N_c^2}{2\pi^2} \frac{\mu_1}{a_1} \frac{1}{\sqrt{1 + 8\mu_2 (x^+)^2 x^-}}, \quad \text{for } 0 < x^- < a_1. \quad (4.4)$$

Eq. (4.4) is illustrated in Fig. 5, in which one can see that the proton loses all of its light cone momentum over a rather short time.

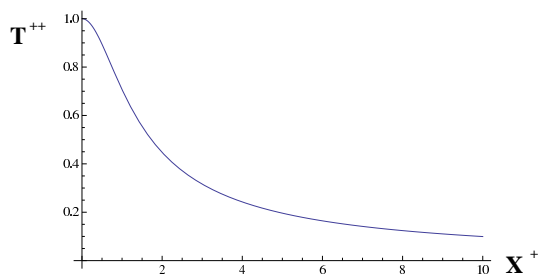


Fig. 5. T^{++} component of the proton’s energy-momentum tensor after the collision as a function of light cone time x^+ (arbitrary units).

We thus conclude that the collision of two nuclei at strong coupling leads to a necessary stopping of the two nuclei shortly after the collision. If the nuclei stop completely in the collision, the strong interactions between them are almost

certain to thermalize the system, probably leading to Landau hydrodynamics.¹⁴⁾ (Rapidity-independent Bjorken hydrodynamics³⁾ seems to be unlikely after stopping. Even before stopping the energy-momentum tensor in Eq. (3.2) is strongly rapidity-dependent.)

§5. Thermalization: Trapped Surface Analysis

While the exact solution of Einstein equations for the colliding shock waves remains elusive, one can infer whether a black hole will be created in the bulk following such collisions by performing a trapped surface analysis.^{15),16)} A trapped surface analysis for shock waves with sources in the bulk has been carried out before in.^{17)–19)} However, with the resulting trapped surface being centered around the sources, it is not clear to what extent the trapped surface is the property of these bulk sources, and what happens to the trapped surface when there are no bulk sources, as is the case for our shock wave (1.1).

Performing a trapped surface analysis for a shock wave without sources (1.1) does not allow one to uniquely fix the position of the trapped surface.²⁰⁾ We therefore start with a shock wave having an extended source in the bulk with the only non-zero component of the bulk energy-momentum tensor being $J_{--} = \frac{E}{z_0 L} \delta(x^-) \delta(z - z_0)$. The corresponding metric is¹⁸⁾

$$ds^2 = \frac{L^2}{z^2} \left\{ -2 dx^+ dx^- + \phi(z) \delta(x^-) dx^{-2} + dx_{\perp}^2 + dz^2 \right\} \quad (5.1)$$

with

$$\phi(z) = \frac{2\pi^2 E L^2}{N_c^2} \begin{cases} \frac{z^4}{z_0^4}, & z \leq z_0 \\ 1, & z > z_0. \end{cases} \quad (5.2)$$

As one can readily see, the metric (5.1) reduces to that in Eq. (1.1) if we send the source to the IR infinity in the bulk by taking $z_0 \rightarrow \infty$ limit of the metric in Eqs. (5.1) and (5.2) keeping μ defined by

$$\mu = \frac{2\pi^2 E L^2}{N_c^2} \frac{1}{z_0^4} \quad (5.3)$$

fixed.²⁰⁾

Marginal trapped surface for a collision of two shock waves given by Eq. (5.1) was found in.¹⁸⁾ In the $z_0 \rightarrow \infty$, μ fixed limit that trapped surface reduces to

$$x^+ = 0, \quad x^- = -\frac{\mu}{2} \left[z^4 - \mu^{-4/3} 2^{-2/3} \right] \quad (5.4)$$

with an analogous expression for the other shock wave obtained by interchanging $x^+ \leftrightarrow x^-$ in Eq. (5.4). (For simplicity we work in the center-of-mass frame where $\mu_1 = \mu_2 = \mu$.) The trapped surface in Eq. (5.4) is independent of the shape of the

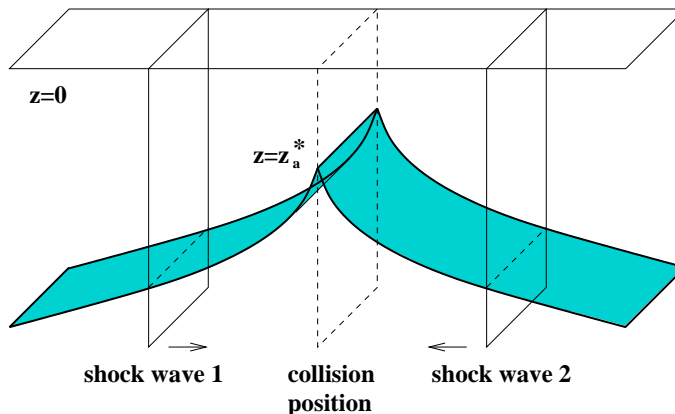


Fig. 6. An illustration of the trapped surface in the collision of two sourceless shock waves. Vertical axis is the bulk z -direction, the horizontal left-right axis can be thought of either as the collision axis or as the time direction. The trapped surface is shaded.

source being sent to deep IR, as shown in.²⁰⁾ The trapped surface from Eq. (5.4) is depicted in Fig. 6.

The existence of trapped surface proves that gravitational collapse is inevitable. That is a black hole will be created in a bulk following a collision of two sourceless shock waves. In the boundary theory this means that a thermal medium is created, which is described by ideal hydrodynamics.

The (lower bound for the) produced entropy per unit transverse area A_{\perp} can be found by calculating the area of the trapped surface, which yields²⁰⁾

$$\frac{S}{A_{\perp}} = \frac{N_c^2}{2\pi^2} (2\mu_1\mu_2)^{1/3}. \quad (5.5)$$

Since the trapped surface analysis does not “know” anything about shock wave thickness (e.g. a_1), we conclude that the thermalization time is only a function of μ_1 and μ_2 , which gives

$$\tau_{th} \sim \frac{1}{(\mu_1\mu_2)^{1/6}}, \quad (5.6)$$

in agreement with the thermalization time suggested in.¹⁰⁾ While numerically this thermalization time is too short to be relevant for RHIC data, it is parametrically shorter than the stopping time (4.3), making our model somewhat more relevant for description of real-life collisions. As $\mu_1\mu_2 \sim s$ with s the center of mass energy of the collision, we get

$$\frac{S}{A_{\perp}} \propto s^{1/3} \quad (5.7)$$

in agreement with the result obtained in.¹⁷⁾ Finally note that, since, as follows from Eq. (2.3), μ_1 and μ_2 are N_c -independent, the produced entropy scales $\propto N_c^2$, in agreement with N_c -counting in a perturbative QCD calculation of particle production for a collision of two nuclei with N_c^2 “valence gluons” in their nucleons.

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