Three-point correlation functions from semiclassical circular strings

Rafael Hernández

Departamento de Física Teórica I
Universidad Complutense de Madrid
28040 Madrid, Spain
rafael.hernandez@fis.ucm.es

Abstract

The strong-coupling limit of three-point correlation functions of local operators can be analyzed beyond the supergravity regime using vertex operators representing spinning string states. When two of the vertex operators correspond to heavy string states having large quantum numbers, while the third operator corresponds to a light state with fixed charges, the correlator can be computed in the large string tension limit by means of a semiclassical approximation. We study the case when the heavy string states are circular string solutions with one AdS_5 spin and three different angular momenta along S^5, for several choices of the light string state.

1 Introduction

Complete resolution of a conformal field theory implies determining the whole spectrum of two and three-point correlation functions of primary operators. Higher order correlation functions can then be written in terms of these two lower ones. In the case of four-dimensional Yang-Mills with $\mathcal{N} = 4$ supersymmetry the spectrum of planar anomalous dimensions of single-trace gauge invariant operators has been exhaustively explored, both in the weak and strong-coupling regimes, after the uncovering of integrable structures in the AdS/CFT correspondence [1]-[4]. It is however unclear whether integrability will also illuminate the evaluation of three-point correlation functions. In the weak-coupling regime three-point functions can be evaluated perturbatively [5]. In the strong-coupling realm a computation at hand within the AdS/CFT correspondence is that of three-point functions for chiral operators, which can be evaluated in the supergravity regime [6]. But in general the calculation of three-point functions requires dealing with primary operators dual to massive string states, which is not a tractable problem within our level of understanding of string theory on $AdS_5 \times S^5$. However, a case beyond the the supergravity limit and still reachable from the correspondence should be that of non-protected operators with large quantum charges, dual to semiclassical spinning string solutions.

The evaluation on the string theory side of the correspondence of correlation functions of single-trace gauge invariant operators is performed by inserting closed string vertex operators in the path integral for the string partion function. Vertex operators scale exponentially with the energy and the quantum charges of the corresponding string state and therefore when the charges are as large as the string tension $\sqrt{\lambda}/2\pi$ the string path integral can be evaluated through a saddle point approximation. The leading contribution to the corresponding correlation functions is thus governed by a semiclassical string configuration. This observation was employed in [7]-[10] to compute two-point correlation functions. The extension to three-point functions has been recently explored in a series of appealing papers [11]-[14], where two of the vertex operators in the correlation function were taken to be semiclassical, or heavy, while the remaining light one was chosen as a massless mode, corresponding to a protected chiral state [11]-[14], or as a massive mode, dual to general non-protected states [14].

The leading order contribution to the correlator of three string vertex operators is then dominated in the large string tension regime by the semiclassical string trajectory coming from the semiclassical operators. The quantum numbers of the heavy vertex operators are much larger than those of the light operator, and thus the contribution to the saddle point from the light operator can be neglected. Therefore, in order to evaluate $\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(x_3)\rangle$ it suffices to obtain the leading classical string configuration saturating the correlation function of the two heavy vertices, $\langle V_{H_1}(x_1)V_{H_2}(x_2)\rangle$, and then evaluate the contribution of the light vertex operator $V_L(x_3)$ on this classical solution,

$$\langle V_{H_1}(x_1)V_{H_2}(x_2)V_L(x_3)\rangle \;= V_L(x_3)_{\text{classical}} \;. \tag{1.1}$$

This observation was employed in reference [14] to suggest a general method able to cover diverse choices of either massless or massive string states for the light vertex operator $V_L(x_3)$. In this note we will closely follow this proposal to explore the case where the classical states associated to the heavy vertex operators in the three-point function are circular string solutions rotating with one AdS_5 spin and three different angular momenta along S^5. The remaining part of the letter is organized as follows. In section 2 we will review some relevant features of the corresponding spinning string solutions. In section 3 we will compute the three-point function coefficients for several choices of light vertex operators. We conclude in section 4 with some prospects and remarks.

2 Circular rotating strings

Semiclassical circular string solutions rotating with several spins and angular momenta in the $AdS_5 \times S^5$ background were analyzed in [15]-[18]. Following notation in there, it will prove useful to parameterize the embedding coordinates of the ten-dimensional background in terms of the global AdS_5 and S^5 angles,

$$Y_1 + iY_2 = \sinh\rho\sin\theta e^{i\phi_1} \;,\;\; Y_3 + iY_4 = \sinh\rho\cos\theta e^{i\phi_2} \;,\;\; Y_5 + iY_0 = \cosh\rho e^{i\,t} \;, \tag{2.1}$$

$$X_1 + iX_2 = \sin\gamma\cos\psi e^{i\varphi_1} \;,\;\; X_3 + iX_4 = \sin\gamma\sin\psi e^{i\varphi_2} \;,\;\; X_5 + iX_6 = \cos\gamma e^{i\varphi_3} \;. \tag{2.2}$$

The Y_M coordinates are related to the Poincaré coordinates in AdS_5 through

$$Y_m = \frac{x_m}{z} \;, \quad Y_4 = \frac{1}{z}\big(-1 + z^2 + x^m x_m\big) \;, \quad Y_5 = \frac{1}{2z}\big(1 + z^2 + x^m x_m\big) \;, \tag{2.3}$$

where $x^m x_m = -x_0^2 + x_i x_i$, with $m = 0, 1, 2, 3$ and $i = 1, 2, 3$. Euclidean rotation allows the classical geodesics to reach the boundary, and comes from continuation of the time-like coordinates to

$$t_e = it \;, \quad Y_{0e} = iY_0 \;, \quad x_{0e} = ix_0 \;. \tag{2.4}$$

The two commuting isometries along the ϕ_i directions and the three along φ_i allow a general ansatz with two spins in AdS_5 and three angular momenta in S^5 [18],

$$Y_1 + iY_2 = b_1 e^{iw_1\tau + ik_1\sigma} \ , \ \ Y_3 + iY_4 = b_2 e^{iw_2\tau + ik_2\sigma} \ , \ \ Y_5 + iY_0 = b_0 e^{i\,t} \ , \tag{2.5}$$

$$X_1 + iX_2 = a_1 e^{i\omega_1\tau + im_1\sigma} \ , \ \ X_3 + iX_4 = a_2 e^{i\omega_2\tau + im_2\sigma} \ , \ \ X_5 + iX_6 = a_3 e^{i\nu\tau} \ , \tag{2.6}$$

where

$$t = \kappa\tau \ , \quad w_a^2 = \kappa^2 + k_a^2 \ , \quad b_0^2 - b_1^2 - b_2^2 = 1 \ , \tag{2.7}$$

with $a = 1, 2$, and

$$\omega_i^2 = m_i^2 + \nu^2 \ , \quad a_1^2 + a_2^2 + a_3^2 = 1 \ , \tag{2.8}$$

with $i = 1, 2, 3$ and $m_3 = 0$, together with the constraints

$$E - \kappa \sum_{a=1}^{2} \frac{S_a}{w_a} = \sqrt{\lambda}\kappa \ , \quad \sum_{i=1}^{3} \frac{J_i}{\omega_i} = \sqrt{\lambda} \ , \tag{2.9}$$

$$2\kappa E - 2\sum_{a=1}^{2} w_a S_a - \sqrt{\lambda}\kappa^2 = 2\sum_{i=1}^{3} \omega_i J_i - \sqrt{\lambda}\nu^2 \ , \tag{2.10}$$

$$\sum_{a=1}^{2} k_a S_a + \sum_{i=1}^{3} m_i J_i = 0 \ . \tag{2.11}$$

Spins along AdS_5 are $S_a = \sqrt{\lambda} b_a^2 w_a$, and the angular momenta along S^5 are $J_i = \sqrt{\lambda} a_i^2 \omega_i$. When all spins and angular momenta are of the same order of magnitude and large, we can solve for ν and κ in the above equations as power series expansions [18],

$$\nu^2 = \frac{J^2}{\lambda} - \sum_{i=1}^{3} m_i^2 \frac{J_i}{J} + \cdots \ , \quad \kappa^2 = \frac{J^2}{\lambda} + \frac{1}{J}\Big(\sum_{i=1}^{3} m_i^2 J_i + 2\sum_{a=1}^{2} k_a^2 S_a\Big) + \cdots \tag{2.12}$$

where we have introduced the total angular momentum $J \equiv J_1 + J_2 + J_3$. The energy is then

$$E = J + S + \frac{\lambda}{2J^2}\Big(\sum_{i=1}^{3} m_i^2 J_i + \sum_{a=1}^{2} k_a S_a\Big) + \cdots \tag{2.13}$$

In what follows we will simply treat the case of a circular string rotating with a single spin S along AdS_5, and three different momenta J_i along S^5. [1]Choosing

$$b_0 = \cosh\rho_0 \ , \quad b_1 = \sinh\rho_0 \ , \quad b_2 = 0 \ , \tag{2.14}$$

[1]The general case of a circular string with two spins along AdS_5 and three angular momenta along S^5 could in principle be also considered. However it leads to

$$Y_4 + Y_5 = \cosh(\kappa\tau_e)\cosh\rho_0 + \sin(-iw_2\tau_e + k_2\sigma)\sinh\rho_0\cos\theta_0 \ ,$$

and thus provides hard to evaluate integrals in the three-point vertices below.

the euclidean continuation in Poincaré coordinates of this solution becomes

$$x_1 = \frac{\cos(-iw_1\tau_e + k_1\sigma)}{\cosh(\kappa\tau_e)} \tanh\rho_0 \ , \quad x_{0e} = \tanh(\kappa\tau_e) \ , \tag{2.15}$$

$$x_2 = \frac{\sin(-iw_1\tau_e + k_1\sigma)}{\cosh(\kappa\tau_e)} \tanh\rho_0 \ , \quad z = \frac{1}{\cosh(\kappa\tau_e)\cosh\rho_0} \ , \tag{2.16}$$

The choice $b_0 = \cosh\rho_0$ fixes where the string is located in the radial coordinate of AdS_5, while rotating in the remaining angular directions.

Our analysis along this note can be easily truncated to cover two different configurations. Setting $\rho_0 = 0$ corresponds to the case of a circular string with three different angular momenta J_i along S^5,

$$\begin{aligned} Y_5 + iY_0 &= e^{i\kappa\tau} \ , \quad X_1 + iX_2 = \sin\gamma_0 \cos\psi e^{i\omega_1\tau + im_1\sigma} \ , \\ X_3 + iX_4 &= \sin\gamma_0 \sin\psi e^{i\omega_2\tau + im_2\sigma} \ , \quad X_5 + iX_6 = \cos\gamma_0 e^{i\nu\tau} \ . \end{aligned} \tag{2.17}$$

The case of a string with just a single spin S and a single momentum J is

$$Y_1 + iY_2 = \sinh\rho_0 e^{iw\tau + ik\sigma} \ , \quad Y_5 + iY_0 = \cosh\rho_0 e^{i\kappa\tau} \ , \quad X_1 + iX_2 = e^{i\omega\tau + im\sigma} \ . \tag{2.18}$$

The contribution of this semiclassical solution to a three-point correlator function was also considered in reference [12].

3 Semiclassical three-point functions

We will now evaluate the leading contribution in the large string tension limit to a three-point correlation function with two complex conjugate heavy vertex operators carrying quantum charges of the order of the string tension and one light operator with order one charges.

Conformal invariance completely fixes the dependence on the location of the vertex operators in a three-point function, up to some coefficent C_{123}. The value of these coefficients can be obtained from a convenient choice for the values of the positions x_1, x_2 and x_3, namely $|x_1| = |x_2| = 1$ and $x_3 = 0$ [10, 14]. [2] Then, as the conformal weights of the heavy operators, $\Delta_{H_1} = \Delta_{H_2}$, are much larger than that of the light operator, Δ_L,

$$\langle V_{H_1}(x_1) V_{H_2}(x_2) V_L(0)\rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_{H_1}}} \ . \tag{3.1}$$

[2] This is indeed the case for the semiclassical trajectory (2.15)-(2.16) at the $\tau_e = \pm\infty$ boundaries.

The three-point correlator reduces to the light vertex operator evaluated on the classical solution saturating the two-point correlation function of the heavy operators, and the value of $\mathcal{C}_3 \equiv C_{123}/C_{12}$ can then be determined through

$$\mathcal{C}_3 = c_\Delta V_L(0)_{\text{classical}} \ , \tag{3.2}$$

where c_Δ is the normalization constant of the light vertex operator. In what follows we will employ the proposal and conventions in reference [14] in order to evaluate the normalized three-point coefficients $\mathcal{C}_3$. The classical states corresponding to the heavy vertex operators will be the circular string solutions described in the previous section, and for the light vertex operators we will consider several different choices.

3.1 Dilaton operator

We will first analyze the case of a light vertex chosen to be the massless dilaton operator,

$$V^{(\text{dilaton})} = (Y_+)^{-\Delta_d} \, (X_z)^j \left[z^{-2}(\partial x_m \bar{\partial} x^m + \partial z \bar{\partial} z) + \partial X_k \bar{\partial} X_k \right] \ , \tag{3.3}$$

where $Y_+ \equiv Y_4 + Y_5$, $X_z \equiv X_5 + iX_6$ and the derivatives are $\partial \equiv \partial_+$ and $\bar{\partial} \equiv \partial_-$. To leading order in the strong-coupling regime the scaling dimension is $\Delta_d = 4 + j$, where we have denoted by j the Kaluza-Klein momentum of the dilaton. The corresponding gauge invariant operator on the gauge theory side is $\text{Tr}(F_{\mu\nu}^2 Z^j + \cdots)$. There is also a fermionic contribution to the dilaton vertex operator, but it is subleading in the large string tension expansion, and thus we can safely take into account only the bosonic terms in all the vertex operators that we will consider. The corresponding gauge invariant operator on the gauge theory side is $\text{Tr}(F_{\mu\nu}^2 Z^j + \cdots)$.

The coefficient of the three-point correlator becomes

$$\mathcal{C}_3^{(\text{dilaton})} = c_\Delta^{(\text{dilaton})} \int_{-\infty}^{\infty} d\tau_e \int_0^{2\pi} d\sigma \, (Y_+)^{-\Delta_d} \, (X_z)^j \left[z^{-2}(\partial x_m \bar{\partial} x^m + \partial z \bar{\partial} z) + \partial X_k \bar{\partial} X_k \right] \ , \tag{3.4}$$

where the normalization constant of the dilaton vertex operator is [19]

$$c_\Delta^{(\text{dilaton})} = \frac{2^{-j/2-1}}{\pi^2}(j+3) \ . \tag{3.5}$$

The contribution from the AdS_5 piece of the circular string ansatz is just κ^2,

$$z^{-2}(\partial x_m \bar{\partial} x^m + \partial z \bar{\partial} z) = \kappa^2 \ , \tag{3.6}$$

while from (2.6) the S^5 contribution is

$$\partial X_k \bar{\partial} X_k = -\nu^2 \ , \tag{3.7}$$

and thus, using that $Y_+ = 1/z$,

$$\mathcal{C}_3^{(\text{dilaton})} = 2\pi c_\Delta^{(\text{dilaton})} \tilde{a}^2 \int_{-\infty}^{\infty} d\tau_e \frac{e^{j\nu\tau_e}}{(\cosh(\kappa\tau_e))^{4+j}} \ , \tag{3.8}$$

where we have defined

$$\tilde{a}^2 \equiv \frac{(\kappa^2 - \nu^2)(1 - a_1^2 - a_2^2)^{j/2}}{(1 + b_1^2)^{2+j/2}} \ , \tag{3.9}$$

with κ^2 and ν^2 as in equation (2.12). The integral over τ_e has been evaluated in [14], and thus our analysis here follows directly from the discussion in there. In the $\nu = 0$ limit the string does not rotate in the (56)-directions, the angular momenta reduce to $J_1 = |m_1| a_1^2$, $J_2 = |m_2| a_2^2$ and $J_3 = 0$, and the three-point vertex is simply

$$\mathcal{C}_{3,\nu=0}^{(\text{dilaton})} = 4\pi^{3/2} c_\Delta^{(\text{dilaton})} \frac{(1 - a_1^2 - a_2^2)^{j/2}}{(4 + j)(1 + b_1^2)^{2+j/2}} \frac{\Gamma\big((j+6)/2\big)}{\Gamma\big((j+5)/2\big)} \kappa \ . \tag{3.10}$$

In the case when $\nu \neq 0$ we find

$$\mathcal{C}_3^{(\text{dilaton})} = 2^{j+5} \pi c_\Delta^{(\text{dilaton})} \tilde{a}^2 \frac{b_+^{(4)} F_-^{(4)} + b_-^{(4)} F_+^{(4)}}{(4+j)^2 \kappa^2 - j^2 \nu^2} \kappa \ , \tag{3.11}$$

where we have defined

$$b_\pm^{(\alpha)} \equiv j + \alpha \pm \frac{j\nu}{\kappa} \quad \text{and} \quad F_\pm^{(\alpha)} \equiv {}_2F_1\left(j + \alpha, \frac{b_\pm^{(\alpha)}}{2}, 1 + \frac{b_\pm^{(\alpha)}}{2}, -1 \right) \ , \tag{3.12}$$

with $\alpha = 4$. In the limit $j = 0$ the coupling is just to the lagrangian, and we get

$$\mathcal{C}_{3,j=0}^{(\text{dilaton})} = \frac{8\pi c_\Delta^{(\text{dilaton})}}{3(1 + b_1^2)^2} \frac{\kappa^2 - \nu^2}{\kappa} \ . \tag{3.13}$$

Let us now concentrate for compactness in the case when the string is moving just along S^5, with three different angular momenta. Equation (3.13) is then [3]

$$\mathcal{C}_{3,j=0}^{(\text{dilaton})} = \frac{8}{3} \pi c_\Delta^{(\text{dilaton})} \frac{\sqrt{\lambda}(2m_1^2 J_1 + 2m_2^2 J_2)}{J\sqrt{J^2 + \frac{\lambda}{J}\left(m_1^2 J_1 + m_2^2 J_2\right)}} \ , \tag{3.14}$$

[3]Comparison with [14] is immediate if we use the condition implied by the Virasoro constraint on the ansatz (2.6), $\kappa^2 = 2\sum_{i=1}^3 a_i^2(\omega_i^2 + m_i^2)$. The three-point function can then be written

$$\mathcal{C}_{3,j=0}^{(\text{dilaton})} = \frac{8}{3} \pi c_\Delta^{(\text{dilaton})} \frac{2a_1^2 m_1^2 + 2a_2^2 m_2^2}{\sqrt{2a_1^2 m_1^2 + 2a_2^2 m_2^2 + \nu^2}} \ .$$

Recalling now that

$$E = \sqrt{J^2 + \frac{\lambda}{J}(m_1^2 J_1 + m_2^2 J_2)}\ , \tag{3.15}$$

our result extends to the case of three different angular momenta the observation in references [12] and [14] that the three-point function is proportional to the derivative with respect to λ of the strong-coupling limit of the anomalous dimension for the corresponding operator,

$$\lambda \frac{\partial E}{\partial \lambda} = \frac{\lambda(m_1^2 J_1 + m_2^2 J_2)}{2J\sqrt{J^2 + \frac{\lambda}{J}(m_1^2 J_1 + m_2^2 J_2)}}\ . \tag{3.16}$$

A similar argument also holds in the more general case of non-vanishing spin along AdS_5. This behavior seems to be a general feature in the case of the light dilaton vertex operator, as argued in [12] from a renormalization group point of view, or in [20] by means of a thermodynamical reasoning.

3.2 Primary scalar operator

The dual to the BMN operator $\mathrm{Tr}Z^j$ is the superconformal primary scalar operator, and the corresponding vertex operator is [19, 13, 14]

$$V^{(\mathrm{primary})} = (Y_+)^{-\Delta_p}(X_z)^j\left[z^{-2}(\partial x_m \bar{\partial} x^m - \partial z \bar{\partial} z) - \partial X_k \bar{\partial} X_k\right]\ , \tag{3.17}$$

where the scaling dimension is now $\Delta_p = j$. The cases where the classical solution is a BMN geodesic or a folded string rotating in S^5 have been considered in [13], while that of a folded spin rotating in AdS_5 has been analyzed in [14]. In this section we will extend the analysis to the case under study in this note, where the semiclassical solution is a circular string rotating in AdS_5 with a single spin S, and with three different angular momenta along S^5. The ansatz (2.5)-(2.6) leads now to

$$\mathcal{C}_3^{(\mathrm{primary})} = 2\pi c_\Delta^{(\mathrm{primary})} \tilde{b}^2 \int_{-\infty}^{\infty} d\tau_e \frac{e^{j\nu\tau_e}}{(\cosh(\kappa\tau_e))^j}\mathcal{I}(\tau_e)\ , \tag{3.18}$$

where

$$\tilde{b}^2 \equiv \left(\frac{1 - a_1^2 - a_2^2}{1 + b_1^2}\right)^{j/2} \quad \text{and} \quad \mathcal{I}(\tau_e) \equiv \frac{2\kappa^2}{\cosh^2(\kappa\tau_e)} - \frac{2}{J}(\sum_{i=1}^{2} m_i^2 J_i + k\, S)\ . \tag{3.19}$$

In the $\nu = 0$ limit we get

$$\mathcal{C}_{3,\nu=0}^{(\mathrm{primary})} = \pi^{3/2} c_\Delta^{(\mathrm{primary})} \tilde{b}^2 \frac{(j-1)\Gamma(j/2)}{\Gamma((j+3)/2)}\kappa\ . \tag{3.20}$$

When $\nu \neq 0$,

$$\mathcal{C}_3^{(\text{primary})} = 2^{j+4}\pi c_\Delta^{(\text{primary})}\tilde{b}^2 \left(\frac{\kappa^2\big(b_+^{(2)}F_-^{(2)} + b_-^{(2)}F_+^{(2)}\big)}{(j+2)^2\kappa^2 - j^2\nu^2} - \frac{\big(b_+^{(0)}F_-^{(0)} + b_-^{(0)}F_+^{(0)}\big)}{8j^2} \right)\kappa\ , \qquad (3.21)$$

where $b_\pm^{(\alpha)}$ and $F_\pm^{(\alpha)}$ are as defined in equation (3.12), with $\alpha = 0, 2$. It is illuminating to consider the limiting case when the classical trajectories from the heavy vertex operators approach BMN geodesics, which correspond to point like-strings. If we take $J_1 = J_2 = S = 0$ and $J_3 = \sqrt{\lambda}\kappa$, relation (3.21) simplifies to

$$\mathcal{C}_3^{(\text{primary})} = 2^{j+3}\pi \frac{j-1}{(j+1)j} c_\Delta^{(\text{primary})}\kappa\ . \qquad (3.22)$$

Recalling now the normalization constant for the BPS operator [13],

$$c_\Delta^{(\text{primary})} = \frac{(j+1)\sqrt{j}}{2^{j+3}\pi N}\sqrt{\lambda}\ , \qquad (3.23)$$

the correlator becomes

$$\mathcal{C}_3^{(\text{primary})} = \frac{1}{N}\sqrt{j}J\ , \qquad (3.24)$$

in agreement with the coefficient for the correlator of three chiral primary operators [6].

3.3 Singlet massive scalar operator

Let us now consider the case where the light vertex operator is taken to be a singlet massive scalar operator, made out of derivatives of the S^5 coordinates [14, 21],

$$V^{(\text{singlet})} = (Y_+)^{-\Delta_r}\big((\partial X_k \partial X_k)(\bar{\partial} X_l \bar{\partial} X_l)\big)^{r/2}\ , \quad \text{with} \ \ r = 2\ , 4\ , \ldots \qquad (3.25)$$

where the scaling dimension is $\Delta_r = 2\sqrt{(r-1)}\lambda^{1/4}$. When $r = 2$ the operator corresponds to a massive string state on the first excited level, and the corresponding dual gauge theory operator is contained within the Konishi multiplet. Higher values of r label the remaining $(r-1)$-th excited levels in the tower of string states.

Using the contribution in (2.5) for the circular string with three different angular momenta along S^5 we easily get

$$V^{(\text{singlet})} = \frac{\kappa^{2r}}{(\cosh(\kappa\tau_e)\cosh\rho_0)^{\Delta_r}}\ , \qquad (3.26)$$

Therefore the coefficient in the three-point function is

$$\mathcal{C}_3^{(\text{singlet})} = \frac{4\pi^{3/2} c_{\Delta_r}^{(\text{singlet})}}{\Delta_r(1+b_1^2)^{\Delta_r/2}} \frac{\Gamma(\Delta_r/2+1)}{\Gamma((\Delta_r+1)/2)}\kappa^{2r-1}\ , \qquad (3.27)$$

as in [14] when $\rho_0 = 0$, and the three-point function behaves also as the $(2r-1)$-th power of the level number of the light string state in the correlator.

The AdS_5 counterpart of this operator produces again an identical result, because

$$\left((\partial Y_K \partial Y_K)(\bar{\partial} Y_L \bar{\partial} Y_L)\right)^{k/2} = \kappa^{2k} \ , \quad \text{with} \ \ k = 2\ , 4\ , \ldots \tag{3.28}$$

The simple structure of the light vertex contribution in both cases happens because the singlet scalar operators are made out of chiral components of the stress tensor, and thus when evaluated on *any* classical trajectory they imply a constant result [14].

4 Concluding remarks

Exhaustive spectroscopy of anomalous dimensions for single-trace gauge invariant operators and energies for the corresponding dual strings rotating in the $AdS_5 \times S^5$ background proved essential in order to uncover the integrable structure of the AdS/CFT correspondence. In this sense, extending the study of three-point functions to general correlators could contribute to clarify whether integrability should also play a role in the evaluation of three-point correlators, and thus in the complete resolution of planar $\mathcal{N} = 4$ Yang-Mills.

In this note we have employed the general proposal in [14] in order to deal with heavy vertex operators corresponding to semiclassical strings rotating in the $AdS_5 \times S^5$ background, and general light vertices. The study of additional spinning string solutions contributing to the heavy vertices, as well as different light vertex operators, is a natural extension of the present approach to three-point functions at strong-coupling. An additional question is the analysis of quadratic fluctuations around the saddle point approximation. Understanding this problem, that could hopefully be treated in generality at least in some restricted sector of the theory, should also help to clarify the general structure of three-point correlators.

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