Fine structure of the Poynting-Robertson effect for a luminous spinning relativistic star

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Abstract

As a sequel to our recent works challenging toward the systematic inclusion of the effect of radiation on the trajectory of a test particle orbiting around a luminous spinning relativistic star eventually aiming at its application to the accretion flow. We explore in the present work the fine structure of the trajectory of test particle just entering the "suspension orbit" under the purpose of a detailed investigation of test particle's trajectory in the vicinity of the "suspension orbit". We end up with a rather puzzling behavior that, contrary to our expectation, the specific angular momentum of the test particle instantly rises instead of decreasing monotonically just before the test particle enters the "suspension orbit".

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I. INTRODUCTION

Astrophysical accretion flow onto massive or compact stars is one of the major concerns in Astronomy and Astrophysics. In the current treatment of the accretion process, the effect of radiation on the inflow has been poorly addressed. Therefore in our recent works([1], [2]), we have been challenging toward the systematic inclusion of the effects of radiation in the accretion process. To this end, we explored the effect of the radiation from a central star on the motion of a single test particle when the central star has the angular momentum and a finite radius to realize that there exists the "suspension orbit" that corresponds to the "critical point" in [3]. There ([1]), the "suspension orbit" has been discovered for the first time and it can be defined as an orbit in which the test particle hovers around the central star at uniform velocity (for more detail, see [1]).

In the present work, we would like to pursue this effort further and in this sense, the present work can be regarded as a sequel to our earlier works ([1], [2]). To be more specific, we explore the fine structure of the trajectory of test particle just entering the "suspension orbit" under the purpose of a detailed investigation of test particle's trajectory in the vicinity of the "suspension orbit". To summarize the main result of our present work, as we shall see shortly in the text, we encounter a rather puzzling behavior that, contrary to our expectation, the specific angular momentum of the test particle instantly rises instead of decreasing monotonically just before the test particle enters the "suspension orbit". Indeed, we find it anomalous as it contradicts to the Kepler's law which is obviously the first principle. In the text of this paper, we will attempt to address the relevant physical interpretation of it.

II. FINE STRUCTURE OF THE TRAJECTORY NEAR THE "SUSPENSION ORBIT"

As we mentioned in the introduction above, in this section we now would like to report on the fine structure of the test particle's trajectory near the "suspension orbit" which exhibits some interesting features. To summarize the motivation behind our current research, so far we have been exploring the dynamics of a test particle orbiting around a luminous relativistic stars ([1] and [2]). And the purpose of such study is to have some insight into the behavior of the accretion flow onto the relativistic stars emitting radiation such as the AGN or the X-ray binaries. In other words, we would like to understand the effect of radiation on the accretion inflow toward stars with strong gravity which has not been addressed in a systematic manner in the literature. To be more specific, based upon our earlier discovery of the "suspension orbit" ([1]) which can be thought of as the generalization of the critical point ([3]) for the case of sufficiently luminous non-rotating relativistic stars and the detailed study of the advent and the effect of the counter drag forces in our recent works ([1] and [2]), in the present work, we studied the behavior of the trajectory of the test particle just entering the "suspension orbit" that we have discovered in our earlier study ([1]). This exploration can be thought of as a detailed investigation of test particle's trajectory in the vicinity of the "suspension orbit". We hope such a study of the fine structure would help our understanding of the nature of "suspension orbit" ([1]). As can be seen in a moment, we will consider the co-rotating since we are interested in the dynamics of a test particle approaching a sufficiently luminous spinning relativistic star.

Now, the geodesic equation for the azimuthal component of test particle's velocity is given below:

$$\begin{aligned}
\frac{dU_\phi}{d\tau} &= -\frac{2}{(1+\cos\alpha)}\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_t^2U_\phi \\
&- 2\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_tU_rU_\phi \\
&- \frac{2(1+\cos\alpha+\cos^2\alpha)}{3(1+\cos\alpha)}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_r^2U_\phi \\
&- \frac{(2-\cos\alpha-\cos^2\alpha)}{3(1+\cos\alpha)}\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)\left(\frac{r^2+U_\phi^2}{r^2}\right)U_\phi \\
&- \frac{4}{(1+\cos\alpha)}\omega(r)\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_tU_\phi^2 \\
&- 2\omega(r)\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_rU_\phi^2 \\
&- \frac{2(2-\cos\alpha-\cos^2\alpha)}{3(1+\cos\alpha)}\left(\frac{\mathcal{J}(r)}{r}\right)\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_tU_\phi^2 \\
&- \frac{(2-\cos\alpha-\cos^2\alpha)}{3(1+\cos\alpha)}(r\mathcal{J}(r))\left(1-\frac{2M}{r}\right)^{-2}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)U_t \\
&- \frac{\sin^2\alpha}{4}(r\mathcal{J}(r))\left(1-\frac{2M}{r}\right)^{-1}\left(\frac{M}{r^2}\right)\left(\frac{L^\infty}{L^\infty_{Edd}}\right)\left(\frac{r^2+2U_\phi^2}{r^2}\right)U_r, \qquad (1)
\end{aligned}$$

where $\sin\alpha = \left(\frac{R}{r}\right)\left(\frac{1-2M/r}{1-2M/R}\right)^{1/2}$ (see [3]) for the radius of the star $R \geq 3M$, the Eddington luminosity $L^\infty_{Edd} \equiv 4\pi mM/\sigma$ is the luminosity of a spherically symmetric source such that at infinity the outward radiation force balances the inward gravity (see [4]), L^∞ is the

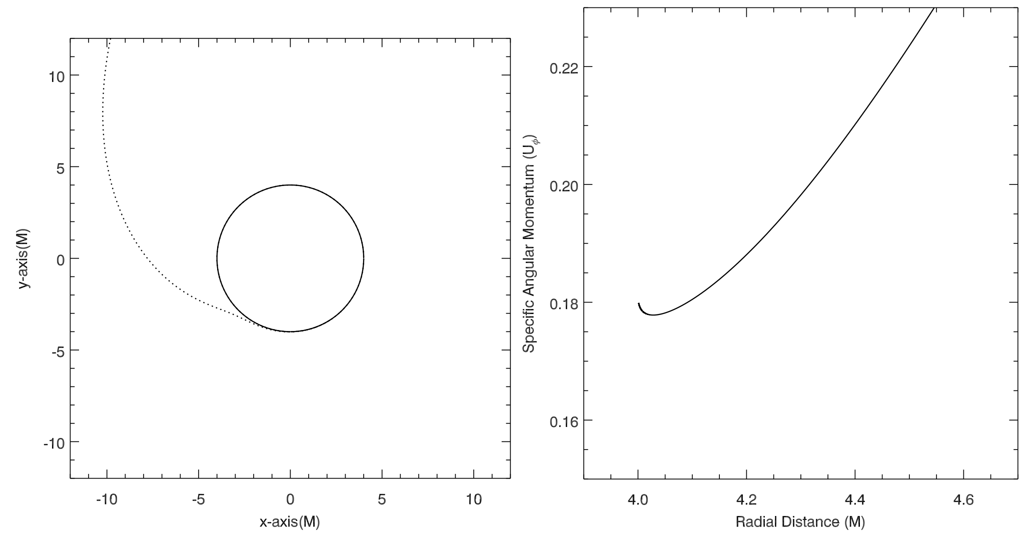

FIG. 1: Left panel shows the trajectory (dotted line) of the (co-rotating) test particle just entering the suspension orbit just outside the surface (solid circle) of the star with luminosity $\left(\frac{L^\infty}{L^\infty_{Edd}}\right) \simeq 0.71$. Right panel shows the profile of the specific angular momentum of the test particle. The central star is spinning counter-clockwise with an uniform angular momentum $j = 0.1$.

luminosity of the star as measured by an observer at infinity, and $\mathcal{J}(r)$ is given by

$$\mathcal{J}(r) = 8j\left(\frac{r}{M}\right)\left(\frac{M^3}{R^3} - \frac{M^3}{r^3}\right) + 4v\left(\frac{r}{R}\right)(1 - 2M/R)^{1/2},$$

where $j \equiv cJ/(GM^2)$ is the dimensionless angular momentum of the star and the average azimuthal velocity v of the radiation source as measured by an observer in the LNRF (Locally Non-Rotating Frame; see [5]; [6]) is calculated to be

$$v = \frac{1}{\pi}j\left(\frac{M^2}{R^2}\right)\left(1 - \frac{2M}{R}\right)^{-1/2}\left[5\left(\frac{R}{M}\right) - 4\right].$$

We now turn to the numerical solutions of the test particle's specific angular momentum that can be represented by the plots (Fig.1). Interestingly enough, it turns out that the numerical solutions reveal contrasting characteristic features between the case when the "suspension orbit" develops just above or below the surface of the star and the other case when the "suspension orbit" emerges at a distance from the star. Therefore, we now start with the first case when the "suspension orbit" develops just above or below the surface of the star.

It is rather puzzling that contrary to our expectation, the specific angular momentum U_ϕ of the test particle instantly rises instead of decreasing monotonically just before the test particle enters the "suspension orbit". This behavior of the fine structure can be interpreted as follows. And to this end, we refer to the azimuthal component of the particle's geodesic equation given above in equation (1). The right hand side of this geodesic equation consists of three groups of terms. The first group represents the well-known Poynting-Robertson effect and among the four terms in this group the second and the third terms are negligible because they are multiplied by the radial component U_r and the azimuthal component U_ϕ which reduce to nearly zero as the test particle enters the "suspension orbit" (recall the defining conditions for the "suspension orbit" first given in [1] according to which U_r drops to zero). As a result, the first and the last terms dominate. The second group appears to encode the frame dragging effect of the central star as each term in this group involves the Lense-Thirring angular velocity $\omega(r)$. The terms in this second group are small enough to be neglected as they are multiplied by U_ϕ^2 which is nearly zero as the test particle enters the "suspension orbit". Lastly, the third group represents the radiation counter drag carefully studied in detail in [1]. This group consists of three terms and each term indicates different origin and hence different physical interpretation. To be more concrete, the first term is negligible as it is multiplied by U_ϕ^2 which is nearly zero as the test particle enters the "suspension orbit" and the third term is nearly zero again as it is multiplied by U_r. As a result, the first and the third terms are both negligible. The second term which appears to have its origin in the frame dragging like the two terms in the second group is non-vanishing and makes dominant contribution as the test particle enters the "suspension orbit". To summarize, the terms on the right hand side of this azimuthal component of the geodesic equation compete with one another governing the behavior of the solution $U_\phi(r)$ and to be more precise, the first and the fourth terms in the first group that can be identified with the Poynting-Robertson effect or drag and the second term in the third group which can be identified with the radiation counter drag compete. As a result of this competition between two terms in the Poynting-Robertson effect represented by the first group and the term in the radiation counter drag represented by the third group, we end up with the puzzling behavior of the test particle's specific angular momentum demonstrated by the plot given above (Fig.1). To be more concrete, it is puzzling as it contradicts to the Kepler's law which dictates that as the test particle falls into the central star, its angular velocity increases monotonically whereas its angular momentum decreases monotonically. We suspect that the advent of such puzzling behavior can be attributed to the role played by the finite size

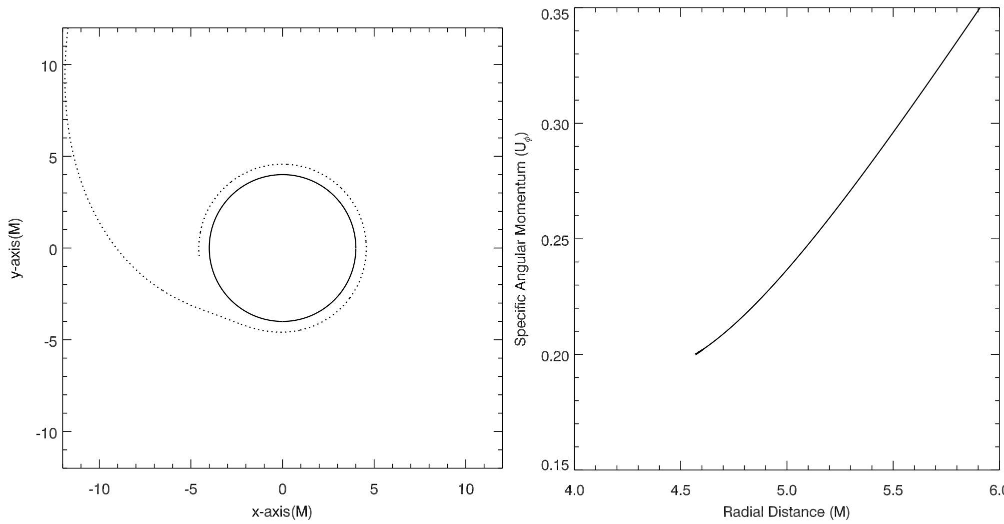

FIG. 2: Left panel shows the trajectory (dotted line) of the (co-rotating) test particle just entering the suspension orbit at some distance from the surface (solid circle) of the star with luminosity $\left(\frac{L^\infty}{L^\infty_{Edd}}\right) \simeq 0.75$. Right panel shows the profile of the specific angular momentum of the test particle. The central star is spinning counter-clockwise with an uniform angular momentum $j = 0.1$.

effect of the source as well as its luminosity and spin because if we neglect the size of the source in the absence of the luminosity and spin, such anomalous behavior would not happen in the first place.

Now, we turn to the other case when the suspension orbit is at a distance from the star's surface. As can be seen in the plots (Fig.2) of the numerical solution, the test particle's specific angular momentum decreases monotonically as the test particle falls into the "suspension orbit" in consistent with our general expectation based upon the Kepler's law stated above. This case, therefore, does not involve problematic issues that call for our careful investigation.

III. DISCUSSION

To summarize, in the present work, as a sequel to our earlier work ([1]) where we studied the trajectory of test particle near luminous rotating relativistic star, we explored the fine structure of the trajectory of test particle just entering the "suspension orbit" under the purpose of a detailed investigation of test particle's trajectory in the vicinity of the "suspension orbit". Indeed, as a main result of our present work, we ended up with a rather puzzling behavior that, contrary to our expectation, the specific angular momentum U_ϕ of the test particle instantly rises instead of decreasing monotonically just before the test particle enters the "suspension orbit". We suspect that the advent of such puzzling behavior can be attributed to the role played by the finite size effect of the source as well as its luminosity and spin because if we neglect the size of the source in the absence of the luminosity and spin, such anomalous behavior would not happen in the first place. Lastly, therefore since the comprehensive physical interpretation and the complete understanding of this anomalous behavior is not available at the moment, we would like to pursue further investigation along this line in our future works.

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