Interplay between the small and the large scale structure of spacetime

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Abstract

Existence of frame invariant, maximum, time interval T, length L, and mass M is postulated. In the de Sitter universe - (1) the life span of universe, (2) the circumference of universe at the point of maximum expansion, and (3) the mass of the universe - are candidates for T, L and M respectively. Impact of such invariant global parameters, on the definition of local physical quantities, such as velocity is discussed.

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1 Introduction: the maxim of maximum, invariant - time, length and mass

Segal [1] noted that a possible route to investigations of nature, is to assign a finite magnitude to a quantity, which tacitly is assigned an infinite value. This was motivated by the observation that the maximum possible speed - namely that of light - is finite (rather than infinite) and invariant in all reference frames. A variation on this theme, is ascribing non-zero values to certain quantities, such as the Plank's constant in quantum mechanics. A zero Plank's constant takes away the concept of uncertainty principles and the noncommuting variables, of the quantum mechanics, and relegates it to classical mechanics. Other examples are discrete spacetimes, in which the infinitesmal space and time intervals, are replaced by finite intervals - Snyder spacetime [2] being one such case.

The Newtonian universe has R^3 spatial topology, and R^1 temporal topology. Both the space and time coordinates, range in the interval $(-\infty, \infty)$. There is no maximum space or time interval - and no maximum speed, in the Newtonian universe. The Minkowski spacetime with $R^3 \times R^1$ topology and indefinite metric, had a maximum invariant speed - namely that of light, but no maximal spatial or temporal interval. The Einstein universe has S^3 spatial topology and R^1 temporal topology. Thus, while the maximum spatial interval was restricted by the size of the universe there however, was no maximum time interval, in the Einstein universe. Segal suggested $S^3 \times S^1$ topology for spacetime, i.e., Einstein's universe with a finite periodic time. The cosmological red shift in Segal's model is obtained via the *uni-energy* operator. In the context of periodic time, we note parenthetically that, it is fairly routine to have an imaginary periodic time, in finite temperature field theory [3, 4] and Euclidean quantum gravity [5, 6].

Time, length and mass are among the fundamental physical dimensions. Physicists routinely employ dimensional check for terms in an equation, to ensure their validity. Could it be that there exist, fundamental, maximum, and frame invariant - time interval T, spatial length L, and mass M? In the de Sitter universe, it does makes sense to talk about - (1) a maximum physical time interval T equal to the life span[1] of the universe, (2) a maximum

[1]The life span of the universe is the period between the big bang, and the big crunch. It is different from the age of universe, measured at a specific time - except the age at the time of the demise of the universe.

spatial interval or length L equal to the circumference of universe L at the point of maximum expansion, and (3) a maxium mass M equal to the mass of the universe. They could also be suitably defined in other cosmological models. Consider the maxim that - *The global parameters T, L and M, of structure of spacetime, are frame invariant.* It is shown in this paper that, this maxim requres a new definition of velocity v_{TLM}, depending upon these global parameters.

2 Interplay between the local and the global

From an operational view point, velocity is measured by noting location of an object at the spatial point s_1 on time instant t_1, - and then at a spatial point s_2, at time instant t_2. This gives the physically measured velocity v -

$$v = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t} \tag{1}$$

Δs and Δt are subject to measurement uncertainties of classical and quantum mechanical nature. Newton essentially initiated the invention of calculus in the process of defining velocity v_{Newton} as the ratio of vanishingly small space interval Δs, and time interval Δt -

$$v_{Newton} = \frac{\lim_{\Delta s \to 0} \Delta s}{\lim_{\Delta t \to 0} \Delta t} = \frac{ds}{dt} \tag{2}$$

This definition is local and independent of any large scale, global parameters of the structure of spacetime. Now, the Lorentz transformation linking a frame $\mathcal{F}'$, moving with relative velocity v with respect to a frame $\mathcal{F}$, actually depend upon the velocity defined in equation [2]. Time intervals, spatial lengths, and mass - measured in the two frames are related by the well known expression -

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

$$\Delta s' = \frac{\Delta s}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

$$m' = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

Here, superscript $'$ refers to quantities defined in $\mathcal{F}'$, while those without this superscript refer to quantities defined in $\mathcal{F}$. Invariance of T, L, and M between $\mathcal{F}$ and $\mathcal{F}'$ is given by the conditions -

$$T = (\Delta t')_{max} = (\Delta t)_{max} \tag{6}$$

$$L = (\Delta s')_{max} = (\Delta s)_{max} \tag{7}$$

$$M = m'_{max} = m_{max} \tag{8}$$

These conditions demand a modification of the factor γ_{STR} of Special Theory of Relativity (STR) -

$$\gamma_{STR} = \frac{1}{\sqrt{1 - v^2_{Newton}/c^2}}. \tag{9}$$

as otherwise there would exists - time intervals longer than T, spatial intervals larger than L, and masses larger than M, i.e., -

$$\Delta t' = \gamma_{STR}(\Delta t = T) > T \tag{10}$$

$$\Delta s' = \gamma_{STR}(\Delta s = L) > L \tag{11}$$

$$m' = \gamma_{STR}(m = M) > M \tag{12}$$

Lets first consider the largest physical time interval measurable in the de Sitter universe - namely its life span. Consider a set of clocks which start ticking at big bang, and go on ticking till the big crunch. No matter how the clocks move, i.e., with different velocities, our maxim requires that, they all show the same elapsed time T, between the big bang and the big crunch. What we are posing is - *a cosmological version of twins paradox, in which the time interval of travel equals the life span of the universe*[2]. Evidently, this requires that, the constant T, the measured time variable Δt and Δs, appear in any equation which relates time intervals between different reference frames. As the velocity v is not time dependent (i.e., we are not considering frames which are accelerating with respect to each other), equation [3] can be re-written as -

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{1}{c^2}\left(\frac{\Delta s}{\Delta t}\right)^2}} \tag{13}$$

where, $\Delta s = v\Delta t$ is the distance travelled in the time interval Δt. One way to obtain this frame independence of T, is by modifying equation [3] to -

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{1}{c^2}\left[\left(\frac{\Delta s}{\Delta t}\right)\left(1 - \frac{\Delta t}{T}\right)\right]^2}} \tag{14}$$

To retain the form of equation [3], following definition for the velocity v_T (for an invariant maximum time interval T), can be adopted -

$$v_T = v_{Newton}\left(1 - \frac{\Delta t}{T}\right) \tag{15}$$

Note that,

$$v_{Newton} = \lim_{T\to\infty} v_T(\Delta s, \Delta t) \tag{16}$$

[2]In this context, it would be relevant to mention work of Boblest, Müller and Wunner [7], on twin paradox in de Sitter universe, and that of Barrow and Levin [8] on twin paradox in compact spaces.

and,

$$v_T(\Delta s, \Delta t = T) = 0 \tag{17}$$

Equation [17] essentially implies that the change in position of a particle, over a period T (recurrence period) is zero, i.e., the position of object in snap shots seperated by the time interval T, appears unchanged - alternatively, the object appears stationary. This, thus satisfies the criterion for recurrence in a periodic universe, indicated in [9]. Clearly for non-vanishing Δs, and finite T,

$$v_T(\Delta s, \Delta t) < v_{Newton} \tag{18}$$

Note also that, Δt is greater than T for superlumical velocities $v(> c)$,

$$\Delta t' > T \Rightarrow v > c\sqrt{\frac{T + \Delta t}{T - \Delta t}} \tag{19}$$

Proceeding as in case of T above, frame invariance of L and M, can be achieved by defining velocity as -

$$v_L(\Delta s, \Delta t) = v_{Newton}\left(1 - \frac{\Delta s}{L}\right) \tag{20}$$

$$v_M(\Delta s, \Delta t, m) = v_{Newton}\left(1 - \frac{m}{M}\right) \tag{21}$$

Equations [15], [20] and [21] can be combined to arrive at the following definition of velocity v_{TLM} -

$$v_{TLM}(\Delta s, \Delta t, m) = v_{Newton}\left(1 - \frac{\Delta t}{T}\right)\left(1 - \frac{\Delta s}{L}\right)\left(1 - \frac{m}{M}\right) \tag{22}$$

The corresponding special relativistic factors for transforming between the frames $\mathcal{F}$ and $\mathcal{F}'$ are -

$$\gamma_T = \frac{1}{\sqrt{1 - \frac{1}{c^2}\left[\left(\frac{\Delta s}{\Delta t}\right)\left(1 - \frac{\Delta t}{T}\right)\right]^2}} \tag{23}$$

$$\gamma_L = \frac{1}{\sqrt{1 - \frac{1}{c^2}\left[\left(\frac{\Delta s}{\Delta t}\right)\left(1 - \frac{\Delta s}{L}\right)\right]^2}} \tag{24}$$

$$\gamma_M = \frac{1}{\sqrt{1 - \frac{1}{c^2}\left[\left(\frac{\Delta s}{\Delta t}\right)\left(1 - \frac{m}{M}\right)\right]^2}} \tag{25}$$

$$\gamma_{TLM} = \frac{1}{\sqrt{1 - \frac{1}{c^2}\left[\left(\frac{\Delta s}{\Delta t}\right)\left(1 - \frac{\Delta t}{T}\right)\left(1 - \frac{\Delta s}{L}\right)\left(1 - \frac{m}{M}\right)\right]^2}} \tag{26}$$

The modified special relativistic factors γ_T, γ_L, γ_M and v_{TLM} may be contrasted with the usual γ_{STR} of standard Special Theory of Relativity (STR).

In equation [15], the term $\Delta t/T$ can be replaced by any general function satisfying the equations [16] and [17]. It would be interesting to find criteria which would lead to specification of such a function. Similar considerations apply in equations [20] and [21] and for γ_{TLM} of equation [26].

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