Modified general relativity as a model for quantum gravitational collapse

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Abstract. We study a class of Hamiltonian deformations of the massless Einstein-Klein-Gordon system in spherical symmetry for which the Dirac constraint algebra closes. The system may be regarded as providing effective equations for quantum gravitational collapse. Guided by the observation that scalar field fluxes do not follow metric null directions due to the deformation, we find that the equations take a simple form in characteristic coordinates. These are amenable to numerical evolution using standard methods.

1. Introduction

Einstein's theory of general relativity predicts the existence of black holes. According to the Penrose-Hawking singularity theorems they can form when matter undergoes gravitational collapse. However, the singularities inside black holes are unwelcome from a physical point of view as they render space-time incomplete. We have no knowledge about what happens near the singularity. This is a regime where quantum gravity effects are expected to come into play.

The canonical formulation of general relativity is one approach to formulating a theory of quantum gravity. It can be traced back to the work of Arnowitt, Deser, and Misner (ADM) [1] who gave the first Hamiltonian formulation. This was used by DeWitt to formulate the quantization program [2]. In practice, there are many technical and conceptual difficulties in this approach [3], and so far no complete theory is available.

In the absence of a full theory it is important to ask whether there are relevant physical situations with symmetries where the canonical quantization can be completed. Technically the easiest is that of a homogeneous cosmology where field theory is reduced to a system with finitely many degrees of freedom. Such reductions have been studied since the 1970s using the Wheeler-DeWitt approach [4], and again in the recent past using the loop quantum gravity (LQG) methods [5].

The next level of problem, beyond quantum mechanics, is dimensional reduction to a field theory. Two important problems of physical interest that fall into this category are cosmology with inhomogeneities and spherically symmetric gravitational collapse. Our interest in this paper is the second problem. We address the question of formulating neighbors of Einstein equations in spherical symmetry, and discuss how these can be interpreted as providing "effective" quantum gravity corrections using some input from LQG.

Classical spherically symmetric collapse equations have been extensively researched. Given our aim, we recap some important publications on this subject. Using Schwarzschild coordinates, Christodoulou showed [6] that there are two classes of inital data for the collapse. For one of these ("weak data") matter bounces at the centre of the coordinate system and evolves back towards radial infinity. The other class of data however gives rise to a black hole. This was numerically confirmed not only by Choptuik [7] but also by Goldwirth and Piran [8], who used infalling null coordinates [9]. For a comparison of the two approaches see [10]. The next important development in the history of classical spherically symmetric gravitational collapse was by Choptuik [11]. Among other things, he found numerically a scaling law, which relates the amplitude of the scalar field to the mass of the black hole. This was later reproduced by Garfinkle [12] based on collapse equations in double null coordinates. Such a form of the equations was also used in [13] to find a similar scaling law with non-zero cosmological constant and in higher dimensions.

The investigation of quantum corrected spherically symmetric collapse equations was the natural next step to take. In [14] it was shown that certain corrections motivated by the LQG program introduce a mass gap in the black hole scaling law. This means that zero-mass black holes are no longer contained in the solution space. Using similar corrections in Painlevé-Gullstrand coordinates [15], Ziprick and Kunstatter [16] also found the mass gap, and importantly, were able to observe evolution beyond the horizon.

In none of the works mentioned in the previous paragraph have the quantum corrections been introduced in such a way that the Dirac constraint algebra closes. Rather, the corrections were introduced directly into the classical evolution equations yielding a new (and consistent) set of equations. It is interesting to ask whether it is possible to arrive at effective equations with quantum gravity corrections that also have a Hamiltonian formulation. This forms part of the motivation for the present work.

Following work on loop quantum cosmology (LQC) by Bojowald and his collaborators [17, 18], where a class anomaly-free effective constraints are presented, Reyes [18] obtained a closed constraint algebra in spherical symmetry using the connection-triad variables of LQG. However, the resulting equations are sufficiently complicated as to make a numerical investigation quite difficult.

The equations we derive in this paper follow the same general idea, but are obtained directly in the ADM formulation. The main obstacle faced in the connection-traid variables is overcome to yield a remarkably simple form of the equations, that can be compared directly to the classical double null formulation. Along the way we note a useful physical insight crucial for the form of the equations we obtain: the null coordinates of the metric do not coincide with the characteristic lines of the scalar field due to the deformation. This is precisely what is needed for singularity avoidance, since it signals a violation of the dominant energy condition.

The paper is structured as follows. In the next section we present the canonical theory and introduce a class of deformations that are to model quantum corrections. In section 3 we derive the corresponding Hamilton equations of motion in the Schwarzschild gauge. The transformation to characteristic coordinates that yield the simple equations is given in section 4. We conclude in section 5 with a summary and outlook.

2. Canonical formulation and its deformation

In metric ADM variables the line element for a spherically symmetric space-time may be written as

$$ds^2 = -[N^2 - (N^r)^2]\mathrm{d}t^2 + 2N^r A^2 \mathrm{d}t\mathrm{d}r + A^2\mathrm{d}r^2 + B^2 d\Omega^2, \tag{1}$$

where N is the lapse function and N^r the radial component of the shift vector. If, to this metric, we minimally couple a massless scalar field with matter Lagrangian density $-(8\pi)^{-1}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ we get the action

$$S = \int_{-\infty}^{\infty}\int_0^{\infty}(P_A\dot{A} + P_B\dot{B} + P_\phi\dot{\phi} - \mathscr{H})\,\mathrm{d}r\,\mathrm{d}t + S_\infty \tag{2}$$

with S_∞ the Gibbons-Hawking-York boundary term. The Poisson brackets are

$$\{A(t,r), P_A(t,\tilde{r})\} = \{B(t,r), P_B(t,\tilde{r})\} = \{\phi(t,r), P_\phi(t,\tilde{r})\} = \delta(r-\tilde{r}),$$

and the total Hamiltonian density $\mathscr{H} = N\mathscr{H}_\perp + N^r\mathscr{H}_{\|r}$ is made up of the Hamiltonian constraint density

$$\begin{aligned}\mathscr{H}_\perp = {}& \frac{G_{\mathrm{N}}P_A}{2B^2}(P_A A - 2P_B B) - \frac{1}{2G_{\mathrm{N}}A^2}[A'B^{2\prime} - A(2BB'' + B'^2) + A^3] \\ & + \frac{P_\phi^2}{2AB^2} + \frac{B^2\phi'^2}{2A},\end{aligned} \tag{3}$$

and the diffeomorphism constraint density

$$\mathscr{H}_{\|r} = -P_A' A + P_B B' + P_\phi\phi'.$$

From this well known set-up [19] we now introduce the classical deformation that is designed to incorporate a certain type of quantum effect. As noted above, the way we arrive at the correction is based on the ideas of Bojowald and his collaborators [17], which was also applied by Reyes [18]. One of the differences here is that we are not using the triad-connection variables [20].

The insight motivating the deformation is that the inverse powers of A and B in (3) can be conveniently quantized using the Thiemann prescription for the inverse triad operator [21]‡. Doing so, we can turn $H_\perp = \int_0^\infty N\mathscr{H}_\perp\,\mathrm{d}r$ into a well-defined operator

‡ We note that this operator is not the only source of quantum corrections; the other is the curvature written using holonomy operators. It is not known how to get a deformed anomaly-free effective system that includes this correction in general, but some special cases have been studied [18, 22].

$\hat{H}_\perp$. If we proceed by computing the expectation value of $\hat{H}_\perp$ for certain classes of states, we get an effective $H_\perp^{\rm ef}$, and the afore cited work by Bojowald and others suggests that it can be of the form $H_\perp^{\rm ef} = \int_0^\infty N \mathscr{H}_\perp^{\rm ef}\, {\rm d}r$, where the effective iltonian constraint density is

$$\mathscr{H}_\perp^{\rm ef} = \sum_{i=1}^{3} Q_{(i)} \mathscr{H}_\perp^{(i)} \tag{4}$$

with

$$\mathscr{H}_\perp^{(1)} = \frac{G_{\rm N} P_A}{2B^2}(P_A A - 2P_B B) - \frac{1}{2G_{\rm N} A^2}[A' B^{2\prime} - A(2BB'' + B'^2) + A^3],$$
$$\mathscr{H}_\perp^{(2)} = \frac{P_\phi^2}{2AB^2}, \qquad \mathscr{H}_\perp^{(3)} = \frac{B^2 \phi'^2}{2A}.$$

In this definition, the $Q_{(i)}$ depend only on B and the $\mathscr{H}_\perp^{(i)}$ describe, in increasing order of i, the extrinsic curvature plus the Ricci scalar of constant coordinate time t hypersurfaces, the kinetic energy of the scalar field, and its gradient energy. This means that the deformation is entirely contained in the functions $Q_{(i)}$. Moreover, by restricting the $Q_{(i)}$ to depend only on B, modifications due to terms involving inverse powers of A are disregarded. Lastly, the term stemming from the extrinsic curvature and the one from the Ricci scalar are treated equally even though their overall dependence on B is different. Despite the concerns, there is a very good reason to investigate the physical consequences of this ansatz. Namely, if we impose the condition

$$Q_{(1)}^2 = Q_{(2)} Q_{(3)} \tag{5}$$

it can be shown (as in [18]) that the Dirac constraint algebra closes. There are no anomalies if (5) holds true. This has far reaching consequences. Among them is that all degrees of freedom are captured by the given canonical variables. Furthermore, it implies that energy is conserved, which is attractive from a physical point of view.

3. Equations of motion in the Schwarzschild gauge

To get a first idea of the effect of the quantum corrections introduced in the previous section, we implement the second class gauges $G_{\parallel r} = r - B = 0$ and $G_\perp = P_A = 0$ in the given order to freeze out some of the degrees of freedom [23]. Concretely, the consistency condition $\dot{G}_{\parallel r} = 0$ gives $N^r = G_{\rm N} N P_A / r$ and the equation $\mathscr{H}_{\parallel r} = 0$ yields

$$P_B = P_A' A - P_\phi \phi'. \tag{6}$$

Note that the gauge choice $G_{\parallel r} = 0$ renders the $Q_{(i)}$ non-dynamical by turning them into functions that solely depend on r. Proceeding with the gauge fixing, the choice of $G_\perp$ forces N^r to vanish, which not only turns the line element (1) into the familiar Schwarzschild form

$$ds^2 = -N^2 {\rm d}t^2 + A^2 {\rm d}r^2 + r^2 d\Omega^2 \tag{7}$$

but also, from (6) and the definition of P_B, implies the trivial equation

$$\dot{A} = G_{\rm N}\frac{Q_{(1)}NP_\phi\phi'}{r}. \tag{8}$$

The consistency condition $\dot{G}_\perp = 0$ leads to

$$\frac{N'}{N} = \frac{A'}{A} + \frac{A^2-1}{r} - \frac{Q'_{(1)}}{Q_{(1)}} \tag{9}$$

and solving $\mathscr{H}_\perp^{\rm ef} = 0$ for A gives

$$\frac{A'}{A} = \frac{1-A^2}{2r} + \frac{G_{\rm N}}{2r^3Q_{(1)}}(Q_{(2)}P_\phi^2 + r^4Q_{(3)}\phi'^2). \tag{10}$$

Finally, Hamilton's form of the scalar field equation is given by

$$\dot{\phi} = \frac{Q_{(2)}NP_\phi}{r^2A}, \qquad \dot{P}_\phi = \left(\frac{r^2Q_{(3)}N\phi'}{A}\right)'. \tag{11}$$

If we set all $Q_{(i)}$ equal to 1, the deformation is switched off. This allows us to recover in (8-11) the familiar collapse equations in Schwarzschild coordinates. This is the form of the equations that has been heavily investigated analytically and numerically (see [24] for an extensive list of references).

However, to reproduce Choptuik's results in these coordinates is a rather complex task. The conceptual reason is that near $r = 0$, where black holes are expected to form, a numerical code has to allow for a very high resolution but as we move away from the origin there is no need for the same accuracy. Trying to keep the computing time low, Choptuik decided to use a so-called adaptive mesh refinement (AMR) algorithm of Berger and Oliger [25]. Unfortunately, and despite its beauty, the method is rather involved so that the conceptual problem is effectively replaced by a practical one. Because of this we seek a different form of the deformed equations, one that is more user-friendly.

4. Equations in characteristic coordinates

Christodoulou [9] showed that Einstein's field equations for the model defined by (2) can be given in a very compact form if infalling null coordinates (u, r) are used. Some years later, Garfinkle [12] derived the same equations in double null coordinates (u, v). Namely, in terms of a line element of the form

$$ds^2 = -W^2{\rm d}u{\rm d}v + r^2d\Omega^2,$$

Einstein's field equations are given by [26]

$$\begin{aligned}
&W^2 = -2\partial_u\partial_v r^2, \\
&2\partial_u r\frac{\partial_u W}{W} = \partial_u^2 r + G_{\rm N} r(\partial_u\phi)^2, \\
&2\partial_v r\frac{\partial_v W}{W} = \partial_v^2 r + G_{\rm N} r(\partial_v\phi)^2, \\
&\partial_u r\partial_v\phi + \partial_v r\partial_u\phi = -r\partial_u\partial_v\phi
\end{aligned} \tag{12}$$

and, due to the Bianchi identities, the second and third equation are redundant [9]. Dropping the second, Garfinkle used the reparametrization $W^2 = 2\partial_v rF$ to arrive at

$$\begin{aligned} \partial_u r &= -\frac{f}{2}, \\ \partial_u \Phi &= \frac{1}{2r}(f-F)(\phi-\Phi), \\ F &= F_u \exp\left(G_{\mathrm{N}} \int_u^v \frac{\partial_{\tilde{v}} r}{r} (\phi-\Phi)^2 \, \mathrm{d}\tilde{v} \right), \end{aligned} \tag{13}$$

where $F_u = 2(\partial_v r)(u,u)$ is determined from the boundary condition $r(u,u) = 0$ [26], and

$$f = \frac{1}{r}\int_u^v \partial_{\tilde{v}} rF \, \mathrm{d}\tilde{v}, \qquad \phi = \frac{1}{r}\int_u^v \partial_{\tilde{v}} r\Phi \, \mathrm{d}\tilde{v}. \tag{14}$$

These equations are of the same compact form as those presented by Christodoulou for infalling null coordinates.

Equations (13) and (14) have several advantages compared to those in Schwarzschild coordinates (8-11) (with the deformation switched off). The most important is that in double null coordinates (u,v) we have $\partial_u r < 0$, which follows from the first equation in (12). This implies that the computational grid becomes smaller as $r = 0$ is approached so that a higher accuracy can be naturally obtained by inserting additional points into constant u slices [12, 13, 14]. An AMR algorithm in the spirit of that used by Choptuik is no longer necessary. We therefore intend to transform our deformed equations to double null coordinates. However, as we will see, this will not be enough to obtain simple equations due to a null-characteristic mismatch.

To simplify the notation, we introduce the functions $Q = (Q_{(2)}/Q_{(1)})^{1/2}$ and $\sigma = A/(Q_{(1)}N)$ as well as the matter variables

$$U = (\sigma\dot{\phi} - \phi')/Q, \qquad V = (\sigma\dot{\phi} + \phi')/Q.$$

This allows us to write the trivially solved equation (8) in the form

$$\frac{\sigma\dot{A}}{A} = -\frac{G_{\mathrm{N}} r}{4}(U^2 - V^2) \tag{15}$$

and the remaining equations (9-11) as

$$\begin{aligned} &\frac{\sigma'}{\sigma} = \frac{1-A^2}{r}, \qquad \frac{A'}{A} = \frac{1-A^2}{2r} + \frac{G_{\mathrm{N}} r}{4}(U^2+V^2), \\ &\sigma\dot{U} + U' = -\ln(r/\sigma)'U + \ln(r/Q)'V, \\ &\sigma\dot{V} - V' = -\ln(r/Q)'U + \ln(r/\sigma)'V. \end{aligned} \tag{16}$$

The last two equations imply that we can denote by $c^\mu_\pm \partial_\mu = \sigma\partial_t \pm \partial_r$ the characteristic directions [27] of the scalar field equation. By definition, the variable U describes the change of ϕ along c_- with speed $-\sigma$, and V does so along c_+ with speed σ. Therefore, the last two equations in (16) tell us how a consecutive change of ϕ along c_- and c_+, and vice versa, looks like.

Note, however, that *the characteristics do not correspond to the null directions* $Q_{(1)}\sigma\partial_t \pm \partial_r$ *of the physical metric* (7). In fact, we have

$$ds(c_\pm, c_\pm)^2 = -\frac{1 - Q_{(1)}^2}{Q_{(1)}^2} A^2, \tag{17}$$

which vanishes if and only if $Q_{(1)} = 1$. This makes sense because it is only in general relativity that the massless scalar field flux follows null geodesics. (The same conclusion can be gained by noting that there is no scalar field potential that can be added to the Lagrangian matter density to reproduce (15) and (16) by means of Einstein's equations. This follows from the fact that a potential not only gives a contribution to the scalar field equation but also to the consistency condition and the constraint equation. However, in the given variables the latter two are effectively unaffected by the quantum corrections (Q only appears in the last two equations in (16)).)

The important implication from (17) is that only if $Q_{(1)}^2 \leq 1$ are we dealing with matter whose fluxes are timelike or null. If $Q_{(1)}^2 > 1$ the scalar field propagates along space-like directions giving a violating of the dominant energy condition. In this case, we can hypothesise that the formation of singularities can be avoided since the axioms of the singularity theorems [28] do not apply in such a situation.

The equations for U and V in (16) clearly suggest that we need to introduce coordinates u and v adapted to the characteristics $c_\pm$. We therefore set

$$D_u = -\sigma\partial_t + \partial_r, \qquad D_v = \sigma\partial_t + \partial_r, \tag{18}$$

where $D_u = (\partial_u r)^{-1}\partial_u$ and $D_v = (\partial_v r)^{-1}\partial_v$. This gives

$$U = -D_u\phi/Q, \qquad V = D_v\phi/Q,$$

which shows that u and v correspond to c_- and c_+, respectively. In fact, since $\partial_u r < 0$ and $\partial_v r > 0$ (see below) the coordinates u and v respectively parametrize future pointing infalling and outgoing characteristics if $Q_{(1)}^2 < 1$. In (18) we have chosen $\partial_u t = -\sigma\partial_u r$ and $\partial_v t = \sigma\partial_v r$, which implies

$$-Q_{(1)}^2 N^2 \mathrm{d}t^2 + A^2 \mathrm{d}r^2 = -W^2 \mathrm{d}u\mathrm{d}v \tag{19}$$

if the function W satisfies

$$A^2 = -\frac{W^2}{4\partial_u r \partial_v r}. \tag{20}$$

Since the characteristics $c_\pm$ to which we adapted the coordinates u and v do not agree with the null lines of the physical relevant line element (7), we of course cannot expect u and v to correspond to the double null coordinates of this metric. To the contrary, as is shown by (19), u and v correspond to the double null coordinates of that metric, for which the null directions agree with the characteristics.

If we factor out A^2 in (19) and use (20) we can show that $\sigma'/\sigma = -\partial_u\partial_v r/(\partial_u r \partial_v r)$. This, together with the first equation in (16), implies

$$W^2 = -2\partial_u\partial_v r^2. \tag{21}$$

A comparison with (12) shows that the deformation has no impact on this equation. From (21) it follows that we again have $\partial_u r < 0$ and since (20) implies that $\partial_u r \partial_v r < 0$ we get $\partial_v r > 0$, as desired. Constructing linear combinations $\sigma \dot{A} \pm A'$ from (15) and (16), equation (20) gives

$$\begin{aligned} 2\partial_u r \frac{\partial_u W}{W} &= \partial_u^2 r + G_{\mathrm{N}} r (\partial_u \phi / Q)^2, \\ 2\partial_v r \frac{\partial_v W}{W} &= \partial_v^2 r + G_{\mathrm{N}} r (\partial_v \phi / Q)^2. \end{aligned} \tag{22}$$

Since we are no longer dealing with Einstein's field equations, the Bianchi identities cannot be used to show a redundancy in (22). Luckily, it is the trivially solved equation (15) which does this job for us. Note that the equations in (22) do not differ very much from the corresponding ones in (12). Finally, the equations for U and V in (16) give the scalar field equation

$$\partial_u (r/Q) \partial_v \phi + \partial_v (r/Q) \partial_u \phi = -(r/Q) \partial_u \partial_v \phi \tag{23}$$

and it is this equation, in which the quantum corrections are most prominent§.

As expected, the characteristic speed σ does not appear in any of (21-23). It cannot be seen along the characteristics. Therefore, the only quantum corrections left are encoded in Q. This implies that we can switch them off not only by setting $Q_{(1)}$, $Q_{(2)}$, and therefore $Q_{(3)}$ equal to 1 but also by means of the relation $Q_{(1)} = Q_{(2)} = Q_{(3)}$. This is not very surprising since, as we have seen, the gauge $G_\perp$ turns the $Q_{(i)}$ into non-dynamical functions and if they are the same, (2) and (4) show that we can effectively remove them by reparametrizing the lapse function. The class of quantum corrections where all $Q_{(i)}$ agree are therefore uninteresting from a physical point of view.

To arrive at a form of the evolution equations that resembles the one given in (13) and (14) we set $W^2 = 2\partial_v r F$. Dropping the first equation in (22), we can focus on the consistency and the constraint equation

$$\partial_v r F = -\partial_u \partial_v r^2, \qquad \partial_v r \frac{\partial_v F}{F} = G_{\mathrm{N}} r (\partial_v \phi / Q)^2, \tag{24}$$

as well as on (23). The key now is to realize that the relevant radial coordinate in (24) is r, whereas the one in (23) is r/Q. To remove this asymmetry we set

$$Q = \frac{r}{q}. \tag{25}$$

The function q behaves like Q in the sense that it is non-dynamical with respect to the Schwarzschild coordinates (t, r). In terms of the characteristic coordinates (u, v) its dynamic is implicitly determined by that of r. The reparametrization (25) can be obtained from the definition $q = (q_{(1)}/q_{(2)})^{1/2}$ with $q_{(1)} = Q_{(1)}$ and $q_{(2)} = Q_{(2)}/r^2$. In order to be able to replace all $Q_{(i)}$ we further define $q_{(3)} = r^2 Q_{(3)}$. This gives

$$q_{(1)}^2 = q_{(2)} q_{(3)}$$

§ It is also apparent that one could obtain a modified dispersion relation from this equation, which is a topic of much recent interest.

and the Dirac constraint algebra remains closed. Since $Q_{(3)}$ does not appear in the definition of Q we are free to reparametrize it without affecting any of the consistency, the constraint, or the scalar field equation. Finally, since $\dot{q}=0$ implies $D_u q = D_v q = q'$, we get the sought after equations. They are

$$\begin{aligned}
\partial_u r &= -\frac{f}{2}, \\
\partial_u \Phi &= \frac{1}{2r}[r\ln(r/q')'f - F](\phi - \Phi), \\
F &= F_u \exp\left(G_{\mathrm{N}}\int_u^v \frac{\partial_{\tilde{v}} r}{r} q'^2 (\phi-\Phi)^2\,\mathrm{d}\tilde{v}\right),
\end{aligned} \tag{26}$$

where

$$f = \frac{1}{r}\int_u^v \partial_{\tilde{v}} r F\,\mathrm{d}\tilde{v}, \qquad \phi = \frac{1}{q}\int_u^v \partial_{\tilde{v}} r q'\Phi\,\mathrm{d}\tilde{v}, \tag{27}$$

and $F_u = 2(\partial_v r)(u,u)$. A comparison with (13) and (14) shows that the modifications due to the deformation are elegantly captured with relatively minimal modifications.

It is now straightforward to produce an evolution scheme for these equations following Garfinkle's method. This would require only one additional input, which is the selection of the function q such that it gives modifications to classical behavior in regions of sufficiently large matter density.

5. Discussion

We derived a set of effective equations for the massless scalar field in spherical symmetry. The equations may be viewed as "neighbors" of Einstein's equations for which the constraint algebra closes. They also have an interpretation as providing effective quantum gravity corrections based on an analogy with the inverse triad operator in LQG. Although this analogy is not exact, it is nevertheless of sufficient interest to probe quantum effects in gravitational collapse numerically.

An interesting fact leading to the final form of our equations is that the characteristics of the scalar field do not coincide with the null directions of the metric. In the deformed theory, the characteristics can be time-like, null, or space-like, depending on the form of the corrections. They are null if and only if there are no quantum corrections. This is the central reason why the equations in double null metric coordinates cannot be put into a useful form. However, when coordinates adapted to the scalar characteristics are used, the equations simplify dramatically.

A physical interpretation of the mismatch between characteristics and null metric directions is that the deformation can introduce violations of the dominant energy condition depending on the choice of deformation functions. This may be seen directly by noting that we can define (in the notation of (4)) an effective energy density

$$\rho_{\mathrm{ef}} = \frac{1}{Q_{(1)}}(Q_{(2)}\mathscr{H}_\perp^{(2)} + Q_{(3)}\mathscr{H}_\perp^{(3)}).$$

Then the dominant energy condition is

$$\rho_{\rm ef} \geq |P_\phi \phi'|,$$

since the diffeomorphism constraint is not modified. It is evident that this can be violated depending on the functions $Q_{(i)}$. We speculate that this is the feature that should be common to all effective quantum equations, however derived, because there can be no singularity avoidance without violation of the dominant energy condition. Our example provides one class of deformed anomaly-free equations with this feature.

Finally, we note that to be interpreted as quantum effects, the deformation functions should come with an associated length scale. In a forthcoming work we investigate these equations numerically, for judiciously chosen functions, with the aim of seeing how black hole formation is modified from the classical results [29].

Acknowledgments

This work was supported in part by the Natural Science and Engineering Research Council of Canada. We thank Martin Bojowald and Juan Reyes for providing us with a double null form of the scalar field-gravity equations in triad-connection variables which motivated this work.

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