

# Double Power Law Decay of the Persistence in Financial Markets

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## Abstract

The persistence phenomenon is studied in the Japanese financial market by using a novel mapping of the time evolution of the values of shares quoted on the Nikkei Index onto Ising spins. The method is applied to historical end of day data from the Japanese stock market during 2002. By studying the time dependence of the spins, we find clear evidence for a double-power law decay of the proportion of shares that remain either above or below ‘starting’ values chosen at random. The results are consistent with a recent analysis of the data from the London FTSE100 market. The slopes of the power-laws are also in agreement. We estimate a long time persistence exponent for the underlying Japanese financial market to be 0.5.

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## I. INTRODUCTION

In its most generic form, the persistence problem is concerned with the fraction of space which persists in its initial ( $t = 0$ ) state up to some later time  $t$ . It is a classic problem which falls into the general class of so-called ‘first passage’ problems [1] and has been extensively studied over the past decade or so for model spin systems by physicists [2-6]. Persistence has been investigated at both zero [2-5] and non-zero [6] temperatures.

Typically, in the non-equilibrium dynamics of spin systems at zero-temperature [2-5], the system is prepared initially in a random state and the fraction of spins,  $P(t)$ , that persists in the same state as at  $t = 0$  up to some later time  $t$  is monitored. At a finite temperature, on the other hand, one is interested in the global persistence behaviour and one monitors the change in the sign of the magnetization in a collection of non-interacting systems [6]. It is now well established that the persistence probability decays algebraically [2-6]

$$P(t) \sim t^{-\theta(d,q)}, \quad (1)$$

where  $\theta(d, q)$  is the non-trivial persistence exponent. Note that the value of  $\theta$  depends not only on the spatial [4] ( $d$ ) and the spin [7] ( $q$ ) dimensionalities, but also on whether the temperature,  $T$ , is zero or finite. It is only for  $T = 0$  and  $d = 1$  that  $\theta(1, q)$  is known exactly [5]; see Ray [8] for a recent review. We merely mention here that at criticality,  $T = T_c$ , [6],  $\theta(2, 2) \sim 0.5$  for the pure two-dimensional Ising model.

More recently it has been discovered that disorder [9-11] also alters the persistence behaviour. A key finding [9-10, 12] is the appearance of ‘blocking’ in systems containing disorder. ‘Blocked’ spins are effectively isolated from the behaviour of the rest of the system in the sense that they *never* flip. As a result,  $P(\infty) > 0$ .

The persistence exponent has also been obtained from a wide range of experimental systems and the values range from 0.19 - 1.02, depending on the system [13-15]. A considerable amount of time and effort has been taken up in trying to obtain estimates of  $\theta(d, q)$  for different models and systems.

In this work, we present one of the first estimates of the persistence exponent from financial data from the Japanese market. The main motivation behind the present study is to compare and contrast the behaviour of the Japanese and UK markets. It was recently [16] found that the persistence behaviour of the London market displays an interesting double power law behaviour. As we shall see, we find very similar features in the Japanese market.

In the next section we give a brief outline of the methodology used. Section III presents our results and we finish with a brief conclusion in Section IV.

## II. METHODOLOGY

Financial markets contain many of the features found in model systems that have been studied over the past 50 years or so in statistical physics. For example, share values of most banks would tend to move in the same direction if the central bank rate is raised or reduced. This behaviour is analagous to the tendency of spins connected by a ferromagnetic coupling to align themselves at low temperature. Market behaviour analagous to antiferromagnetic coupling is also seen in real markets. For example, the above news regarding the central bank rate would have the opposite effect on the values of shares in manufacturing companies. Furthermore, interactions between different companies exist and are highly non-trivial to model. In our data analysis we make no attempt to model these interactions. We simply work with historical data from the Nikkei Index from 2002.

The pre-processed financial dataset was kindly supplied to us by T. Kaizoji and L. Pichl (International Christian University, Tokyo, Japan). For the purposes of this study we extracted end-of-day (EOD) data from the database to enable a direct comparison with the earlier study made with the data from the London FTSE100 market [16].

As mentioned, we work with EOD share prices quoted in Japanese yens. We follow the earlier work on the London market and map the share values onto Ising spins. The dataset for 2002 was first partitioned into quarterly data (Jan-Mar, Apr-Jun, Jul-Sep, Oct-Dec). In figure 1 we display the behaviour of the share values of 3 typical companies over a randomly chosen quarter. The share values are in Japanese Yen. The dashed lines indicate the respective share values at  $t = 0$ . In this work we restrict ourselves to the first passage time, that is the first time a given share value crosses it's value at  $t = 0$ .

The start of each quarter is designated as 'Day 0' and the share values take on their 'Base values'. The share values at the end of trading on the next day ('Day 1') are compared with the corresponding base values. A spin  $S_i(t)$  is associated with each share value  $i$ . We allocate the values as follows

$$S_i(t) = \begin{cases} +1, & \text{if Base price} \leq \text{EOD price on day 1;} \\ -1, & \text{if Base price} > \text{EOD price on day 1.} \end{cases} \quad (2)$$

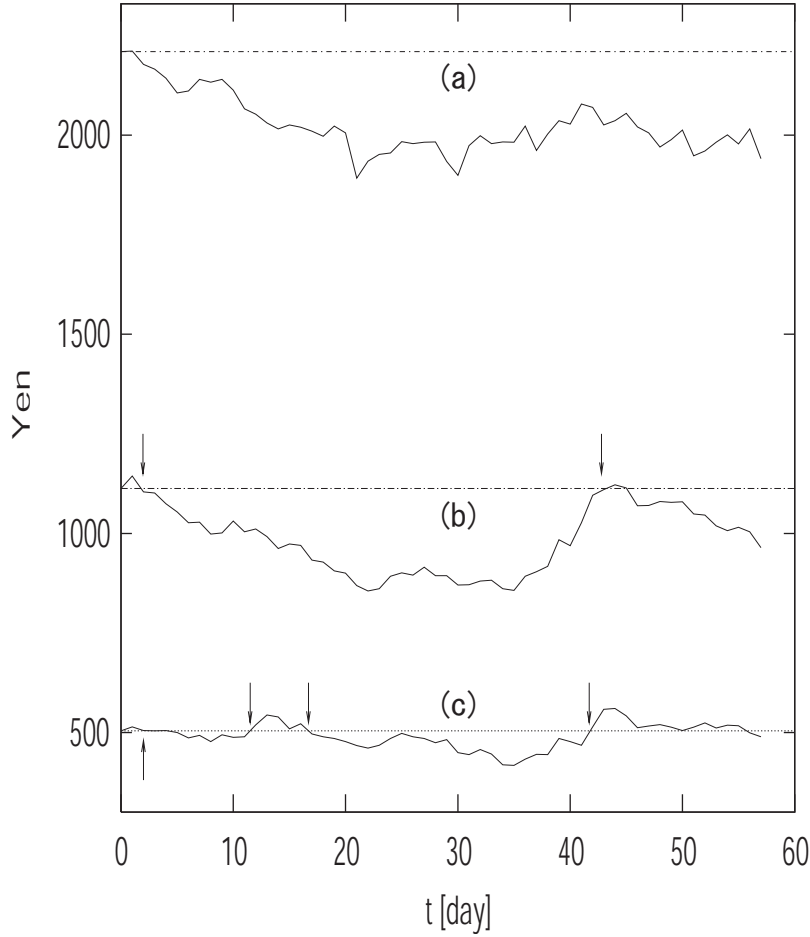


FIG. 1: The time series of the share values of 3 typical companies making up the Nikkei Index. The three dashed horizontal lines indicate the respective share values at  $t = 0$ . The top plot (a) indicates that the share value remains below the ‘base’ price for the duration of the quarter considered. In the middle plot (b) the time at which the share value returns to the base price for the first time is indicated by the first arrow. Similarly, the arrows in the bottom plot (c) indicate the first 4 times at which the share value returns to the price at  $t = 0$ .

Table 1 shows a typical mapping.

It should be emphasised that the value of the spin is always with reference to the share value at  $t = 0$  and *not* the EOD price at the previous day. Consequently, in this work we disregard all fluctuations which take place during the day. In our example in Table 1, the spin has ‘flipped’ at  $t = 3$ . The values of  $\{S_i(t = 0); 1 \leq i \leq 225\}$  form the initial configuration for our spin system. Each 3-monthly dataset is converted into possible values of Ising spins,  $S_i(t)$  with the share values at  $t = 0$  as the reference prices. This allows us

Day	$t$	EOD value	$S_i(t)$
0	-	257	Base price
1	0	239	-1
2	1	228	-1
3	2	235	-1
4	3	245	+1

TABLE I: A typical example of the mapping used. Note that the value of  $S_i(t)$  is determined with reference to the price at  $t = 0$ .

to track the share values relative to their prices at  $t = 0$ . In figure 1 the first passage time is indicated by the first arrow for any given share. Subsequent arrows indicate the second (and later) passage times.

To complete the analogy with the persistence problem as studied in statistical physics, we look for the *first time*  $S_i(t) \neq S_i(t = 0)$  as this corresponds to the underlying share value either going above (if  $S_i(t) = +1$ ) or below (if  $S_i(t) = -1$ ) the price at  $t = 0$ , also for the first time. For an analysis of the first passage properties, all subsequent values of  $S_i(t)$  are disregarded.

At each time step, we count the number of spins that still remain in their initial ( $t = 0$ ) state and the total number of spins,  $n(t)$ , which have never flipped until time  $t$  is given by [12]

$$n(t) = \sum_i (S_i(t)S_i(0) + 1)/2. \quad (3)$$

The density of non-flipping spins,  $R(t)$ , is the key measure of interest here, namely

$$R(t) = n(t)/N, \quad (4)$$

where  $N = 225$ , the number of companies appearing in the Nikkei Index. [In the actual analysis discussed in this paper, we had to work with  $N = 224$  as the share values of one of the constituent companies of the Nikkei Index were not quoted due to some financial irregularity.]

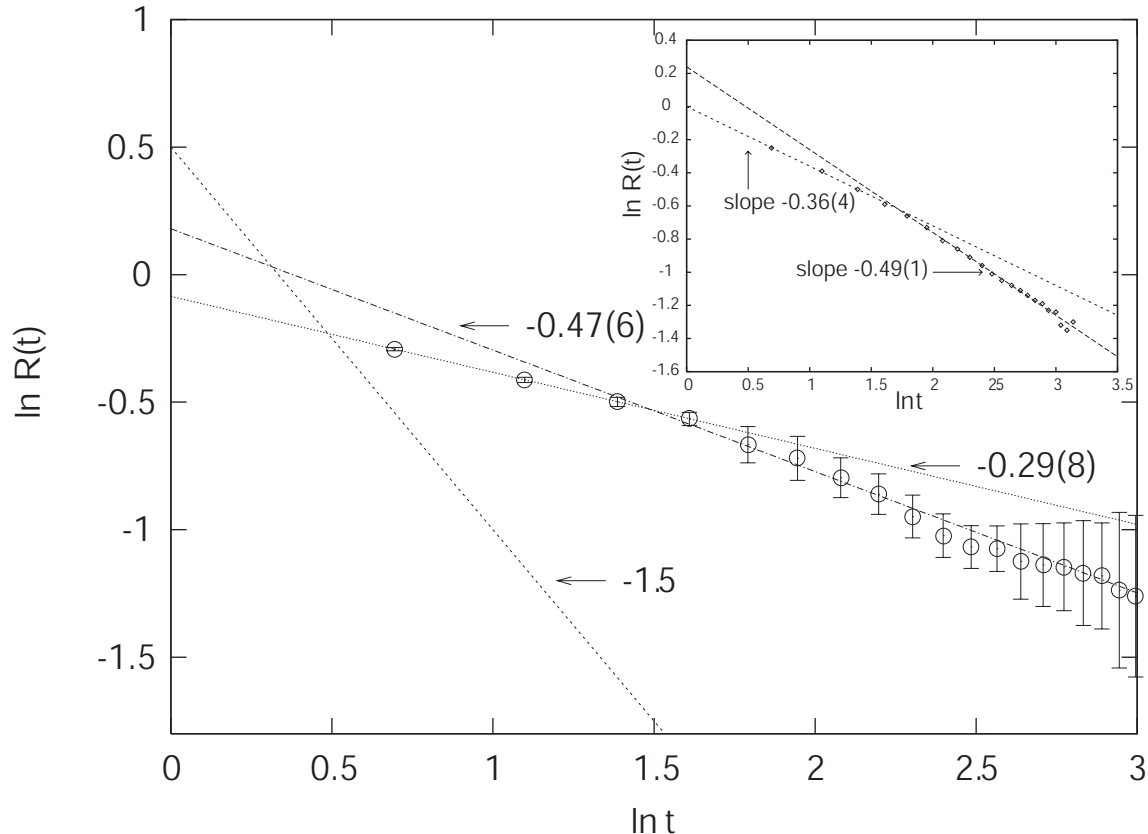


FIG. 2: A log-log plot of  $R(t)$  against  $t$  for the Japanese data. We can clearly see the appearance of a double power law with the presence of a distinctive ‘shoulder’. The inset shows the corresponding plot for the data from the London market [16]. The line with slope  $-1.5$  is for reference purposes only and is the exact result in 1 dimension for a symmetric random walk.

### III. RESULTS

We now discuss our main results. Figure 2 shows a log-log plot of the density of non-flipping spins against time. The data has been averaged over all of the samples.

We can clearly see the presence of a double power law with a distinctive ‘shoulder’. Whereas for short times we estimate the slope to be  $-0.29(8)$ , for longer times we find  $-0.47(6)$ . For information we also display the corresponding plot [16] from the London FTSE100 data as an inset in figure 2. In the earlier analysis of the London data [16] it was found that the short-time (typically up to about a week) persistence exponent is  $\sim 0.36(4)$  and for longer times it’s  $\sim 0.49(1)$ . Note that as here we are averaging over only 4 samples with the Nikkei data, the error-bars are much bigger than those obtained for the London data

which was averaged over 25 samples. Nevertheless, the plot confirms that the behaviour of the Japanese data is consistent with that seen on the London FTSE100 market. A financial market contains different types of traders. There are usually both short-term (speculative) and long-term traders. Clearly, those traders speculating on the market are likely to react over a much shorter time scale than those trading for the long-haul. The double power law discovered in our analysis could be a signature of the presence of both types of traders. Of course, in addition, there are intraday traders hoping to make a profit by trading at even higher frequency (eg hourly). As we work with EOD data, we do not expect to see the behaviour of such traders in our data. Both the Japanese Nikkei and the London FTSE100 markets are well developed and yield very similar features.

We note that the corresponding global persistence exponent for a one-dimensional symmetric random walk is given from [1]

$$R(t) = \begin{cases} t^{d/2-2}, & d < 2 \\ \frac{1}{t \ln^2 t}, & d = 2 \\ t^{-d/2}, & d > 2 \end{cases} \quad (5)$$

For reference, the value of the exponent for the  $1d$  case is shown in figure 2. Clearly, from our data we can exclude such a model.

#### IV. CONCLUSION

To conclude we have used a novel mapping to map EOD share values on the Nikkei Index onto Ising spins. The mapping can be re-interpreted in term of the first passage crossing time. We find clear evidence for a double power law decay with different exponents for short and long time behaviour. Our observation is consistent with the recent findings on the London FTSE100 market. We suggest that the double-power is the signature of the presence of different types of traders in the market. Namely, traders speculating on a daily basis and those taking a long-term view. It would be interesting to see whether the presence of really ‘high frequency’ intraday traders can be deduced from minute-by-minute tick data.

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