Improved ground-state estimate by thermal resummation

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Abstract

For the deconfining phase of SU(2) Yang-Mills thermodynamics and for high temperatures we point out that a linear dependence on temperature of a one-loop selfconsistently resummed thermal correction to the pressure and the energy density takes place despite a quartic dependence arising from an unsummed two-loop correction. This linearity is hierarchically smaller than the one belonging to the tree-level estimate of the thermal ground-state. We discuss and interpret this result.

Introduction. In addressing the deconfining (and preconfining) thermodynamics of an SU(2) Yang-Mills theory the notion of a thermal ground state proves useful [1, 2, 3]. Composed of interacting calorons and anticalorons of topological charge modulus $|Q| = 1$ [5, 6, 7] an a priori estimate for this ground state emerges upon a spatial coarse-graining over noninteracting (anti)calorons of trivial holonomy [8] down to a a certain resolution. The latter selfconsistently and in dependence of temperature and an integration constant is set by the emerging, spacetime-homogeneous modulus $|\phi|$ of an adjoint, nonpropagating scalar field ϕ.

The entirety of hard quantum fluctuations (higher resolving power than $|\phi|$) associated with fundamental field configurations of trivial topology, whose influence on (anti)calorons is void of full analytical access beyond the semiclassical approximation [4], is encoded in terms of a pure-gauge configuration $a_\mu^{\rm gs}$ after coarse-graining. The field configuration $a_\mu^{\rm gs}$ introduces finite ground-state pressure and energy density thus lifting the pure BPS situation described by ϕ alone. Conceptually, there is some resemblence with the treatment of quantum fluctions in perturbation theory where the subtraction of infinities by counter terms, respecting the form of the classical Yang-Mills action, consistently ignores the influence of ultraviolet physics at a given resolution and at an arbitrary loop order [13, 14]. We assume the existence of the Yang-Mills partition function. The latter could be formulated as a weighted average over fundamental fields at a high resolution. The according weight should then be derivable from the action of the infinitely resolved system in the sense of a Borel resummable, approximating series obtained by consistent order-by-order removal of (or an all-order average over) the ultraviolet physics. Then the effective action at resolution $|\phi|$ neatly splits into the classical Yang-Mills term describing the propagation and interactions among topologically trivial fluctuations, an interaction term between $Q = 0$ and $|Q| = 1$ configurations given by the square of the covariant derivative acting on ϕ, $(D_\mu\phi)^2$ (Higgs mechanism for two out of three propagating directions in the SU(2) algebra), and a potential V for the field ϕ [2]. It is conceivable that a formulation of the partition function at a higher resolution than $|\phi|$ exhibits that the interaction between topological and field configurations with $Q = 0$ effectively cuts off the number of higher dimensional operators [9, 10, 11, 12] in the corresponding effective action, limits the number of irreducible $Q = 0$ loops, and poses a limit to the topological charge modulus $|Q|$ of relevant field configurations. The usefulness of the effective theory [1, 2, 3] at resolution $|\phi|$ is linked to the fact that this is explicitly demonstrable.

In the effective theory [1, 2, 3] the temperature dependence of the gauge coupling e is dictated by the Legendre transformation between the pressure and energy density for tree-level quasiparticles (Higgs mechanism) fluctuating freely above the estimate of the thermal ground state expressed by ϕ, its potential V, and $a_\mu^{\rm gs}$. At high temperature one has $e \equiv \sqrt{8}\pi$. A direct contribution to the linear temperature dependence of the pressure $P^{\rm gs}$ and the energy density $\rho^{\rm gs}$ of the thermal ground state arises via the potential $V(\phi)=\mathrm{tr}\,\frac{\Lambda^6}{\phi^2}=4\pi\Lambda^3 T$. Thus

$$\frac{\rho^{\mathrm{gs,V}}}{T^4}=-\frac{P^{\mathrm{gs,V}}}{T^4}=2(2\pi)^4\,\lambda^{-3}\sim 3117.09\,\lambda^{-3}\,, \tag{1}$$

where $\lambda\equiv\frac{2\pi T}{\Lambda}$, and the Yang-Mills scale Λ is related to the critical temperature T_c of the deconfining-preconfining transition as $\Lambda=\frac{2\pi}{13.87}\,T_c$. At high temperature there are in addition linear contributions $\Delta P^{\mathrm{gs,1\text{-}loop}}$ and $\Delta\rho^{\mathrm{gs,1\text{-}loop}}$ to P^{gs} and ρ^{gs}, respectively, arising from the free fluctuations of tree-level massive modes [16]. Namely, one has

$$\frac{\Delta\rho^{\mathrm{gs,1\text{-}loop}}}{T^4}=\frac{\Delta P^{\mathrm{gs,1\text{-}loop}}}{T^4}=-\frac{1}{4}\,a^2=-2(2\pi)^4\,\lambda^{-3}\sim -3117.09^4\,\lambda^{-3}\,, \tag{2}$$

where $a\equiv\frac{2e|\phi|}{T}=\frac{8\sqrt{2}\pi^2}{\lambda^{3/2}}$. Notice that by virtue of the linear dependence on temperature of $P^{\mathrm{gs,V}}+\Delta P^{\mathrm{gs,1\text{-}loop}}$ the Legendre transformation yields $\rho^{\mathrm{gs,V}}+\Delta\rho^{\mathrm{gs,1\text{-}loop}}=0$ which, indeed, is the case, compare Eqs. (1) and (2).

The main purpose of this report is to demonstrate that at high temperature the ground-state inherent linear T dependence in certain contributions to the total energy density and the total pressure is also generated by the selfconsistent one-loop propagation of the massless modes albeit subject to a hierarchically smaller coefficient compared to that of Eqs. (1) and (2).

Recalling that there is a leading quartic dependence on high temperature [17, 18] of the two-loop correction to the pressure, $\Delta P^{\mathrm{2\text{-}loop}}$, this may come as a surprise. Specifically, one has [17, 18]

$$\Delta P^{\mathrm{2\text{-}loop}}=-\frac{4\pi^2}{45}\times 4.39\times 10^{-4}\,T^4 \tag{3}$$

for $T\gg T_c$. While the correction $\Delta P^{\mathrm{2\text{-}loop}}$ of Eq. (3) is interpreted as the loss in pressure of a thermal gas of tree-level massless, effective gauge modes due to the emergence of large-holonomy calorons and their subsequent dissociation into screened and long-lived monopole-antimonopole pairs [18] the above mentioned linear dependence, originating from a selfconsistent resummation of the one-loop irreducible insertion of the one-loop polarization tensor, represents a positive contribution to the pressure. That is, a *radiatively* induced effect with ground-state characteristics (linear T dependence) emerges as a result of the selfconsistent propagation of tree-level massless modes through a sea of stable and screened monopole-antimonopole pairs.

Massless quasiparticles after resummation. In Fig. 1 the diagrammatic basis for the resummation of the one-loop contribution to the on-shell polarization tensor of the massless mode is shown. This resummation into a selfconsistently modified dispersion law for transverse, propagating, tree-level massless modes was performed in [19] and is summarized in terms of the screening function $G(T,|\mathbf{p}|)$:

$$p_0^2=\mathbf{p}^2+G(T,|\mathbf{p}|)\;\Leftrightarrow\;Y^2=X^2+\frac{G}{T^2}(\lambda,X)\,, \tag{4}$$

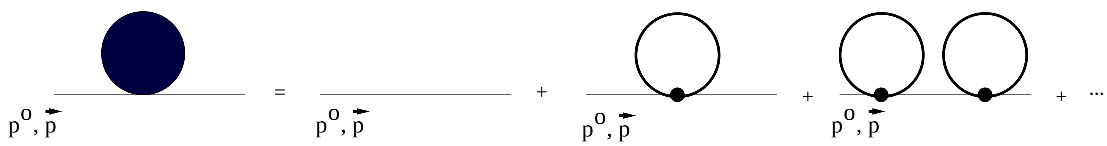


Figure 1: The Dyson series for the resummation of the one-loop irreducible contribution to the polarization tensor of the massless mode. A thin line refers to the propagation of the tree-level massless, a thick line to the propagation of tree-level massive modes.

where p_0 is the mode's energy, $\mathbf{p}$ its spatial momentum, and $Y \equiv \frac{p_0}{T}$, $X \equiv \frac{|\mathbf{p}|}{T}$. Details on the properties of G are presented in [19], here we mention only that G is positive for low momenta $|\mathbf{p}|$ (screening) and that it turns negative above some critical, temperature dependent momentum (antiscreening). For increasingly large $|\mathbf{p}|$ there is an exponentially fast decay of $|G|$. On the 'mass' shell of Eq. (4) there is no one-loop irreducible, imaginary contribution to the polarization tensor arising from the diagram depicted in Fig. 2. Resummation of the polarization tensor into a contribution $P^{\text{1-loop,res}}$ to the total pressure arising from free but radiatively shifted tree-level massless quasiparticles is performed by connecting the external legs on the left-hand side of Fig. 1. For dimensionless pressure and energy density one has:

$$
\begin{aligned}
\frac{P^{\text{1-loop,res}}}{T^4} &= \frac{1}{\pi^2}\int_0^\infty dX\,\frac{X^2\sqrt{X^2+G/T^2}}{\mathrm{e}^{\sqrt{X^2+G/T^2}}-1}\,,\\
\frac{\rho^{\text{1-loop,res}}}{T^4} &= -\frac{1}{\pi^2}\int_0^\infty dX\,X^2\log\left(1-\mathrm{e}^{-\sqrt{X^2+G/T^2}}\right).
\end{aligned}
\tag{5}
$$

We define the dimensionless corrections $\frac{\Delta P^{\text{1-loop,res}}}{T^4}$ and $\frac{\Delta P^{\text{1-loop,res}}}{T^4}$ to the contributions of free, massless particles as

$$
\frac{\Delta P^{\text{1-loop,res}}}{T^4} \equiv \Delta\bar{P} \equiv \frac{P^{\text{1-loop,res}}}{T^4} - \frac{\pi^2}{45}\,,\qquad \frac{\Delta \rho^{\text{1-loop,res}}}{T^4} \equiv \Delta\bar{\rho} \equiv \frac{\rho^{\text{1-loop,res}}}{T^4} - \frac{\pi^2}{15}\,. \tag{6}
$$

In Fig. 3 both $\Delta\bar{P}$ and $\Delta\bar{\rho}$ are depicted as a function of λ within the high-temperature interval $100 \le \lambda \le 500$ (recall that $\lambda_c = 13.87$). An excellent fit to the following power dependences

$$
\Delta\bar{P} \equiv c_P\lambda^\delta\,,\qquad \Delta\bar{\rho} \equiv c_\rho\lambda^\gamma \tag{7}
$$

reveals that

$$
c_P = 8.49627\,,\quad \delta = -3.00904\,,\quad c_\rho = 3.9577\,,\quad \gamma = -3.02436\,. \tag{8}
$$

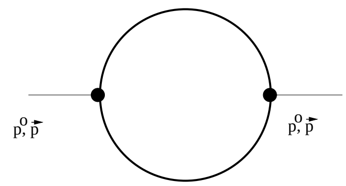


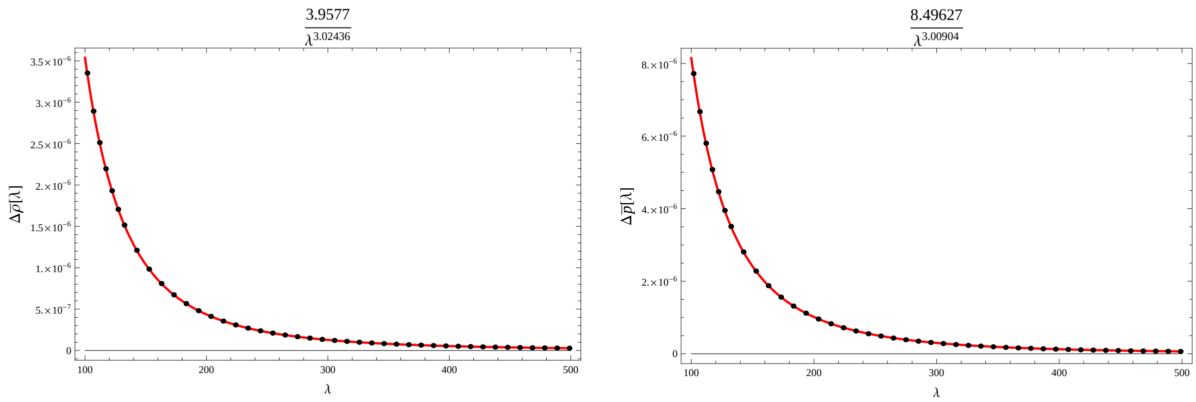
Figure 2: Potential one-loop contribution to the screening function G. On the 'mass' shell, generated by the series in Fig. 1, this contribution, however, vanishes [19].

Eq. (8) represents the main result of our present work: Through infinite and selfconsistent one-loop resummation the power and the sign in front of this power in temperature of a fixed-order correction to pressure and energy density is profoundly changed! Specifically, by resumming the selfconsistent one-loop polarization of the massless modes, we generate a linear correction which is about three orders of magnitude smaller than the a priori estimate given by $P^{\mathrm{gs,V}}$ and $\rho^{\mathrm{gs,V}}$, compare Eqs. (1), (7), and (8). That this correction is linear to a very good approximation is not a chance-result: Selfconsistent propagation, which is indirectly influenced by the a priori estimate of the thermal ground state through the quasiparticle mass of tree-level heavy modes, yields a correction improving this very estimate. In calculating the contribution of higher irreducible loop orders to the polarization tensor we observe a hierarchic decrease and expect a termination at a finite order [15, 20] such that a consideration of those contributions in the resummation process practically does not change our result obtained here by one-loop resummation.

Corrections $\Delta P^{\text{1-loop,res}}$ and $\Delta\rho^{\text{1-loop,res}}$ are not thermodynamically selfconsistent. Namely, defining $\Delta\rho_L^{\text{1-loop,res}}$ to be the Legendre transformation of $\Delta P^{\text{1-loop,res}}$,

$$\Delta\rho_L^{\text{1-loop,res}} \equiv T\frac{d\Delta P^{\text{1-loop,res}}}{dT} - \Delta P^{\text{1-loop,res}}\,, \tag{9}$$

we observe from Fig. 4 that by no means $\Delta\rho_L^{\text{1-loop,res}} = \Delta\rho^{\text{1-loop,res}}$ (or $\Delta\bar{\rho}_L = \Delta\bar{\rho}$); even their signs are different. But in contrast to the correction $\Delta P^{\text{2-loop}}$ of Eq. (3), which is by far dominating the two-loop correction to the pressure at high temperatures and thus needs to be thermodynamically selfconsistent by itself no such relation exists between $\Delta P^{\text{1-loop,res}}$ and $\Delta\rho^{\text{1-loop,res}}$. On the contrary, thermodynamical selfconsistency after one-loop resummation is only expected to occur when corrections due to a shift in dispersion law of the tree-level massive modes are taken into account at resummed one-loop level. Small deviations from thermodynamical selfconsistency after taking these contributions into account arise from the use of tree-level quasiparticle consistency at a higher radiative order. Since the contributions to the pressure arising from radiative corrections follow a large hierarchy [15, 20] we are assured that



(a) (b)

Figure 3: The high-temperature λ dependence of (a) $\Delta\bar{\rho}$ and (b) $\Delta\bar{P}$. Dots correspond to computed values, the line is a fit to this data.

the demand for thermodynamical selfconsistency at a higher loop order introduces small changes to the evolution of e which in turn induce very small changes to the radiative corrections themselves. For practical purposes it thus suffices to work at tree-level selfconsistency to judge the selfconsistency of radiative corrections. The shift in disperion law for tree-level massive modes is harder to compute than for tree-level massive modes since: (i) The resummation invokes two separate one-loop contributions to the polarization tensor arising from a massive and a massless tadpole. (ii) The polarization tensor is not transverse for propagating massive modes, and thus more than just a single screening function need to be considered. We leave this to future investigation.

Summary and conclusions. To summarize, we have shown numerically that for high temperatures the selfconsistent resummation of the one-loop polarization tensor of the tree-level massless mode generates a linear dependence on temperature in the according correction to the pressure and the energy density of the thermal gas of massless particles. This is remarkable because a two-loop correction to the pressure, which dominates all fixed-order radiative corrections at high-temperature, depends quartically on temperature. While this fixed-order correction is interpreted as an investment of energy by massless gauge modes into the emergence of screened but stable monopole-antimonopole pairs upon strong (anti)calorons deformation (large temporary holonomy) and subsequent dissociation [4], the correction due to selfconsistent resummation describes the propagation in a preexisting (yet unresolvable [18]) sea of stable, screened monopole-antimonopole pairs. A linear dependence on temperature of the pressure and energy-density correction is suggestive for the ground-state estimate [2, 3, 16] being improved by a radiative dressing of tree-level massless modes.

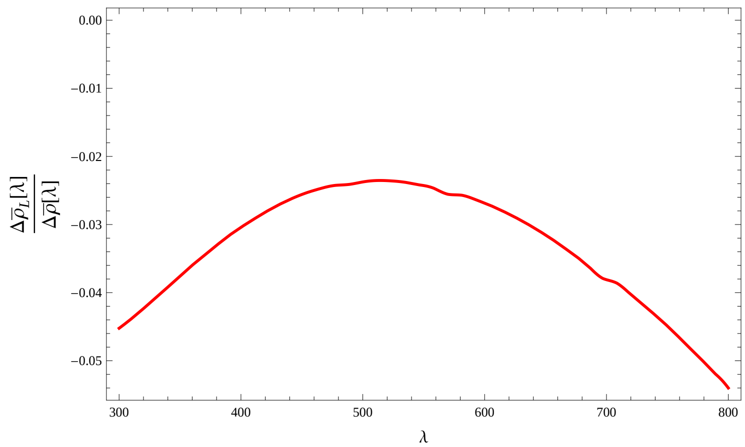


Figure 4: The λ dependence of the ratio $\frac{\Delta\bar{\rho}_L}{\Delta\bar{\rho}}$, where $\Delta\rho_L^{\text{1-loop,res}} \equiv T^4\,\Delta\bar{\rho}_L$ is defined in Eq. (9), and $300 \le \lambda \le 800$. The figure clearly expresses that the corrections to pressure and energy density of a thermal gas of massless particles due to one-loop resummation are thermodynamically not selfconsistent because radiative corrections shifting the dispersion law of tree-level massive modes are not considered.

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