A Novel Symmetry Constraint Of The Super cKdV System

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Abstract

A new (1+1)-dimensional integrable system, i. e. the super coupled Korteweg-de Vries (cKdV) system, has been constructed by a super extension of the well-known (1+1)-dimensional cKdV system. For this new system, a novel symmetry constraint between the potential and eigenfunction can be obtained by means of the binary nonlinearization of its Lax pairs. The constraints for even variables are explicit and the constraints for odd variables are implicit. Under the symmetry constraint, the spacial part and the temporal parts of the equations associated with the Lax pairs for the super cKdV system can be decomposed into the super finite-dimensional integrable Hamiltonian systems on the supersymmetry manifold $R^{4N|2N+2}$, whose integrals of motion are explicitly given.

Key words: explicit symmetry constraints, implicit symmetry constraints, super Hamiltonian system, Liouville integrable.

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1 Introduction

The super-extensions of the classical integrable systems lead to super integrable systems and they have undergone extensive development in the past years. There are many super integrable systems in literatures, such as the super AKNS system [1]-[3], the super KdV equation [4]-[7], the super KP hierarchy [8]-[11], etc. It was known that super systems contained the odd variables which would provide more prolific fields for mathematical researchers and physical ones. Darboux transformation [12]-[14], bi-Hamiltonian structure [15]-[17], Painlevé analysis [18] and so on, have been widely studied. Very recently, nonlinearization of the super AKNS system and the super Dirac system have been investigated in Refs. [19]-[21].

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It is well known that mono-nonlinearization technique was firstly proposed by Cao in Ref. [22], and binary-nonlinearization technique was proposed by Ma in Ref. [23]. Both mono-nonlinearization and binary-nonlinearization have the following characteristics. Firstly, the advantage of nonlinearizaton method is to decompose infinite dimensional systems into finite ones. Secondly, one of the essential steps of nonlinearization method is to calculate the variational derivative. Lastly, the key of the nonlinearization method is to find symmetry constraints between the potential and the eigenfunction by means of variational derivative. On the one hand, nonlinearization of Lax pairs is valid for many classical integrable systems [24]-[27]. On the other hand, binary nonlinearization has been applied to the super AKNS system and the super Dirac system in Refs.[19]-[21]. However, is nonlinearization method valid for the other super integrable systems? For the cKdV system, the answer is affirmative in this paper. The cKdV system firstly proposed by Hirota and Satsuma in Ref.[28] is very important in the classical integrable systems. Its mono-nonlinearization and Darboux transformation were studied in Refs.[29, 30].

The paper is organized as follows. In the next section, the cKdV system is to be extended into the super one, and the super Hamiltonian structure will be obtained for new system by means of the supertrace identity. In section 3, variational derivative of the spectral parameter with respect to the potential is calculated by Lemma 2.1 in Ref. [21], and a symmetry constraint between the potential and the eigenfunction can be found. The symmetry constraint is an interesting constraint, and it is explicit for even elements, but it is implicit for odd elements. Then in section 4, after introduction of two new odd variables, the novel symmetry constraint is substituted into the Lax pairs and the adjoint Lax pairs of the super cKdV system while considering the two new variables. And we find that the constrained Lax pairs and the adjoint Lax pairs of the super cKdV system are super Hamiltonian systems, and are completely integrable systems in the Liouville sense. Integrals of motion with odd eigenfunctions are given explicitly. The conclusions and discussions are given in section 5.

2 The super cKdV soliton hierarchy

Let's begin with the following spectral problem

$$\phi_x = U(u,\lambda)\phi, \quad U(u,\lambda) = \begin{pmatrix} -\frac{1}{2}\lambda + \frac{1}{2}q & -r & \alpha \\ 1 & \frac{1}{2}\lambda - \frac{1}{2}q & \beta \\ \beta & -\alpha & 0 \end{pmatrix}, \quad u = \begin{pmatrix} q \\ r \\ \alpha \\ \beta \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1}$$

where u is a potential, and λ is a spectral parameter. Set $p(q) = p(r) = p(\lambda) = 0$, and $p(\alpha) = p(\beta) = 1$. Here $p(f)$ means the parity of arbitrary function f. Note that $U \in \mathbf{B}(0,1)$, where $\mathbf{B}(0,1)$ is a Lie superalegra.

Set

$$V=\begin{pmatrix} A & B & \rho \\ C & -A & \delta \\ \delta & -\rho & 0 \end{pmatrix}$$

where $p(A)=p(B)=p(C)=0$, $p(\rho)=p(\delta)=1$. Noting that

$$UV-VU=\begin{pmatrix} -B-rC+\alpha\delta+\beta\rho & -\lambda B+2rA+qB-2\alpha\rho & -\frac{1}{2}\lambda\rho-\alpha A-\beta B+\frac{1}{2}q\rho-r\delta \\ \lambda C+2A-qC+2\beta\delta & B+rC-\alpha\delta-\beta\rho & \frac{1}{2}\delta+\beta A-\alpha C+\rho-\frac{1}{2}q\delta \\ \frac{1}{2}\delta+\beta A-\alpha C+\rho-\frac{1}{2}q\delta & \frac{1}{2}\lambda\rho+\alpha A+\beta B-\frac{1}{2}q\rho+r\delta & 0 \end{pmatrix},$$

then there goes co-adjoint representation equation

$$V_x=[U,V]=UV-VU, \tag{2}$$

it becomes

$$\begin{cases} A_x=-B-rC+\alpha\delta+\beta\rho, \\ B_x=-\lambda B+2rA+qB-2\alpha\rho, \\ C_x=\lambda C+2A-qC+2\beta\delta, \\ \rho_x=-\frac{1}{2}\lambda\rho-\alpha A-\beta B+\frac{1}{2}q\rho-r\delta, \\ \delta_x=\frac{1}{2}\delta+\beta A-\alpha C+\rho-\frac{1}{2}q\delta. \end{cases} \tag{3}$$

On setting $A=\sum\limits_{j\geq 0}A_j\lambda^{-j}$, $B=\sum\limits_{j\geq 0}B_j\lambda^{-j}$, $C=\sum\limits_{j\geq 0}C_j\lambda^{-j}$, $\rho=\sum\limits_{j\geq 0}\rho_j\lambda^{-j}$, $\delta=\sum\limits_{j\geq 0}\delta_j\lambda^{-j}$, then equation (3) is equivalent to

$$\begin{cases} B_0=C_0=\rho_0=\delta_0=0, \\ A_{j,x}=-B_j-rC_j+\beta\rho_j+\alpha\delta_j, \quad j\geq 0, \\ B_{j,x}=-B_{j+1}+2rA_j+qB_j-2\alpha\rho_j, \quad j\geq 0, \\ C_{j,x}=C_{j+1}+2A_j-qC_j+2\beta\delta_j, \quad j\geq 0, \\ \rho_{j,x}=-\frac{1}{2}\rho_{j+1}-\alpha A_j-\beta B_j+\frac{1}{2}q\rho_j-r\delta_j, \quad j\geq 0, \\ \delta_{j,x}=\frac{1}{2}\delta_{j+1}+\beta A_j-\alpha C_j+\rho_j-\frac{1}{2}q\delta_j, \quad j\geq 0. \end{cases} \tag{4}$$

It can be written as the following recurrence relation

$$\begin{pmatrix} A_{n+1} \\ -C_{n+1} \\ 2\delta_{n+1} \\ -2\rho_{n+1} \end{pmatrix}=\mathcal{L}\begin{pmatrix} A_n \\ -C_n \\ 2\delta_n \\ -2\rho_n \end{pmatrix}, \tag{5}$$

where the recursive operator is given by

$$\mathcal{L}=\begin{pmatrix} -\partial+\partial^{-1}q\partial & r+\partial^{-1}r\partial & \frac{1}{2}\alpha+\partial^{-1}\alpha\partial & -\frac{1}{2}\beta+\partial^{-1}\beta\partial \\ 2 & \partial+q & \beta & 0 \\ -4\beta & -4\alpha & 2\partial+q & 2 \\ -4\beta\partial+4\alpha & 4r\beta & 2r-2\alpha\beta & -2\partial+q \end{pmatrix},$$

with $\partial=d/dx$ and $\partial\partial^{-1}=\partial^{-1}\partial=1$.

Owing to $B_0=C_0=\rho_0=\delta_0=0$, we get that $A_{0,x}=0$. So we choose the initial value $A_0=-\frac{1}{2}$. If we set all constants of integration to be zero, all $A_j,B_j,C_j,\rho_j,\delta_j(j>0)$ are uniquely given by (5). For instance

$$A_1=0, B_1=-r, C_1=1, \rho_1=\alpha, \delta_1=\beta,$$

$$A_2=-r+2\alpha\beta, B_2=r_x-qr, C_2=q, \rho_2=-2\alpha_x+q\alpha, \delta_2=2\beta_x+q\beta.$$

Then, consider the auxiliary spectral problem associated with the spectral problem (1)

$$\phi_{t_n}=V^{(n)}\phi, \tag{6}$$

where

$$V^{(n)}=(\lambda^n V)_+ +\Delta_n=\sum_{j=0}^{n}\begin{pmatrix} A_j & B_j & \rho_j \\ C_j & -A_j & \delta_j \\ \delta_j & -\rho_j & 0 \end{pmatrix}\lambda^{n-j}+\begin{pmatrix} \frac{1}{2}C_{n+1} & 0 & 0 \\ 0 & -\frac{1}{2}C_{n+1} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and $(\lambda^n V)_+$ denotes non-negative power of λ in V.

The compatibility conditions of Lax pairs

$$\phi_x=U\phi, \quad \phi_{t_n}=V^{(n)}\phi, \tag{7}$$

determine a hierarchy of super cKdV system

$$\begin{cases} q_{t_n}=C_{n+1,x}, \\ r_{t_n}=B_{n+1}+rC_{n+1}, \\ \alpha_{t_n}=\frac{1}{2}\alpha C_{n+1}-\frac{1}{2}\rho_{n+1}, \\ \beta_{t_n}=\frac{1}{2}\delta_{n+1}-\frac{1}{2}\beta C_{n+1}. \end{cases} \tag{8}$$

The first nonlinear cKdV system in the hierarchy (8) reads as

$$\begin{cases} q_{t_2}=q_{xx}+2qq_x+2r_x-4\alpha_x\beta-4\alpha\beta_x-4\beta\beta_{xx}, \\ r_{t_2}=-r_{xx}+2q_x r+2qr_x+4\alpha\alpha_x-4r\beta\beta_x, \\ \alpha_{t_2}=-2\alpha_{xx}+\frac{3}{2}q_x\alpha+2q\alpha_x+r_x\beta+2r\beta_x-2\alpha\beta\beta_x, \\ \beta_{t_2}=2\beta_{xx}+\frac{1}{2}q_x\beta+2q\beta_x+2\alpha_x, \end{cases} \tag{9}$$

whose Lax pairs are U and

$$V^{(2)}=\begin{pmatrix} -\frac{1}{2}\lambda^2+\frac{1}{2}q_x+\frac{1}{2}q^2-2\beta\beta_x & -r\lambda+r_x-qr & \alpha\lambda-2\alpha_x+q\alpha \\ \lambda+q & \frac{1}{2}\lambda^2-\frac{1}{2}q_x-\frac{1}{2}q^2+2\beta\beta_x & \beta\lambda+2\beta_x+q\beta \\ \beta\lambda+2\beta_x+q\beta & -\alpha\lambda+2\alpha_x-q\alpha & 0 \end{pmatrix}.$$

In what follows, the super Hamiltonian structures of the super cKdV system (8) can be achieved. Using the super trace identity [31, 32]

$$\frac{\delta}{\delta u}\int Str(V\frac{\partial U}{\partial \lambda})dx=(\lambda^{-\gamma}\frac{\partial}{\partial \lambda}\lambda^{\gamma})Str(\frac{\partial U}{\partial u}V), \tag{10}$$

where Str means super trace, we have

$$\begin{pmatrix} \frac{\delta}{\delta q} \\ \frac{\delta}{\delta r} \\ \frac{\delta}{\delta \alpha} \\ \frac{\delta}{\delta \beta} \end{pmatrix}\int -A_{n+1}dx=(\gamma-n)\begin{pmatrix} A_n \\ -C_n \\ 2\delta_n \\ -2\rho_n \end{pmatrix},$$

where γ is an arbitrary constant. Let $n=1$ in above equality, we obtain $\gamma=0$. Therefore, we get the following identity

$$\begin{pmatrix} A_{n+1} \\ -C_{n+1} \\ 2\delta_{n+1} \\ -2\rho_{n+1} \end{pmatrix}=\frac{\delta}{\delta u}H_n, \quad H_n=\int \frac{1}{n+1}A_{n+2}dx.$$

Thus, the super cKdV hierarchy can be written as the following super Hamiltonian form

$$u_{t_n} = \begin{pmatrix} q \\ r \\ \alpha \\ \beta \end{pmatrix}_{t_n} = K_n = J \begin{pmatrix} A_{n+1} \\ -C_{n+1} \\ 2\delta_{n+1} \\ -2\rho_{n+1} \end{pmatrix} = J\frac{\delta H_n}{\delta u}, \tag{11}$$

where the super symplectic operator is given by

$$J = \begin{pmatrix} 0 & -\partial & 0 & 0 \\ -\partial & 0 & \frac{1}{2}\alpha & -\frac{1}{2}\beta \\ 0 & -\frac{1}{2}\alpha & 0 & \frac{1}{4} \\ 0 & \frac{1}{2}\beta & \frac{1}{4} & 0 \end{pmatrix}.$$

3 A novel symmetry constraint

In this section, a symmetry constraint between the potential and the eigenfunction can be obtained. To this end, consider the adjoint spectral problem associated with spectral problem (1)

$$\psi_x = -(U(u,\lambda))^{St}\psi = \begin{pmatrix} \frac{1}{2}\lambda - \frac{1}{2}q & -1 & \beta \\ r & -\frac{1}{2}\lambda + \frac{1}{2}q & -\alpha \\ -\alpha & -\beta & 0 \end{pmatrix}\psi, \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \tag{12}$$

where St means super-transposition.

Using Lemma 2.1 in [21], we can easily get the variational derivative of the spectral parameter λ with respect to the potential u:

$$\frac{\delta\lambda}{\delta u} = \frac{1}{E} \begin{pmatrix} \frac{1}{2}(\psi_1\phi_1 - \psi_2\phi_2) \\ -\psi_1\phi_2 \\ \psi_1\phi_3 + \psi_3\phi_2 \\ \psi_2\phi_3 - \psi_3\phi_1 \end{pmatrix}, \tag{13}$$

where $E = \int \frac{1}{2}(\psi_1\phi_1 - \psi_2\phi_2)dx$. When zero boundary conditions $\lim_{|x|\to\infty}\phi = \lim_{|x|\to\infty}\psi = 0$ are imposed, it satisfies following equation

$$\mathcal{L}\frac{\delta\lambda}{\delta u} = \lambda\frac{\delta\lambda}{\delta u}, \tag{14}$$

where $\mathcal{L}$ is defined as in (5). The above variational derivative will serve as a conserved covariant yielding a specific symmetry used in symmetry constraints.

For Lax pairs (7), we choose the following symmetry constraint

$$\begin{pmatrix} -r + 2\alpha\beta \\ -q \\ 4\beta_x + 2q\beta \\ 4\alpha_x - 2q\alpha \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(<\Psi_1, \Phi_1> - <\Psi_2, \Phi_2>) \\ -<\Psi_1, \Phi_2> \\ <\Psi_1, \Phi_3> + <\Psi_3, \Phi_2> \\ <\Psi_2, \Phi_3> - <\Psi_3, \Phi_1> \end{pmatrix}, \tag{15}$$

where $\Phi_i = (\phi_{i1}, \cdots, \phi_{iN})^T$, $\Psi_i = (\psi_{i1}, \cdots, \psi_{iN})^T (i = 1, 2, 3)$, and $< .,. >$ denotes the standard inner product in R^N. We find that the odd potentials α and β can not be expressed by eigenfunctions explicitly, but the even potentials q and r can be expressed by eigenfunctions explicitly. Therefore, the symmetry constraint (15) is a novel constraint.

Remark 1 *In classical integrable systems, symmetry constraint between potential and eigenfunction is either explicit or implicit. To this day, we haven't got an example with its symmetry constraint that could combine explicit constraint and implicit constraint. Even in super integrable systems, we haven't got it too. Therefore, eq.(15) is absolutely a novel symmetry constraint.*

Then denote the expression of $P(u)$ under the symmetry constraint (15) by $\tilde{P}$. From the property (14) and the recurrence relation (5), we obtain

$$\begin{cases} \tilde{A}_{n+1} = \frac{1}{2}(<\Lambda^{n-1}\Psi_1, \Phi_1> - <\Lambda^{n-1}\Psi_2, \Phi_2>), \quad n \geq 1, \\ \tilde{B}_{n+1} =<\Lambda^{n-1}\Psi_2, \Phi_1>, \quad n \geq 1, \\ \tilde{C}_{n+1} =<\Lambda^{n-1}\Psi_1, \Phi_2>, \quad n \geq 1, \\ \tilde{\rho}_{n+1} = -\frac{1}{2}(<\Lambda^{n-1}\Psi_2, \Phi_3> - <\Lambda^{n-1}\Psi_3, \Phi_1>), \quad n \geq 1, \\ \tilde{\delta}_{n+1} = \frac{1}{2}(<\Lambda^{n-1}\Psi_1, \Phi_3> + <\Lambda^{n-1}\Psi_3, \Phi_2>), \quad n \geq 1, \end{cases} \tag{16}$$

where $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_N)$.

4 Binary nonlinearization

In the last section, we have found a novel symmetry constraint (15). Because the odd potentials α and β can not be explicitly expressed by eigenfunctions, we introduce the following new independent odd variables

$$\phi_{N+1} = \alpha, \quad \psi_{N+1} = 4\beta. \tag{17}$$

Choosing N distinct parameters $\lambda_1, \cdots, \lambda_N$, we obtain the following two spatial and temporal systems

$$\begin{cases} \begin{pmatrix} \phi_{1j} \\ \phi_{2j} \\ \phi_{3j} \end{pmatrix}_x = U(u, \lambda_j) \begin{pmatrix} \phi_{1j} \\ \phi_{2j} \\ \phi_{3j} \end{pmatrix}, \quad j = 1, 2, \cdots, N, \\ \begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}_x = -U^{St}(u, \lambda_j) \begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}, \quad j = 1, 2, \cdots, N, \end{cases} \tag{18}$$

$$\begin{cases} \begin{pmatrix} \phi_{1j} \\ \phi_{2j} \\ \phi_{3j} \end{pmatrix}_{t_n} = V^{(n)}(u, \lambda_j) \begin{pmatrix} \phi_{1j} \\ \phi_{2j} \\ \phi_{3j} \end{pmatrix}, \quad j = 1, 2, \cdots, N, \\ \begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}_{t_n} = -(V^{(n)})^{St}(u, \lambda_j) \begin{pmatrix} \psi_{1j} \\ \psi_{2j} \\ \psi_{3j} \end{pmatrix}, \quad j = 1, 2, \cdots, N. \end{cases} \tag{19}$$

It is easy to verify that the compatibility condition of (18) and (19) is still the nth super cKdV systems $u_{t_n} = K_n$. When the symmetry constraint (15) and new independent variables (17) are considered, systems (18) and (19) become the following finite-dimensional system

$$\left\{\begin{array}{l} \phi_{1j,x}=\frac{1}{2}(-\lambda_j+<\Psi_1,\Phi_2>)\phi_{1j}+\frac{1}{2}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>-\phi_{N+1}\psi_{N+1})\phi_{2j}\\ \qquad\quad +\phi_{N+1}\phi_{3j},\\ \phi_{2j,x}=\phi_{1j}+\frac{1}{2}(\lambda_j-<\Psi_1,\Phi_2>)\phi_{2j}+\frac{1}{4}\psi_{N+1}\phi_{3j},\\ \phi_{3j,x}=\frac{1}{4}\psi_{N+1}\phi_{1j}-\phi_{N+1}\phi_{2j},\\ \phi_{N+1,x}=\frac{1}{4}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)+\frac{1}{2}<\Psi_1,\Phi_2>\phi_{N+1},\\ \psi_{1j,x}=\frac{1}{2}(\lambda_j-<\Psi_1,\Phi_2>)\psi_{1j}-\psi_{2j}+\frac{1}{4}\psi_{N+1}\psi_{3j},\\ \psi_{2j,x}=\frac{1}{2}(-<\Psi_1,\Phi_1>+<\Psi_2,\Psi_2>+\phi_{N+1}\psi_{N+1})\psi_{1j}+\frac{1}{2}(-\lambda_j+<\Psi_1,\Phi_2>)\psi_{2j}\\ \qquad\quad -\phi_{N+1}\psi_{3j},\\ \psi_{3j,x}=-\phi_{N+1}\psi_{1j}-\frac{1}{4}\psi_{N+1}\psi_{2j},\\ \psi_{N+1,x}=<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>-\frac{1}{2}<\Psi_1,\Phi_2>\psi_{N+1}, \end{array}\right. \tag{20}$$

where $1\leq j\leq N$. Then system (20) can be written as follows

$$\left\{\begin{array}{l} \Phi_{1,x}=\frac{1}{2}(-\Lambda+<\Psi_1,\Phi_2>)\Phi_1+\frac{1}{2}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>-\phi_{N+1}\psi_{N+1})\Phi_2\\ \qquad\quad +\phi_{N+1}\Phi_3=\frac{\partial H_1}{\partial\Psi_1},\\ \Phi_{2,x}=\Phi_1+\frac{1}{2}(\Lambda-<\Psi_1,\Phi_2>)\Phi_2+\frac{1}{4}\psi_{N+1}\Phi_3=\frac{\partial H_1}{\partial\Psi_2},\\ \Phi_{3,x}=\frac{1}{4}\psi_{N+1}\Phi_1-\phi_{N+1}\Phi_2=\frac{\partial H_1}{\partial\Psi_3},\\ \phi_{N+1,x}=\frac{1}{4}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)+\frac{1}{2}<\Psi_1,\Phi_2>\phi_{N+1}=\frac{\partial H_1}{\partial\psi_{N+1}},\\ \Psi_{1,x}=\frac{1}{2}(\Lambda-<\Psi_1,\Phi_2>)\Psi_1-\Psi_2+\frac{1}{4}\psi_{N+1}\Psi_3=-\frac{\partial H_1}{\partial\Phi_1},\\ \Psi_{2,x}=\frac{1}{2}(-<\Psi_1,\Phi_1>+<\Psi_2,\Phi_2>+\phi_{N+1}\psi_{N+1})\Psi_1+\frac{1}{2}(-\Lambda+<\Psi_1,\Phi_2>)\Psi_2\\ \qquad\quad -\phi_{N+1}\Psi_3=-\frac{\partial H_1}{\partial\Phi_2},\\ \Psi_{3,x}=-\phi_{N+1}\Psi_1-\frac{1}{4}\psi_{N+1}\Psi_2=\frac{\partial H_1}{\partial\Phi_3},\\ \psi_{N+1,x}=<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>-\frac{1}{2}<\Psi_1,\Phi_2>\psi_{N+1}=\frac{\partial H_1}{\partial\phi_{N+1}}, \end{array}\right. \tag{21}$$

where Hamiltonian function

$$\begin{aligned} H_1 \;\; = \;\; & -\frac{1}{2}<\Lambda\Psi_1,\Phi_1>+\frac{1}{2}<\Lambda\Psi_2,\Phi_2>+\frac{1}{2}<\Psi_1,\Phi_2>(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\\ & +<\Psi_2,\Phi_1>-\frac{1}{2}\phi_{N+1}\psi_{N+1}<\Psi_1,\Phi_2>+\phi_{N+1}(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>)\\ & +\frac{1}{4}\psi_{N+1}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>). \end{aligned}$$

For t_2-part, we have the following spectral problem

$$\phi_{t_2}=V^{(2)}\phi=\begin{pmatrix} -\frac{1}{2}\lambda^2+\frac{1}{2}q_x+\frac{1}{2}q^2-2\beta\beta_x & -r\lambda+r_x-qr & \alpha\lambda-2\alpha_x+q\alpha\\ \lambda+q & \frac{1}{2}\lambda^2-\frac{1}{2}q_x-\frac{1}{2}q^2+2\beta\beta_x & \beta\lambda+2\beta_x+q\beta\\ \beta\lambda+2\beta_x+q\beta & -\alpha\lambda+2\alpha_x-q\alpha & 0 \end{pmatrix}\phi, \tag{22}$$

and its adjoint spectral problem

$$\psi_{t_2}=-(V^{(2)})^{St}\psi=\begin{pmatrix} \frac{1}{2}\lambda^2-\frac{1}{2}q_x-\frac{1}{2}q^2+2\beta\beta_x & -\lambda-q & \beta\lambda+2\beta_x+q\beta\\ r\lambda-r_x+qr & -\frac{1}{2}\lambda^2+\frac{1}{2}q_x+\frac{1}{2}q^2-2\beta\beta_x & -\alpha\lambda+2\alpha_x-q\alpha\\ -\alpha\lambda+2\alpha_x-q\alpha & -\beta\lambda-2\beta_x-q\beta & 0 \end{pmatrix}\psi. \tag{23}$$

Considering N copies of (22) and (23) under the symmetry constraint (15), we obtain the following finite-dimensional system

$$\begin{cases} \phi_{1j,t_2}=(-\frac{1}{2}\lambda_j^2+\frac{1}{2}\tilde{q}_x+\frac{1}{2}\tilde{q}^2-2\tilde{\beta}\tilde{\beta}_x)\phi_{1j}+(-\tilde{r}\lambda_j+\tilde{r}_x-\tilde{q}\tilde{r})\phi_{2j}+(\tilde{\alpha}\lambda_j-2\tilde{\alpha}_x+\tilde{q}\tilde{\alpha})\phi_{3j},\\ \phi_{2j,t_2}=(\lambda_j+\tilde{q})\phi_{1j}+(\frac{1}{2}\lambda_j^2-\frac{1}{2}\tilde{q}_x-\frac{1}{2}\tilde{q}^2+2\tilde{\beta}\tilde{\beta}_x)\phi_{2j}+(\tilde{\beta}\lambda_j+2\tilde{\beta}_x+\tilde{q}\tilde{\beta})\phi_{3j},\\ \phi_{3j,t_2}=(\tilde{\beta}\lambda_j+2\tilde{\beta}_x+\tilde{q}\tilde{\beta})\phi_{1j}+(-\tilde{\alpha}\lambda_j+2\tilde{\alpha}_x-\tilde{q}\tilde{\alpha})\phi_{2j},\\ \psi_{1j,t_2}=(\frac{1}{2}\lambda_j^2-\frac{1}{2}\tilde{q}_x-\frac{1}{2}\tilde{q}^2+2\tilde{\beta}\tilde{\beta}_x)\psi_{1j}-(\lambda_j+\tilde{q})\psi_{2j}+(\tilde{\beta}\lambda_j+2\tilde{\beta}_x+\tilde{q}\tilde{\beta})\psi_{3j},\\ \psi_{2j,t_2}=(\tilde{r}\lambda_j-\tilde{r}_x+\tilde{q}\tilde{r})\psi_{1j}+(-\frac{1}{2}\lambda_j^2+\frac{1}{2}\tilde{q}_x+\frac{1}{2}\tilde{q}^2-2\tilde{\beta}\tilde{\beta}_x)\psi_{2j}+(-\tilde{\alpha}\lambda_j+2\tilde{\alpha}_x-\tilde{q}\tilde{\alpha})\psi_{3j},\\ \psi_{3j,t_2}=(-\tilde{\alpha}\lambda_j+2\tilde{\alpha}_x-\tilde{q}\tilde{\alpha})\psi_{1j}-(\tilde{\beta}\lambda_j+2\tilde{\beta}_x+\tilde{q}\tilde{\beta})\psi_{2j}, \end{cases} \tag{24}$$

where $1\le j\le N$, $\tilde{q}$, $\tilde{r}$, $\tilde{\alpha}$, $\tilde{\beta}$ respectively denote q, r, α, β under the symmetry constraint (15), and $\tilde{q}_x$, $\tilde{r}_x$, $\tilde{\alpha}_x$, $\tilde{\beta}_x$ are given by the following identities

$$\begin{cases} \tilde{q}_x=<\Lambda\Psi_1,\Phi_2>-<\Psi_1,\Phi_2>^2+<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>+\frac{1}{4}\psi_{N+1}(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>),\\ \tilde{r}_x=<\Psi_2,\Phi_1>-\frac{1}{2}<\Psi_1,\Phi_2>(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)+\frac{1}{2}<\Psi_1,\Phi_2>\phi_{N+1}\psi_{N+1},\\ \tilde{\alpha}_x=\frac{1}{4}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)+\frac{1}{2}<\Psi_1,\Phi_2>\phi_{N+1},\\ \tilde{\beta}_x=\frac{1}{4}(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>)-\frac{1}{8}<\Psi_1,\Phi_2>\psi_{N+1}. \end{cases}$$

Thus, the constrained system (24) becomes

$$\begin{cases} \Phi_{1,t_2}=\frac{1}{2}(-\Lambda^2+<\Lambda\Psi_1,\Phi_2>+<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\Phi_1+\frac{1}{2}[(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\Lambda\\ \qquad -\phi_{N+1}\psi_{N+1}\Lambda+2<\Psi_2,\Phi_1>]\Phi_2+\frac{1}{2}(2\phi_{N+1}\Lambda-<\Psi_2,\Phi_3>+<\Psi_3,\Phi_1>)\Phi_3=\frac{\partial H_2}{\partial\Psi_1},\\ \Phi_{2,t_2}=(\Lambda+<\Psi_1,\Phi_2>)\Phi_1+\frac{1}{2}(\Lambda^2-<\Lambda\Psi_1,\Phi_2>-<\Psi_1,\Phi_1>+<\Psi_2,\Phi_2>)\Phi_2+\frac{1}{4}(\psi_{N+1}\Lambda\\ \qquad +2<\Psi_1,\Phi_3>+2<\Psi_3,\Phi_2>)\Phi_3=\frac{\partial H_2}{\partial\Psi_2},\\ \Phi_{3,t_2}=\frac{1}{4}(\psi_{N+1}\Lambda+2<\Psi_1,\Phi_3>+2<\Psi_3,\Phi_2>)\Phi_1-\frac{1}{2}(2\phi_{N+1}\Lambda-<\Psi_2,\Phi_3>+<\Psi_3,\Phi_1>)\Phi_2\\ \qquad =\frac{\partial H_2}{\partial\Psi_3},\\ \phi_{N+1,t_2}=\frac{1}{2}\phi_{N+1}<\Lambda\Psi_1,\Phi_2>+\frac{1}{4}(<\Lambda\Psi_2,\Phi_3>-<\Lambda\Psi_3,\Phi_1>)=\frac{\partial H_2}{\partial\Psi_{N+1}},\\ \Psi_{1,t_2}=\frac{1}{2}(\Lambda^2-<\Lambda\Psi_1,\Phi_2>-<\Psi_1,\Phi_1>+<\Psi_2,\Phi_2>)\Psi_1-(\Lambda+<\Psi_1,\Phi_2>)\Psi_2+\frac{1}{4}(\psi_{N+1}\Lambda\\ \qquad +2<\Psi_1,\Phi_3>+2<\Psi_3,\Phi_2>)\Psi_3=-\frac{\partial H_2}{\partial\Phi_1},\\ \Psi_{2,t_2}=-\frac{1}{2}[(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\Lambda-\phi_{N+1}\psi_{N+1}\Lambda+2<\Psi_2,\Phi_1>]\Psi_1+\frac{1}{2}(-\Lambda^2+<\Lambda\Psi_1,\Phi_2>\\ \qquad +<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\Psi_2-\frac{1}{2}(2\phi_{N+1}\Lambda-<\Psi_2,\Phi_3>+<\Psi_3,\Phi_1>)\Psi_3=-\frac{\partial H_2}{\partial\Phi_2},\\ \Psi_{3,t_2}=-\frac{1}{2}(2\phi_{N+1}\Lambda-<\Psi_2,\Phi_3>+<\Psi_3,\Phi_1>)\Psi_1-\frac{1}{4}(\psi_{N+1}\Lambda+2<\Psi_1,\Phi_3>+2<\Psi_3,\Phi_2>)\Psi_2\\ \qquad =\frac{\partial H_2}{\partial\Phi_3},\\ \psi_{N+1,t_2}=<\Lambda\Psi_1,\Phi_3>+<\Lambda\Psi_3,\Phi_2>-\frac{1}{2}\psi_{N+1}<\Lambda\Psi_1,\Phi_2>=\frac{\partial H_2}{\partial\phi_{N+1}}, \end{cases} \tag{25}$$

where Hamiltonian function is as follows

$$\begin{aligned} H_2 &= -\frac{1}{2}(<\Lambda^2\Psi_1,\Phi_1>-<\Lambda^2\Psi_2,\Phi_2>)+\frac{1}{2}<\Lambda\Psi_1,\Phi_2>(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)\\ &\quad +<\Lambda\Psi_2,\Phi_1>-\frac{1}{2}\phi_{N+1}\psi_{N+1}<\Lambda\Psi_1,\Phi_2>+\frac{1}{4}\psi_{N+1}(<\Lambda\Psi_2,\Phi_3>-<\Lambda\Psi_3,\Phi_1>)\\ &\quad +<\Psi_2,\Phi_1><\Psi_1,\Phi_2>-\frac{1}{2}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>)\\ &\quad +\phi_{N+1}(<\Lambda\Psi_1,\Phi_3>+<\Lambda\Psi_3,\Phi_2>)+\frac{1}{4}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)^2. \end{aligned}$$

Let's construct integrals of motion for (21). An obvious equality $(\tilde{V}^2)_x=[\tilde{U},\tilde{V}^2]$ leads to

$$F_x=(\frac{1}{2}Str\tilde{V}^2)_x=\frac{d}{dx}(\tilde{A}^2+\tilde{B}\tilde{C}+2\tilde{\rho}\tilde{\delta})=0, \tag{26}$$

that is to say, F is a generating function of integrals of motion for the constrained spatial system (21). Since $F=\sum\limits_{n\geq 0}F_n\lambda^{-n}$, we obtain the following expressions

$$F_n=\sum_{i=0}^{n}(\tilde{A}_i\tilde{A}_{n-i}+\tilde{B}_i\tilde{C}_{n-i}+2\tilde{\rho}_i\tilde{\delta}_{n-i}).$$

Using (16), we get

$$\begin{aligned}
F_0 &= \frac{1}{4},\quad F_1=F_2=0,\\
F_3 &= -\frac{1}{2}(<\Lambda\Psi_1,\Phi_1>-<\Lambda\Psi_2,\Phi_2>)-\frac{1}{4}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)\psi_{N+1}\\
&\quad +\frac{1}{2}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>-\phi_{N+1}\psi_{N+1})<\Psi_1,\Phi_2>+<\Psi_2,\Phi_1>\\
&\quad +\phi_{N+1}(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>)=H_1,\\
F_4 &= -\frac{1}{2}(<\Lambda^2\Psi_1,\Phi_1>-<\Lambda^2\Psi_2,\Phi_2>)+\phi_{N+1}(<\Lambda\Psi_1,\Phi_3>+<\Lambda\Psi_3,\Phi_2>)\\
&\quad +\frac{1}{2}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>-\phi_{N+1}\psi_{N+1})<\Lambda\Psi_1,\Phi_2>+<\Lambda\Psi_2,\Phi_1>\\
&\quad -\frac{1}{4}(<\Lambda\Psi_2,\Phi_3>-<\Lambda\Psi_3,\Phi_1>)\psi_{N+1}+\frac{1}{4}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>)^2\\
&\quad -\frac{1}{2}(<\Psi_2,\Phi_3>-<\Psi_3,\Phi_1>)(<\Psi_1,\Phi_3>+<\Psi_3,\Phi_2>)+<\Psi_2,\Phi_1><\Psi_1,\Phi_2>,\\
F_n &= -\frac{1}{2}(<\Lambda^{n-2}\Psi_1,\Phi_1>-<\Lambda^{n-2}\Psi_2,\Phi_2>)+\phi_{N+1}(<\Lambda^{n-3}\Psi_1,\Phi_3>+<\Lambda^{n-3}\Psi_3,\Phi_2>)\\
&\quad +\frac{1}{2}(<\Psi_1,\Phi_1>-<\Psi_2,\Phi_2>-\phi_{N+1}\psi_{N+1})<\Lambda^{n-3}\Psi_1,\Phi_2>+<\Lambda^{n-3}\Psi_2,\Phi_1>\\
&\quad -\frac{1}{4}(<\Lambda^{n-3}\Psi_2,\Phi_3>-<\Lambda^{n-3}\Psi_3,\Phi_1>)\psi_{N+1}+\sum_{i=2}^{n-2}[\frac{1}{4}(<\Lambda^{i-2}\Psi_1,\Phi_1>\\
&\quad -<\Lambda^{i-2}\Psi_2,\Phi_2>)(<\Lambda^{n-i-2}\Psi_1,\Phi_1>-<\Lambda^{n-i-2}\Psi_2,\Phi_2>)+<\Lambda^{i-2}\Psi_2,\Phi_1>\\
&\quad <\Lambda^{n-i-2}\Psi_1,\Phi_2>-\frac{1}{2}(<\Lambda^{i-2}\Psi_2,\Phi_3>-<\Lambda^{i-2}\Psi_3,\Phi_1>)(<\Lambda^{n-i-2}\Psi_1,\Phi_3>\\
&\quad +<\Lambda^{n-i-2}\Psi_3,\Phi_2>)],\quad n\geq 5.
\end{aligned}\tag{27}$$

Here $F_n(n\geq 0)$ are all polynomials of 6N+2 dependent variables ϕ_{ij}, ψ_{ij}, ϕ_{N+1} and ψ_{N+1}, with $i=1,2,3$ and $j=1,\cdots,N$. Note that for temporal part, $V_{t_n}=[V^{(n)},V]$ is true. With the similar discussion, we found that $F=\frac{1}{2}Str\tilde{V}^2$ is also a generating function of integrals of motion for (19). Moreover, when the symmetry constraint (15) and new independent variables (17) are considered, system (19) is constrained as follows

$$\left\{\begin{aligned}
&\phi_{1j,t_n}=(\sum_{m=0}^{n}\tilde{A}_m\lambda_j^{n-m}+\tfrac{1}{2}\tilde{C}_{n+1})\phi_{1j}+\sum_{m=0}^{n}\tilde{B}_m\lambda_j^{n-m}\phi_{2j}+\sum_{m=0}^{n}\tilde{\rho}_m\lambda_j^{n-m}\phi_{3j},\quad 1\leq j\leq N,\\
&\phi_{2j,t_n}=\sum_{m=0}^{n}\tilde{C}_m\lambda_j^{n-m}\phi_{1j}-(\sum_{m=0}^{n}\tilde{A}_m\lambda_j^{n-m}+\tfrac{1}{2}\tilde{C}_{n+1})\phi_{2j}+\sum_{m=0}^{n}\tilde{\delta}_m\lambda_j^{n-m}\phi_{3j},\quad 1\leq j\leq N,\\
&\phi_{3j,t_n}=\sum_{m=0}^{n}\tilde{\delta}_m\lambda_j^{n-m}\phi_{1j}-\sum_{m=0}^{n}\tilde{\rho}_m\lambda_j^{n-m}\phi_{2j},\quad 1\leq j\leq N,\\
&\phi_{N+1,t_n}=\tfrac{1}{2}\phi_{N+1}<\Lambda^{n-1}\Psi_1,\Phi_2>+\tfrac{1}{4}(<\Lambda^{n-1}\Psi_2,\Phi_3>-<\Lambda^{n-1}\Psi_3,\Phi_2>),\\
&\psi_{1j,t_n}=-(\sum_{m=0}^{n}\tilde{A}_m\lambda_j^{n-m}+\tfrac{1}{2}\tilde{C}_{n+1})\psi_{1j}-\sum_{m=0}^{n}\tilde{C}_m\lambda_j^{n-m}\psi_{2j}+\sum_{m=0}^{n}\tilde{\delta}_m\lambda_j^{n-m}\psi_{3j},\quad 1\leq j\leq N,\\
&\psi_{2j,t_n}=-\sum_{m=0}^{n}\tilde{B}_m\lambda_j^{n-m}\psi_{1j}+(\sum_{m=0}^{n}\tilde{A}_m\lambda_j^{n-m}+\tfrac{1}{2}\tilde{C}_{n+1})\psi_{2j}-\sum_{m=0}^{n}\tilde{\rho}_m\lambda_j^{n-m}\psi_{3j},\quad 1\leq j\leq N,\\
&\psi_{3j,t_n}=-\sum_{m=0}^{n}\tilde{\rho}_m\lambda_j^{n-m}\psi_{1j}-\sum_{m=0}^{n}\tilde{\delta}_m\lambda_j^{n-m}\psi_{2j},\quad 1\leq j\leq N,\\
&\psi_{N+1,t_n}=<\Lambda^{n-1}\Psi_1,\Phi_3>+<\Lambda^{n-1}\Psi_3,\Phi_2>-\tfrac{1}{2}\psi_{N+1}<\Lambda^{n-1}\Psi_1,\Phi_2>.
\end{aligned}\right.\tag{28}$$

After a direct calculation, we have

$$\left\{ \begin{array}{l} \Phi_{1,t_n} = \frac{\partial F_{n+2}}{\partial \Psi_1}, \quad \Phi_{2,t_n} = \frac{\partial F_{n+2}}{\partial \Psi_2}, \quad \Phi_{3,t_n} = \frac{\partial F_{n+2}}{\partial \Psi_3}, \quad \phi_{N+1,t_n} = \frac{\partial F_{n+2}}{\partial \Psi_{N+1}}, \\ \Psi_{1,t_n} = -\frac{\partial F_{n+2}}{\partial \Phi_1}, \quad \Psi_{2,t_n} = -\frac{\partial F_{n+2}}{\partial \Phi_2}, \quad \Psi_{3,t_n} = \frac{\partial F_{n+2}}{\partial \Phi_3}, \quad \psi_{N+1,t_n} = \frac{\partial F_{n+2}}{\partial \Phi_{N+1}}, \end{array} \right. \tag{29}$$

which shows that constrained system (28) is a super Hamiltonian system.

In what follows, for 6N+2 dimensional super Hamiltonian systems (21) and (29), we find 3N+1 integrals of motion. It is natural to find that

$$f_k = \psi_{1k}\phi_{1k} + \psi_{2k}\phi_{2k} + \psi_{3k}\phi_{3k}, \quad 1 \leq k \leq N, \tag{30}$$

are integrals of motion for constrained systems (21) and (29). Therefore, for constrained systems (21) and (29), we choose 3N+1 integrals of motion

$$f_1, \cdots, f_N, F_3, F_4, \cdots, F_{2N+3}. \tag{31}$$

After a simple calculation, we get

$$\{F_m, F_{n+2}\} = \frac{\partial}{\partial t_n} F_m = 0, \tag{32}$$

where Poisson bracket is defined by

$$\{F, G\} = \sum_{i=1}^{3}\sum_{j=1}^{N}\left(\frac{\partial F}{\partial \phi_{ij}}\frac{\partial G}{\partial \psi_{ij}} - (-1)^{p(\phi_{ij})p(\psi_{ij})}\frac{\partial F}{\partial \psi_{ij}}\frac{\partial G}{\partial \phi_{ij}}\right) + \frac{\partial F}{\partial \phi_{N+1}}\frac{\partial G}{\partial \psi_{N+1}} + \frac{\partial F}{\partial \psi_{N+1}}\frac{\partial G}{\partial \phi_{N+1}}. \tag{33}$$

The identity (32) means that $\{F_m\}_{m\geq 0}$ are in involution. The property of involution among $\{f_k\}_{k=1}^{N}$ is obvious. About the independence of $\{f_k\}_{k=1}^{N}$ and $\{F_m\}_{m=3}^{2N+3}$, we can refer to the proof of Proposition 1 in [19]. Thus we obtain the following theorem

Theorem 1 *The constrained systems (21) and (29) are Liouville integrable super Hamiltonian systems, whose integrals of motion are given by (31).*

5 Conclusions and Discussions

In this paper, the cKdV system is successfully extended to the super one. For new system, its super Hamiltonian structure is expressed in the form of (11). In our previous papers [19]-[21],the binary nonlinearization has been applied to the super AKNS system and the super Dirac system. For the super AKNS system, two kinds of nonlinearization of Lax pairs, including nonlinearization under an explicit symmetry constraint[19] and nonlinearization under an implicit symmetry constraint[20], have been considered respectively. And for the super Dirac system, we only consider binary nonlinearization under an explicit symmetry constraint[21]. From these three kinds of nonlinearization of Lax pairs, the symmetry constraint is either implicit or explicit. The novelty of the constraint (15) for the super cKdV system is due to the combination of the explicit constraint for even potentials (q, r) and the implicit constraint for odd potentials (α, β). Such combination will make the process of binary nonlinearization complex. It is highly non-trivial to solve (α, β) from implicit constraints (15) because it is related to a coupled differential equations with variable coefficients. We introduce two new odd variables (17) following the technique of implicit constraint[33]. Thus, the spatial part and temporal parts of the super cKdV system are nonlinearized respectively to the constrained spatial system (21) and to the constrained temporal system (29). Then, we see that systems (21) and (29) are super Hamiltonian systems. Furthermore, constrained systems (21) and (29) are integrable in the Liouville sense.

However, we are not able to do this for supersymmetric cKdV system. Because spectral matrix of supersymmetric cKdV system can not be described by a certain Lie super algebra. In a word, how to make nonlinearization of supersymmetric cKdV system is an interesting problem. Furthermore, it is also an interesting problem to find an explicit solution of the super finite dimensional integrable system. We shall consider these problems in the future.

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References

[1] M. G*ü*rses, *Ö*. O*ğ*uz, A super AKNS scheme, Phys. Lett. A 108 (1985), no. 9, 437-440.

[2] Y. S. Li, L. N. Zhang, Super AKNS scheme and its infinite conserved currents, Nuovo. Cimento A 93 (1986), no. 2, 175-183.

[3] Y. S. Li, L. N. Zhang, Hamiltonian structure of the super evolution equation, J. Math. Phys. 31 (1990), no. 2, 470-475.

[4] B. A. Kupershmidt, A super Korteseg-de Vries equation: an integrable system, Phys. Lett. A 102 (1984), no. 5-6, 213-215.

[5] P. Mathieu, Supersymmetric extension of the Korteweg-de Vries equation, J. Math. Phys. 29 (1988), no. 11, 2499-2506.

[6] D. Ashok, R. Shibaji, The zero curvature formulation of the sKdV equations, J. Math. Phys. 31 (1990), no. 9, 2145-2149.

[7] J. C. Shaw, M. H. Tu, Binary Darboux-B*ä*cklund transformations for the Manin-Radul super KdV hierarchy, J. Math. Phys. 39 (1998), no. 9, 4773-4784.

[8] Yu. I. Manin, A. O. Radul, A supersymmetric extension of the Kadomtsev-Petviashvili hierarchy, Comm. Math. Phys. 98 (1985) 65-77.

[9] F. Yu, Bi-Hamiltonian structure of super KP hierarchy, J. Math. Phys. 33 (1992), no. 9, 3180-3189..

[10] I. N. McArthur, Two-reduction of the super-KP hierarchy, Comm. Math. Phys. 159 (1994) 121-131.

[11] J. C. Shaw, M. H. Tu, Hamiltonian structures of generated Manin-Radul super-KdV and constrained super KP hierarchies, J. Math. Phys. 40 (1999), no. 6, 3021-3034.

[12] Q. P. Liu, Darboux transformation for the Manin-Radul supersymmetry KdV equation, M. Ma*ñ*as, Phys. Lett. B 394 (1997) 337-342.

[13] Q. P. Liu, Darboux transformations for super-symmetric KP hierarchies, Phys. Lett. B 485 (2000) 293-300.

[14] M. Siddiq, M. Hassan, U. Saleem, On Darboux transformation of the supersymmetric sine-Gordon equation, J. Phys. A: Math. Gen. 39 (2006) 7313-7318.

[15] Z. Popowicz, Odd bihamiltonian structure of new supersymmetric N=2, 4 Korteweg de Vries equation and odd SUSY Virasoro-like algebra, Phys. Lett. B 459 (1999) 150-158.

[16] F. R. Manuel, Dynamical symmetries, bi-Hamiltonian structures, and superintegrable n=2 systems, J. Math. Phys. 41 (2000), no. 4, 2121-2134.

[17] P. H. M. Kersten, A. S. Sorin, Bi-Hamiltonian structure of the N-2 supersymmetric α=1 KdV hierarchy, Phys. Lett. A 300 (2002) 397-406.

[18] S. Bourque, P. Mathieu, The Painlevé analysis for N=2 super Korteweg-de Vries equations, J. Math. Phys. 42 (2001), no.8, 3517-3539.

[19] J. S. He, J. Yu, R. G. Zhou, Y. Cheng, Binary nonlinearization of the super AKNS system, Modern Phys. Lett. B 22 (2008), no. 4, 275-288.

[20] J. Yu, J. W. Han, J. S. He, Binary nonlinearization of the super AKNS system under an implicit symmetry constraint, J. Phys. A: Math. Theor. 42 (2009) 465201(10pp).

[21] J. Yu, J. S. He, W. X. Ma, Y. Cheng, The Bargmann symmetry constraint and binary nonlinearization of the super Dirac systems, Chin. Ann. Math. B 31 (2010), no. 3, 361-372.

[22] C. W. Cao, A cubic system which generates Bargmann potential and N-gap potential, Chin. Quart. J. Math. 3 (1988), no. 1, 90-96.

[23] W. X. Ma, W. Strampp, An explicit symmetry constraint for the Lax pairs and the adjoint Lax pairs of AKNS systems, Phys. Lett. A 185 (1994), no. 3, 277-286.

[24] Y. B. Zeng, Y. S. Li, The constraints of potentials and the finite-dimensional integrable systems, J. Math. Phys. 30 (1989), no. 8, 1679-1689.

[25] W. X. Ma, R. G. Zhou, Adjoint symmetry constraints leading to binary nonlinearization, J. Nonlinear Math. Phys. 9 (2002) 106-126.

[26] X. X. Xu, A generalized Wadati-Konno-Ichikawa hierarchy and its binary nonlinearization by symmetry constraints, Chaos, Solitons and Fractals 15 (2003) 475-486.

[27] R. G. Zhou, Nonlinearizations of spectral problems of the nonlinear Schrödinger equation and the real-valued modified Korteweg-de Vries equation, J. Math. Phys. 48 (2007), no. 1, 013510.

[28] R. Hirota, J. Satsuma, Soliton solutions of a coupled Korteweg-de Vries equation, Phys. Lett. A 85 (1981), no. 8-9, 407-408.

[29] C. W. Cao, X. G. Geng, C. Neumann and Bargmann systems associated with the coupled KdV soliton hierarchy, J. Phys. A 23 (1990), no. 18, 4117-4125.

[30] Z. Y. Qin, Darboux transformations of the cKdV hierarchy with sources, Chaos Solitons Fractals 21 (2004), no. 4, 989-998.

[31] X. B. Hu, An approach to generate superextensions of integrable systems, J. Phys. A 30 (1997) no. 2, 619-632.

[32] W. X. Ma, J. S. He, Z. Y. Qin, A supertrace identity and its applications to superintegrable systems, J. Math. Phys. 49 (2008), no. 3, 033511.

[33] Y. S. Li, W. X. Ma, Binary nonlinearization of AKNS spectral problem under higher-order symmetry constraints, Chaos, Solitons and Fractals 11(2000), 697-710.