INTERACTION OF THE ELECTROMAGNETIC S–WAVE WITH THE THIN METAL FILM

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Abstract

It is shown that for thin metal films, thickness of which does not exceed a thickness of a skin – layer, the problem allows analytical solution for any boundary conditions. The analysis of transmission, reflection and absorption of an electromagnetic wave coefficients depending on a angle of incidence, thickness of a layer, coefficient of specular reflection and frequency of oscillations of electromagnetic field is carried out.

Key words: degenerate collisional plasma, electromagnetic p-wave, thin metallic film, coefficients of transmission, reflection and absorbtion.

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INTRODUCTION

The problem of interaction of an electromagnetic wave with the metal films attracts attention to itself for a long time [1] - [5]. It is connected with the theoretical interest to this problem, and with numerous practical appendices of it as well.

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Research of interaction of an electromagnetic wave with conducting medium (in particular, metal films) was carried out basically for a case of specular dissipation of electrons on a film surface. It is connected with the fact that for more general boundary conditions the problem becomes essentially complicated and does not allow analytical solution generally. At the same time for the real materials coefficient of specular electron reflection from the surface is far from unit as a rule. For example, in the work [6] on the basis of the analysis of longitudinal magnetic resistance of the thin metal wire it is shown, that for sodium the coefficient of specular reflection is equal to 0.3.

In the present work it is shown that for thin films, thickness of which does not exceed thickness of a skin – layer, the problem allows analytical solution for any boundary conditions.

Let's notice, that the most part of reasonings carrying out below is fair for more general case of conducting (in particular, semi-conductor) films.

PROBLEM STATEMENT

Let's consider a thin slab of metal, which the electromagnetic wave falls on. An angle of falling we will designate as θ . Let's assume that a vector of electric field of the electromagnetic wave is parallel to a slab surface. Such wave is called *s*-wave (see [3] or [1]).

We take the Cartesian system of coordinates with the origin of coordinates on one of the slab surfaces, an axis x, directed deep into a slab. Axis y we direct parallel to electric field vector of electromagnetic wave. Then the behaviour of electric and magnetic fields of a wave in a layer is described by the following system of differential equations [3]:

$$\begin{pmatrix}
\frac{dE_y}{dx} - ikH_z = 0 \\
\frac{dH_z}{dx} + ik(\sin^2\theta - 1)E_y = -\frac{4\pi}{c}j_y.
\end{cases}$$
(1)

Here c is the velocity of light, **j** is the current density, k is the wave number $(k = \frac{\omega}{c})$, $E_y(x)$ and $H_z(x)$ defined from relations for electric and magnetic fields

$$\mathbf{E} = e^{-i\omega t + ik\sin\theta y} \{ 0, E_y(x), 0 \}, \qquad \mathbf{H} = e^{-i\omega t + ikz} \{ H_x(x), 0, H_z(x) \}.$$

Let's designate a thickness of a slab as d.

Transmission coefficient T, reflection coefficient R and slab absorption coefficient A of the electromagnetic wave are described by the following expressions [1], [7]

$$T = \frac{1}{4} \left| P^{(1)} - P^{(2)} \right|^2, \tag{2a}$$

$$R = \frac{1}{4} \left| P^{(1)} + P^{(2)} \right|^2, \tag{2b}$$

$$A = 1 - T - R. \tag{2c}$$

Quantities $P^{(j)}$ (j = 1, 2) are defined by the following expressions

$$P^{(j)} = \frac{Z^{(j)} \cos \theta - 1}{Z^{(j)} \cos \theta + 1}, \qquad j = 1, 2.$$
(3)

Quantities $Z^{(1)}$ and $Z^{(2)}$ correspond to an impedance on the bottom slab surface at antisymmetric by electric field (case 1, when $E_y(0) = -E_y(d), H_z(0) = H_z(d)$) and symmetric by electric field (case 2, when $E_y(0) = E_y(d), H_z(0) = -H_z(d)$) configurations of external fields.

The impedance thus in both cases is defined as follows

$$Z^{(j)} = \frac{E_y(-0)}{H_z(-0)}.$$
(4)

Let's consider a case when the width of a slab d is less than depth of a skin – layer δ . We will note, that depth of a skin – layer depends essentially on frequency of radiation, decreasing monotonously during the process of growth of the last. The quantity δ possesses the minimum value in so-called infra-red case [8]

$$\delta_0 = \frac{c}{\omega_p},$$

where ω_p is the plasma frequency.

For typical metals [8] $\delta_0 \sim 10^{-5}$ cm.

Thus for the films thickness of which d is less than δ_0 our assumption is correct for any frequencies.

Electric and magnetic fields vary little at distances less than depth of a skin – layer. Therefore under fulfilment of the given assumption $d < \delta$ this field will vary a little within a slab. In case 1 when $H_z(0) = H_z(d)$, it is possible to assume, that quantity H_z is constant in the slab. Change of quantity of electric field at the thickness of a slab can be defined from the first equation of system (1)

$$E_y(d) - E_y(0) = ikdH_z.$$

Considering antisymmetric character of electric field in this case we receive

$$E_y(0) = -\frac{ikdH_z}{2}.$$

Accordingly (4) for the impedance we have

$$Z^{(1)} = -\frac{idk}{2}.$$
 (5)

For the case 2 when $E_y(0) = E_y(d)$, it is possible to consider electric field as constant in the slab, which we will designate as E_y . Then magnetic field change at the width of slab can be defined from the second equation of the system (1)

$$H_z(d) - H_z(0) = ikd(\sin^2\theta - 1)E_y - \frac{4\pi}{c} \int_0^d j_y(x)dx.$$
 (6)

Thus

$$j_y(x) = \sigma(x)E_y,$$

where $\sigma(x)$ is the conductivity which depends from coordinate x in general case.

Let's introduce the conductivity averaged by thickness of slab

$$\sigma_d = \frac{1}{E_y d} \int_0^d j_y(x) dx = \frac{1}{d} \int_0^d \sigma(x) dx.$$
(7)

Then the relation (6) according to (7) can be rewritten in the form

$$H_z(d) - H_z(0) = ikd(\sin^2\theta - 1)E_y - \frac{4\pi d\sigma_d}{c}E_y.$$

Considering symmetry of the magnetic field, from here we receive

$$H_z(0) = -\frac{1}{2}ikd(\sin^2\theta - 1)E_y + \frac{2\pi d\sigma_d}{c}E_y.$$

For the impedance (4) we receive

$$Z^{(2)} = \frac{2c}{-ickd(\sin^2\theta - 1) + 4\pi d\sigma_d}$$

Let's assume further, that length of the wave of incoming radiation surpasses essentially thickness of a slab. This assumption is satisfied for the majority of cases when the thickness of a slab is less than the depth of a skin – layer. Then the quantity $kd \ll 1$ and in expressions for impedances it is possible for to neglect it. It is hence received according to (5) $Z^{(1)} = 0$, and $Z^{(2)} = \frac{c}{2\pi d\sigma_d}$. According to (3) we have

$$P^{(1)} = \frac{c\cos\theta - 2\pi d\sigma_d}{c\cos\theta + 2\pi d\sigma_d}, \qquad P^{(2)} = -1.$$

According to the expressions (2a), (2b) and (2c) we receive

$$T = \left| \frac{c \cos \theta}{c \cos \theta + 2\pi \sigma_d d} \right|^2, \tag{8a}$$

$$R = \left| \frac{2\pi\sigma_d d}{c\cos\theta + 2\pi\sigma_d d} \right|^2,\tag{8b}$$

$$A = \frac{4c\pi \operatorname{Re}\left(\sigma_d\right) d\cos\theta}{\left|c\cos\theta + 2\pi\sigma_d d\right|^2}.$$
(8c)

In limiting case of a non-conductive slab, when $\sigma_d \to 0$ from these expressions we have $T \to 1, R \to 0, A \to 0$. At almost tangential falling, when $\theta \to \pi/2$ we receive $T \to 0, R \to 1, A \to 0$.

If we designate

$$B = \frac{2\pi d\sigma_d}{c\cos\theta},$$

then formulas (8) can be written down in the compact form

$$T = \frac{1}{|1+B|^2}, \qquad R = \frac{1}{|1+B^{-1}|^2}, \qquad A = \frac{2 \operatorname{Re} B}{|1+B|^2}.$$
(9)

Let's consider a case of a metal film. Let the relation $kd \ll 1$ to be satisfied. Then in a low-frequency case, when $\omega \to 0$, the quantity σ_d can be presented in the form [9]

$$\sigma_d = \frac{w}{\Phi(w)} \sigma_0, \qquad w = \frac{d}{l}.$$
 (10)

Here

$$\frac{1}{\Phi(w)} = \frac{1}{w} - \frac{3}{2w^2}(1-p)\int_{1}^{\infty} \left(\frac{1}{t^3} - \frac{1}{t^5}\right) \frac{1-e^{-wt}}{1-pe^{-wt}} dt.$$

Here l is the mean free electron path, p is the reflectivity coefficient, $\sigma_0 = \omega_p^2 \tau / (4\pi)$ is the static conductivity of the volume sample, $\tau = l/v_F$ is the time of the mean free electron path, v_F is the electron speed on Fermi's surface. It is supposed, that Fermi's surface has spherical form.

In a low-frequency case when the formula (10) is applicable, coefficients T, R, A according to the formulas (8) do not depend on frequency of the incident radiation.

For any frequencies of the expression (8) will be satisfied under the condition, that it is necessary to use the following expression as quantity $l \rightarrow \frac{v_F \tau}{1 - i\omega\tau}$, and instead of σ_0 we should use the expression $\sigma_0 \rightarrow \frac{\sigma_0}{1 - i\omega\tau}$.

In case of arbitrary frequencies coefficients of transmission, reflection and absorption are calculated also with the help of the formulas (9), in which

$$w = \frac{d}{l}(1 - i\omega\tau),$$

and

$$B = \frac{2\pi d\sigma_0}{c\cos\theta(1-i\omega\tau)} \left[1 - \frac{1.5}{w}(1-p) \int_{1}^{\infty} \left(\frac{1}{t^3} - \frac{1}{t^5}\right) \frac{1-e^{-wt}}{1-pe^{-wt}} dt \right].$$

Let's consider the case of a thin slab of sodium. Then [1] $\omega_p = 6.5 \cdot 10^{15} \text{ sec}^{-1}$, $v_F = 8.52 \cdot 10^7 \text{ cm/sec}$. Frequency of the volume collisions of electrons we take to be equal $\nu = \tau^{-1} = 10^{-3}\omega_p \text{ sec}^{-1}$. On fig. 1, 2 and 3 correspondingly the dependence of coefficients of transmission, reflection and absorption on an angle of falling of an electromagnetic wave on the border, on a thickness of a layer and on reflectivity coefficients is represented. On fig. 4 and 5 dependence of reflectivity on frequency of oscillations of an electromagnetic field under various values of thickness of the slab is represented. On fig. 4 the case when coefficient of specular

reflection is equal to zero is considered. On fig. 5 the case when coefficient of specular reflection is equal to unit is considered.

CONCLUSION

From fig. 1 it is visible, that dependences of all the coefficients $T = T(\theta)$, $R = R(\theta)$ and $A = A(\theta)$ become apparent close to $\theta = \frac{\pi}{2}$. Thus the absorption coefficients has smooth maximum close to the point $\theta = \frac{\pi}{2}$.

It is interesting to note (fig. 2), that the quantity of absorption coefficient practically does not depend on a thickness of the slab d (under change d from 10^{-7} cm to 10^{-6} cm). Under this change d approximately in 2 times the propagation coefficient decreases, and the reflection coefficient increases (also in 2 times).

On fig. 3 for the first time dependence of coefficient T = T(p), R = R(p)and A = A(p) on quantity of coefficient of specular reflection is found out. Coefficients T, R, A discover strong dependence on coefficient of specular reflection that is found out for the first time. With the growth of coefficient of specular reflection the reflection coefficient increases, and the absorption coefficient decreases.

The analysis of graphs on fig. 4 and fig. 5 shows, that with growth of oscillations frequency the reflection coefficient is monotonously decreasing function. With growth of a thickness of the slab the values of reflection coefficient increase, with growth of coefficient of reflection the values of reflection coefficient also increase.



Figure 1. Dependence of transmission coefficient (curve 1), reflecion coefficient (curve 2) and absorption coefficient (curve 3) on quantity of the angle of incidence θ , $0 \le \theta \le \frac{\pi}{2}$, $d = 10^{-7}$ cm, $\omega = 10^{-2}\omega_p \sec^{-1}$, p = 0.5.



Figure 2. Dependence of transmission coefficient (curve 1), reflecion coefficient (curve 2) and absorption coefficient (curve 3) on quantity of the thickness of the slab d, 10^{-7} cm $\leq d \leq 10^{-6}$ cm, at normal falling of the wave ($\theta = 0$), $\omega = 10^{-1}\omega_p \sec^{-1}$, p = 0.5.



Figure 3. Dependence of transmission coefficient (curve 1), reflection coefficient (curve 2) and absorption coefficient (curve 3) on quantity of coefficient of specular reflection p ($0 \le p \le 1$) under normal incidence of the wave ($\theta = 0$), $\omega = 10^{-1}\omega_p c^{-1}$, $d = 10^{-7}$ sm

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Figure 4. Dependence of reflection coefficient R on the quantity of oscillation frequency of the field ω under various values of a thickness slab d and under normal falling of an electromagnetic wave ($\theta = 0$). Curves of 1, 2, 3 correspond to values $d = 10^{-7}$ cm, $2 \cdot 10^{-7}$ cm, $3 \cdot 10^{-7}$ cm The coefficient of specular reflection is equal to zero (p = 0).



Figure 5. Dependence of reflection coefficient R on the quantity of oscillation frequency of the field ω under various values of a thickness slab d and under normal incidence of an electromagnetic wave ($\theta = 0$). Curves of 1, 2, 3 correspond to values $d = 10^{-7}$ cm, $2 \cdot 10^{-7}$ cm, $3 \cdot 10^{-7}$ cm. The coefficient of specular reflection is equal to unit (p = 1).

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