

# Reduction of quantum noise in optical interferometers using squeezed light

Michael Weyrauch\*

*Physikalisch-Technische Bundesanstalt, D-38116 Braunschweig, Germany*

Volodymyr G. Voronov†

*Faculty of Physics, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine*

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## Abstract

We study the photon counting noise in optical interferometers used for gravitational wave detection. In order to reduce quantum noise a squeezed vacuum state is injected into the usually unused input port. Here, we specifically investigate the so called ‘dark port case’, when the beam splitter is oriented close to  $90^\circ$  to the incoming laser beam, such that nearly all photons go to one output port of the interferometer, and only a small fraction of photons is seen in the other port (‘dark port’). For this case it had been suggested that signal amplification is possible without concurrent noise amplification [R. Barak and Y. Ben-Aryeh, J. Opt. Soc. Am. B25(361)2008]. We show that by injection of a squeezed vacuum state into the second input port, counting noise is reduced for large values of the squeezing factor, however the signal is not amplified. Signal strength only depends on the intensity of the laser beam.

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\* michael.weyrauch@ptb.de

† v\_voronov@univ.kiev.ua

## I. INTRODUCTION

For gravitational wave detection with optical interferometers various sources of noise must be carefully controlled and, if possible, minimized. In an effort to reduce quantum-mechanical noise, Caves [1] proposed the squeezed state technique: into the normally unused port of the interferometer a squeezed vacuum state is injected. Details of this technique are analyzed e.g. in Refs. [2, 3] and references therein.

The photon state in the interferometer after passing a beam splitter is not a product state, but the states of the output ports are entangled. In recent papers by Barak and Ben-Aryeh [4] and Voronov and Weyrauch [5], the consequences of this entanglement for the photon statistics of an optical interferometer were studied.

In Ref. [4] it was suggested that under certain conditions, the gravitational wave signal may be amplified without a corresponding increase in counting noise. In Ref. [5] we disputed this surprising prediction, and showed that it was the result of an inaccuracy in the calculations. Furthermore, we calculated photon distributions in the output state for various settings of a beam splitter with respect to weak and strong incoming laser fields. We showed that a squeezed vacuum injected into the other port cannot amplify the signal however may reduce counting noise for large squeezing factors.

In a recent paper [6], Ben-Aryeh specifically reanalyzes the ‘dark port case’, i.e. a configuration where the beam splitter is oriented close to  $90^\circ$  to the incoming laser beam with a squeezed vacuum entering the other port. He confirms our findings in Ref. [5] and sharpens the physical interpretation of the results obtained.

It is the purpose of the present paper to investigate the photon statistics in the dark output port of the interferometer in more detail, and present the calculations and results more succinctly than in our previous paper [5].

In section II we develop formulas for the calculation of the photon number distributions in the dark output port of the interferometer, as well as their mean values and variances. Numerical results and their physical interpretation will be discussed in section 3. A brief summary concludes the paper.

## II. PHOTON STATISTICS IN DARK OUTPUT PORT OF A BEAM SPLITTER

The photon field operators  $\hat{a}_i$  and  $\hat{a}_i^\dagger$  of the input ports and the photon field operators  $\hat{b}_i$  and  $\hat{b}_j^\dagger$  of the output ports of a beam splitter are related through the beam splitter transformation [7]

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}. \quad (1)$$

Both sets of operators fulfill boson commutation relations  $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{i,j}$  and  $[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{i,j}$ , respectively. The parameter  $\gamma$  parameterizes the splitting ratio of the beam splitter with respect to the incoming laser beam.

The incoming laser beam in port 1 is a coherent state, and a squeezed vacuum state is injected in port 2:

$$|\psi_{\text{in}}(\alpha, \zeta)\rangle = \hat{S}_2(\zeta)\hat{D}_1(\alpha)|0, 0\rangle \quad (2)$$

with

$$\hat{D}_1(\alpha) = \exp\left(\alpha\hat{a}_1^\dagger - \alpha^*\hat{a}_1\right), \quad \hat{S}_2(\zeta) = \exp\left(\frac{\zeta^*}{2}\hat{a}_2^2 - \frac{\zeta}{2}\hat{a}_2^{\dagger 2}\right). \quad (3)$$

The coherence parameter  $\alpha$  and the squeezing parameter  $\zeta$  are complex numbers.

The beam splitter transformation allows to write the  $\hat{a}$  operators in terms of the  $\hat{b}$  operators, and one may write the photon state after passing the beam splitter as

$$|\psi_{\text{out}}(\alpha, \zeta, \gamma)\rangle = \exp(|\zeta|\hat{A})\hat{D}_1(\alpha \cos \gamma)\hat{D}_2(\alpha \sin \gamma)|0, 0\rangle \quad (4)$$

with

$$\hat{A} = \hat{s}_1 \sin^2 \gamma + \hat{s}_2 \cos^2 \gamma + \hat{s}_{12} \sin \gamma \cos \gamma \quad (5)$$

and

$$\hat{s}_i = \frac{1}{2|\zeta|}(\zeta^*\hat{b}_i^2 - \zeta\hat{b}_i^{\dagger 2}) \quad \hat{s}_{12} = \frac{1}{|\zeta|}(\zeta\hat{b}_1^\dagger\hat{b}_2^\dagger - \zeta^*\hat{b}_1\hat{b}_2). \quad (6)$$

From the expression for  $\hat{A}$  we see that both output states are entangled by the operator  $\hat{s}_{12}$ . This fact significantly complicates evaluation of the photon statistics of the output state. However, it is possible to use Lie algebraic disentangling techniques in order to rewrite the output state in a way which enables the determination of photon distributions (for details we refer the reader to Ref. [5]).

After disentangling it is possible to write Eq. (4) as follows [8, 9]

$$|\psi_{\text{out}}\rangle = \exp(\sigma_T\hat{t}_{12}) \exp(\sigma_S\hat{s}_{12}) \exp(\sigma_1\hat{s}_1) \exp(\sigma_2\hat{s}_2)\hat{D}_1(\alpha \cos \gamma)\hat{D}_2(\alpha \sin \gamma)|0, 0\rangle \quad (7)$$

with  $\hat{t}_{12} = \hat{b}_1 \hat{b}_2^\dagger - \hat{b}_1^\dagger \hat{b}_2$ . The output state is now expressed in terms of two squeezed coherent states entangled via the operators  $\exp(\sigma_T \hat{t}_{12})$  and  $\exp(\sigma_S \hat{s}_{12})$ . The coefficients  $\sigma_T, \sigma_S, \sigma_1, \sigma_2$  are real functions of the input parameters  $r = |\zeta|$  and  $\gamma$ . A simple method for the numerical determination of these parameters is described in Appendix A of Ref. [5].

The dark port case corresponds  $\gamma = \pi/2 - \delta$ , where  $\delta$  is a small phase shift (we assume it is real), for which one finds to first order in  $\delta$  (see Appendix A in Ref. [5])

$$\sigma_1 = r, \quad \sigma_2 = 0, \quad \sigma_S = -\delta \sinh r, \quad \text{and} \quad \sigma_T = \delta(1 - \cosh r). \quad (8)$$

We furthermore assume a very strong coherent state incoming in port 1, such that the  $\hat{b}_2$  and  $\hat{b}_2^\dagger$  operators can be replaced in the entanglement factors in Eq. (7) by their expectation values  $\alpha$  and  $\alpha^*$ , respectively. The output state can then be written as

$$|\psi_{\text{out}}\rangle = \hat{D}_1(-\alpha\delta(1 - \cosh r)) \hat{D}_1(-\delta\alpha^* e^{i\theta} \sinh r) \hat{S}_1(\zeta) \hat{D}_1(-\alpha\delta) \hat{D}_2(\alpha) |0, 0\rangle \quad (9)$$

with  $\zeta = r e^{i\theta}$ . The operators with index 1 may be combined using the relations

$$\begin{aligned} \hat{D}(\alpha_2) \hat{D}(\alpha_1) &= \hat{D}(\alpha_1 + \alpha_2) \exp\left[\frac{1}{2}(\alpha_2 \alpha_1^* - \alpha_2^* \alpha_1)\right], \\ \hat{D}(\alpha) \hat{S}(\zeta) &= \hat{S}(\zeta) \hat{D}(\alpha \cosh r + \alpha^* e^{i\theta} \sinh r). \end{aligned} \quad (10)$$

We finally obtain

$$|\psi_{\text{out}}\rangle = e^{i|\alpha|^2 \delta^2 \Delta} \hat{S}_1(\zeta) \hat{D}_1(\tilde{\alpha}) \hat{D}_2(\alpha) |0, 0\rangle \quad (11)$$

with  $\alpha = |\alpha| e^{i\phi}$  and

$$\begin{aligned} \tilde{\alpha} &= -\alpha\delta \cosh r - \alpha^* \delta e^{i\theta} \sinh r, \\ \Delta &= \frac{1}{2} \sin(\theta - 2\phi) \sinh(2r). \end{aligned} \quad (12)$$

As one can see from Eq. (11), a strong coherent state with coherence parameter  $\alpha$  exits through port 2 of the interferometer and a weak squeezed coherent state with coherence parameter  $\tilde{\alpha}$  and squeezing parameter  $\zeta$  exits through port 1. Note that in Ref. [5] there are two missprints:  $\Delta$  must be defined without the term  $e^{2i\phi}$  and with opposite sign. Also note, that the coherence parameter  $\tilde{\alpha}$  depends on the squeezing parameter  $\zeta$ , the coherence parameter  $\alpha = |\alpha| e^{i\phi}$  and the phase shift  $\delta$ . The phase factor  $e^{i|\alpha|^2 \delta^2 \Delta}$  is irrelevant for the determination of the photon statistics.

In order to determine the photon statistics of the output state we need to determine its number (Fock) representation. In terms of the number representation of a squeezed coherent state [10]

$$\hat{S}(\zeta)\hat{D}(\tilde{\alpha})|0\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} f_n(\zeta, \tilde{\alpha})|n\rangle \quad (13)$$

with

$$f_n(\zeta, \tilde{\alpha}) = \frac{(e^{i\theta} \tanh r)^{n/2}}{2^{n/2}(\cosh r)^{1/2}} \exp\left(-\frac{1}{2}(|\tilde{\alpha}|^2 - e^{-i\theta}\tilde{\alpha}^2 \tanh r)\right) H_n\left(\frac{\tilde{\alpha}e^{-i\theta/2}}{\sqrt{2 \cosh r \sinh r}}\right), \quad (14)$$

and  $H_n$  the Hermite polynomials, one immediately obtains the distribution function

$$P_{n_1} = \frac{1}{n_1!} |f_{n_1}(\zeta, \tilde{\alpha})|^2. \quad (15)$$

The mean and the variance of this distribution may be obtained analytically [10]

$$\begin{aligned} \langle n_1 \rangle &= |\alpha|^2 \delta^2 + \sinh^2 r, \\ (\Delta n_1)^2 &= |\alpha|^2 \delta^2 (\cosh(2r) - \cos(\theta - 2\phi) \sinh(2r)) + 2 \sinh^2 r \cosh^2 r. \end{aligned} \quad (16)$$

Note, that the mean does not depend on the phases  $\theta$  and  $\phi$ , but the variance does. The results Eq. (16) on first sight appear different from those obtained in Ref. [5], however, it is possible to show that they are equivalent. The form presented here is, however, much more transparent.

Finally, we would like to remark, that the mean and variance Eq. (16) may alternatively be calculated by a method used by Caves [1]. He expresses the output observables, which are described by  $\hat{b}$  operators, in terms of the input operators  $\hat{a}$  using the beam splitter transformation. In this way one obtains the same results Eq. (16) very efficiently, however, the full distribution function is not easily obtained.

### III. NUMERICAL RESULTS AND PHYSICAL INTERPRETATION

We consider squeezing factors up to  $r = 1.5$  in our numerical work, since at the present time the largest squeezing factor experimentally realized is about  $r = 1.3$  corresponding to a maximum squeezing of about -11.5 dB [11].

From the formulas (15) and (16) we see that the mean of the photon distribution does not depend on the phases  $\theta$  and  $\phi$ , but the variance depends on the phase relation  $\theta - 2\phi$ . Obviously, choosing  $\theta - 2\phi = 0$  is optimal in the sense of minimizing the counting noise.

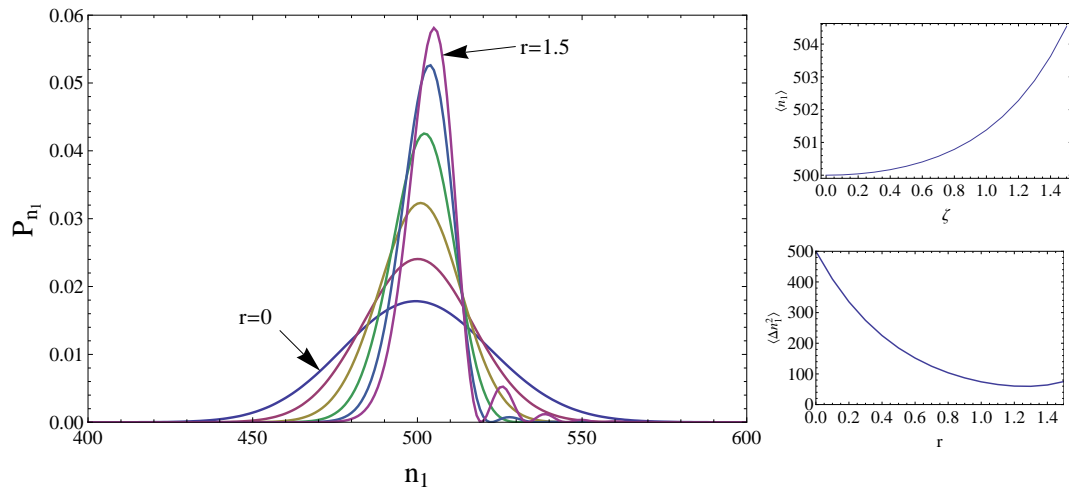


FIG. 1. (Color online) Photon number probability distribution for  $|\delta\alpha|^2 = 500$ ,  $\theta = 2\phi$ , and different values of the squeezing parameter  $r = 0, 0.3, 0.6, 0.9, 1.2, 1.5$  in the dark port. On the right hand side of the plot we show the mean and the variance squared of these distributions.

Substituting  $\theta - 2\phi = 0$  From Eqs. (16) one finds

$$\begin{aligned} \langle n_1 \rangle &= \delta^2 |\alpha|^2 + \sinh^2 r, \\ (\Delta n_1)^2 &= \delta^2 |\alpha|^2 e^{-2r} + 2 \sinh^2 r \cosh^2 r. \end{aligned} \quad (17)$$

For strong coherent input state (large  $|\alpha|$ ) and within the squeezing factor ranges experimentally accessible ( $r$  up to about 1.3) the second terms in both expressions can be neglected. Consequently, amplification of the signal in output port 1 by squeezing the input vacuum state is not possible. However, the width of the distribution  $\sigma = \sqrt{(\Delta n_1)^2}$  decreases  $\sim e^{-r}$  with increasing squeezing parameter  $r$ . Particularly, for a reasonably large squeezing  $r \sim 1$  noise may be reduced by more than 50%.

In Fig. 1 we show for the case  $\theta - 2\phi = 0$  and  $|\delta\alpha|^2 = 500$  the photon number distribution of the output state calculated from Eq. (15). Additionally we determine the mean and the variance of these distributions from Eq. (17). Notice, that for large squeezing parameters the distributions show characteristic oscillations [10].

## IV. SUMMARY AND CONCLUSIONS

In this paper we studied the entanglement effects on the photon distributions in the output of the interferometer for the ‘dark port’ case, when the beam splitter is oriented close to  $90^\circ$  to an incoming coherent state and a squeezed vacuum state is injected into the usually unused second input port.

Our results for the ‘dark port’ case show that squeezing does not influence the mean of the distribution tangibly, thus there is no amplification of the signal. This result contrasts with the findings of Ref. [4]. Signal amplification can only be achieved by increasing the intensity of the input coherent state, that is by increasing of  $|\alpha|$ . Squeezing allows to decrease the noise: for a squeezing factor  $r \sim 1$  the reduction of noise is more than 50%.

Furthermore, our analysis shows that the mean of the distribution does not depend on the phases  $\theta$  (squeezing parameter phase) or  $\phi$  (coherence parameter phase), but the variance of the distribution depends on the phase relation  $\theta - 2\phi$ . The most appropriate choice in order to achieve minimum possible counting noise is  $\theta - 2\phi = 0$ .

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