Automorphism group and ad-invariant metric on all six dimensional solvable real Lie algebras

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Abstract

Using adjoint representation of Lie algebras, we calculate the automorphism group and ad-invariant metric on six dimensional solvable real Lie algebras with 5, 4 and 3 dimensional nilradicals.

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1 Introduction

The automorphisms of real three dimensional Lie algebras [1] is a powerful tool for analyzing the dynamics of 3+1 dimensional Bianchi cosmological models [2]. Furthermore at the classical level, time dependent automorphism inducing diffeomorphisms can be used to simplify the line element and thus Einstein's field equations and also provide an algorithm for counting the number of essential constants (see[3] and[4] for three and four dimensional Lie algebras respectively). On the other hand, the automorphism groups and ad-invariant metrics can be used in the calculation of complex and bi-Hermitian structures [5] and generalized complex structures [6] on Lie algebras and also in the classification of Lie bialgebras [7]. Meanwhile, the calculation of ad-invariant metric on Lie algebras are important in the construction of the physical models such as WZW models [8]. Here in this manner we calculate the automorphism group and ad-invariant metric on six dimensional solvable Lie algebras (with 5 [10] and 4 [11] and 3 (nilpotent) [9] dimensional nilradicals).

2 Mathematical preliminaries

Let L be Lie algebra with the base $\{X_i\}$; then we have

$$[X_i, X_j] = f_{ij}{}^k X_k, \tag{1}$$

where $f_{ij}{}^k$ are the structure constants of the Lie algebra L. An automorphism O is a linear map on L such that preserves the Lie algebra structure i.e.:

$$O[X_i, X_j] = [OX_i, OX_j]. \tag{2}$$

Or, by use of $OX_i = X'_i$ we have

$$[X'_i, X'_j] = f_{ij}{}^k X'_k, \tag{3}$$

i.e. the automorphism O is a linear map which fixes the structure constants. By use of matrix representation for O ; i.e. $OX_i = O_i{}^j X_j$ we have rewritten relation (3) in the following form;

$$O_l{}^i O_m{}^j f_{ij}{}^k = f_{lm}{}^n O_n{}^k, \tag{4}$$

where by use of adjoint representation $(\chi_i)_j{}^k = -f_{ij}{}^k$ or $(\mathcal{Y}^k)_{ij} = -f_{ij}{}^k$ one can rewrite relation (4) in the following matrix form:

$$O_l{}^i O \chi_i = \chi_l O, \tag{5}$$

or

$$O \mathcal{Y}^k O^t = \mathcal{Y}^n O_n{}^k. \tag{6}$$

In this way, by use of the above relations one can calculate the automorphism group O of a Lie algebra L. Furthermore, an ad-invariant symmetric metric[1] on Lie algebra L can be written as follow:

$$\langle X_i, X_j \rangle = g_{ij}, \tag{7}$$

such that

$$ad_{X_j} \langle X_i, X_k \rangle = 0, \tag{8}$$

i.e.

$$\langle ad_{X_j} X_i, X_k \rangle + \langle X_i, ad_{X_j} X_k \rangle = 0, \tag{9}$$

or

$$\langle X_i, [X_j, X_k] \rangle = \langle [X_i, X_j], X_k \rangle, \tag{10}$$

which can be rewritten in the following matrix form:

$$\chi_i g = -(\chi_i g)^t. \tag{11}$$

[1] The Cartan-Killing form $K_{ij} = f_{ik}{}^l f_{jl}{}^k$ is a special case of this metric.

Note that because the automorphism map O fixes the structure constants (see (3)); from above relation (11) one can see that the metric g on Lie algebra L is also fixed under automorphism map, i.e:

$$g'_{ij} = \langle X_i, X_j \rangle = \langle X'_i, X'_j \rangle = g_{ij}, \tag{12}$$

or

$$g_{ij} = O_i{}^k O_j{}^l g_{kl}, \tag{13}$$

with matrix form[2]

$$g = OgO^t. \tag{14}$$

i.e. the ad-invariant metric is also invariant under automorphism group or in other word the automorphism groups are isometries of this metric. Now, one can calculate ad-invariant metric g on Lie algebras according to the relations (11) and (14). In the next section we calculate the automorphism groups and ad-invariant metric on six dimensional solvable real Lie algebras by use of maple program for solving relations (5),(11) and (14).

3 Automorphism groups and ad-invariant metrics

Classification of six dimensional solvable real Lie algebras with 3 dimensional nilradical (i.e. six dimensional nilpotent Lie algebras) are obtained by Morozov [9] then Lie algebras with 5 dimensional nilradical are mainly classified by Mubarakzyanov [10] and were finished by Turkowski with the classification of these Lie algebras with 4 dimensional nilradical [11]; (for a good bibilography see [12]). Here we use the Mubarakzyanov [10] classification of six dimensional solvable real Lie algebras with nilradical 5 (see also [13] for correction of some misprint of [10]). The automorphism groups[3] and structure constants of these 99 Lie algebras are written in table 1. For 40 six dimensional solvable real Lie algebras with nilradical 4 [11], these are written in table 2. Tables 3 contains the automorphism groups for 22 six dimensional nilpotent Lie algebras [9](and also use [14]). The ad-invariant metrics of all six dimensional real solvable Lie algebras are written in table 4.

[2] This relation can also be obtain by replacing (5) into (11) directly.

[3] Note that we choose automorphism groups, (i.e. matrices that connected to the identity matrix) from the solutions of the relation 5 or 6.

TABLE 1: The automorphism groups of six dimensional solvable real Lie algebras with nilradical 5

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,1}$ $0 <\mid \delta \mid\leq\mid \gamma \mid\leq\mid \beta \mid\leq\mid \alpha \mid\leq 1$	$f_{16}^1 = 1, f_{26}^2 = \alpha, f_{36}^3 = \beta, f_{46}^4 = \gamma, f_{56}^5 = \delta$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,2}$ $0 <\mid \delta \mid\leq\mid \gamma \mid\leq 1$	$f_{16}^1 = \alpha, f_{26}^1 = 1, f_{26}^2 = \alpha, f_{36}^3 = 1, f_{46}^4 = \gamma, f_{56}^5 = \delta$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,3}$ $0 <\mid \delta \mid\leq 1$	$f_{16}^1 = \alpha, f_{26}^1 = 1, f_{26}^2 = \alpha, f_{36}^2 = 1, f_{36}^3 = \alpha$ $, f_{46}^4 = 1, f_{56}^5 = \delta$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,4}$	$f_{16}^1 = \alpha, f_{26}^1 = 1, f_{26}^2 = \alpha, f_{36}^2 = 1, f_{36}^3 = \alpha$ $, f_{46}^3 = 1, f_{46}^4 = \alpha, f_{56}^5 = 1$	$\begin{pmatrix} a_4 & 0 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,5}$	$f_{16}^1 = 1, f_{26}^1 = 1, f_{26}^2 = 1, f_{36}^2 = 1, f_{36}^3 = 1$ $, f_{46}^3 = 1, f_{46}^4 = 1, f_{56}^4 = 1, f_{56}^5 = 1$	$\begin{pmatrix} a_5 & 0 & 0 & 0 & 0 & 0 \\ a_4 & a_5 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & a_5 & 0 & 0 & 0 \\ a_2 & a_3 & a_4 & a_5 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,6}$ $\alpha \leq \beta$	$f_{16}^1 = 1, f_{26}^2 = \alpha, f_{36}^2 = 1, f_{36}^3 = \alpha, f_{46}^4 = \beta$ $, f_{56}^4 = 1, f_{56}^5 = \beta$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,7}$ $\alpha^2 + \beta^2 \neq 0$	$f_{16}^1 = \alpha, f_{26}^1 = 1, f_{26}^2 = \alpha, f_{36}^2 = 1, f_{36}^3 = \alpha$ $, f_{46}^4 = \beta, f_{56}^4 = 1, f_{56}^5 = \beta$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,8}$ $0 <\mid \gamma \mid \leq \mid \beta \mid \leq \mid \alpha \mid$	$f_{16}^1 = \alpha, f_{26}^2 = \beta, f_{36}^3 = \gamma, f_{46}^4 = p, f_{46}^5 = -1$ $, f_{56}^4 = 1, f_{56}^5 = p$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & -a_5 & a_4 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,9}$ $\alpha \neq 0$	$f_{16}^1 = \alpha, f_{26}^2 = \beta, f_{36}^2 = 1, f_{36}^3 = \beta, f_{46}^4 = p$ $, f_{46}^5 = -1, f_{56}^4 = 1, f_{56}^5 = p$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & -a_5 & a_4 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,10}$	$f_{16}^1 = \alpha, f_{26}^1 = 1, f_{26}^2 = \alpha, f_{36}^2 = 1, f_{36}^3 = \alpha$ $, f_{46}^4 = p, f_{46}^5 = -1, f_{56}^4 = 1, f_{56}^5 = p$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & -a_5 & a_4 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,11}$ $\alpha s \neq 0$	$f_{16}^1 = \alpha, f_{26}^2 = p, f_{26}^3 = -1, f_{36}^2 = 1, f_{36}^3 = p$ $, f_{46}^4 = q, f_{46}^5 = -s, f_{56}^4 = s, f_{56}^5 = q$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2 & a_3 & 0 & 0 & 0 \\ 0 & -a_3 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & -a_4 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,12}$ $\alpha \neq 0$	$f_{16}^1 = \alpha, f_{26}^2 = p, f_{26}^3 = -1, f_{36}^2 = 1, f_{36}^3 = p, f_{46}^2 = 1$ $, f_{46}^4 = p, f_{46}^5 = -1, f_{56}^3 = 1, f_{56}^4 = 1, f_{56}^5 = p$	$\begin{pmatrix} a_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_4 & a_5 & 0 & 0 & 0 \\ 0 & -a_5 & a_4 & 0 & 0 & 0 \\ 0 & a_2 & a_3 & a_4 & a_5 & 0 \\ 0 & -a_3 & a_2 & -a_5 & a_4 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,13}$ if $\alpha^2 + \beta^2 = 0$ then $h = 0$ but if $h \neq 0$ then α or $\beta \neq 0$	$f_{23}^1 = 1, f_{16}^1 = \alpha + \beta, f_{26}^2 = \alpha, f_{36}^3 = \beta$ $, f_{46}^4 = 1, f_{56}^5 = h$	$\begin{pmatrix} a_2a_4 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_6 & 0 \\ a_7 & \frac{a_3\alpha}{a_4} & -\frac{a_1\beta}{a_2} & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,14}$	$f_{23}^1 = 1, f_{16}^1 = \alpha + \beta, f_{26}^2 = \alpha, f_{36}^3 = \beta$ $, f_{46}^4 = 1, f_{56}^1 = 1, f_{56}^5 = \alpha + \beta$	$\begin{pmatrix} a_2a_4 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ a_6 & 0 & 0 & 0 & a_2a_4 & 0 \\ a_7 & \frac{a_3\alpha}{a_4} & -\frac{a_1\beta}{a_2} & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,15}$	$f_{23}^1 = 1, f_{16}^1 = 1 + h, f_{26}^2 = 1, f_{26}^4 = 1$ $, f_{36}^3 = h, f_{36}^5 = 1, f_{46}^4 = 1, f_{56}^5 = h$	$\begin{pmatrix} a_1a_3 & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_1a_7}{h} & a_1 & 0 & a_2 & 0 & 0 \\ a_3a_6 & 0 & a_3 & 0 & a_4 & 0 \\ 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_3 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,16}$	$f_{23}^1 = 1, f_{16}^1 = 1, f_{26}^2 = 1, f_{26}^4 = 1$ $, f_{36}^5 = 1, f_{46}^1 = 1, f_{46}^4 = 1$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & a_3 & 0 & 0 \\ a_6 & 0 & 1 & 0 & a_4 & 0 \\ a_2a_7 + a_3 & 0 & 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,17}$ $\alpha^2 + \epsilon^2 \neq 0$ $, \alpha\epsilon = 0$	$f_{23}^1 = 1, f_{16}^1 = \alpha, f_{26}^2 = \alpha, f_{36}^4 = 1$ $, f_{46}^1 = \epsilon, f_{56}^5 = 1$	$\begin{pmatrix} a_2a_3 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ -\frac{-\alpha a_3a_7 + \alpha\epsilon a_4 + \epsilon a_3 - \epsilon a_2a_3}{\alpha^2} & 0 & a_3 & a_4 & 0 & 0 \\ \frac{\epsilon a_3(a_2-1)}{\alpha} & 0 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & 0 & a_8 & a_9 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,18}$ $\beta \neq 0$	$f_{23}^1 = 1, f_{16}^1 = 1 + \alpha, f_{26}^2 = \alpha, f_{36}^3 = 1, f_{36}^4 = 1$ $, f_{46}^4 = 1, f_{56}^5 = \beta$	$\begin{pmatrix} a_1a_2 & 0 & 0 & 0 & 0 & 0 \\ -a_1a_7 & a_1 & 0 & 0 & 0 & 0 \\ \frac{a_2a_6}{\alpha} & 0 & a_2 & a_3 & 0 & 0 \\ 0 & 0 & 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,19}$	$f_{23}^1 = 1, f_{16}^1 = 1 + \alpha, f_{26}^2 = \alpha, f_{36}^3 = 1, f_{36}^4 = 1$ $, f_{46}^4 = 1, f_{56}^1 = 1, f_{56}^5 = 1 + \alpha$	$\begin{pmatrix} a_1a_2 & 0 & 0 & 0 & 0 & 0 \\ -a_1a_7 & a_1 & 0 & 0 & 0 & 0 \\ \frac{a_2a_6}{\alpha} & 0 & a_2 & a_3 & 0 & 0 \\ 0 & 0 & 0 & a_2 & 0 & 0 \\ a_4 & 0 & 0 & 0 & a_1a_2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,20}$ $\beta \neq 0$	$f_{23}^1 = 1, f_{16}^1 = 1, f_{36}^3 = 1, f_{36}^4 = 1, f_{46}^1 = 1$ $, f_{46}^4 = 1, f_{56}^5 = \beta$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ -a_7 & 1 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & a_3 & 0 & 0 \\ -a_2a_6 + a_3 & 0 & 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,21}$ $h \neq 0$	$f_{23}^1 = 1, f_{16}^1 = 2\alpha, f_{26}^2 = \alpha, f_{26}^3 = 1$ $, f_{36}^3 = \alpha, f_{46}^4 = 1, f_{56}^5 = h$	$\begin{pmatrix} a_1^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6 + \alpha a_2a_6 - \alpha a_1a_7}{\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{a_1a_6}{\alpha} & 0 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,22}$	$f_{23}^1 = 1, f_{16}^1 = 2\alpha, f_{26}^2 = \alpha, f_{26}^3 = 1$ $, f_{36}^3 = \alpha, f_{46}^4 = 1, f_{56}^1 = 1, f_{56}^5 = 2\alpha$	$\begin{pmatrix} a_1^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6 + \alpha a_2a_6 - \alpha a_1a_7}{\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{a_1a_6}{\alpha} & 0 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 & a_1^2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,23}$ $\epsilon h = 0$	$f_{23}^1 = 1, f_{16}^1 = 2\alpha, f_{26}^2 = \alpha, f_{26}^3 = 1, f_{36}^3 = \alpha$ $, f_{36}^4 = 1, f_{46}^4 = \alpha, f_{56}^1 = \epsilon, f_{56}^5 = 2\alpha + h$	$\begin{pmatrix} a_2^2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ \frac{a_2a_7}{\alpha} & 0 & a_2 & a_3 & 0 & 0 \\ 0 & 0 & 0 & a_2 & 0 & 0 \\ -\frac{\epsilon(a_2^2 - a_5)}{h} & 0 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & \frac{a_2a_7 + \alpha a_3a_7 - a_1\alpha^2}{a_2\alpha} & a_8 & a_9 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,24}$	$f_{23}^1=1, f_{26}^3=1, f_{36}^4=1, f_{46}^1=h, f_{56}^5=1$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ a_1 & 1 & a_2 & a_3 & 0 & 0 \\ ha_3+a_7-a_2a_6 & 0 & 1 & a_2 & 0 & 0 \\ ha_2-a_6 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,25}$	$f_{23}^1=1, f_{16}^1=1+h, f_{26}^2=1, f_{36}^3=h$ $, f_{46}^4=\beta, f_{46}^5=1, f_{56}^5=\beta$	$\begin{pmatrix} a_1a_2 & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_1a_7}{h} & a_1 & 0 & 0 & 0 & 0 \\ a_2a_6 & 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & a_4 & 0 \\ 0 & 0 & 0 & 0 & a_3 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,26}$	$f_{23}^1=1, f_{16}^1=1+h, f_{26}^2=1, f_{36}^3=h, f_{46}^4=1+h$ $, f_{46}^5=1, f_{56}^1=1, f_{56}^5=1+h$	$\begin{pmatrix} a_1a_2 & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_1a_7}{h} & a_1 & 0 & 0 & 0 & 0 \\ a_2a_6 & 0 & a_2 & 0 & 0 & 0 \\ a_3 & 0 & 0 & a_1a_2 & a_4 & 0 \\ a_4 & 0 & 0 & 0 & a_1a_2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,27}$ $\alpha^2+\beta^2\neq 0$ $,\alpha\epsilon=0$	$f_{23}^1=1, f_{16}^1=\alpha+\beta, f_{26}^2=\alpha, f_{36}^3=\beta, f_{36}^4=1$ $, f_{46}^4=\beta, f_{46}^5=1, f_{56}^1=\epsilon, f_{56}^5=\beta$	$\begin{pmatrix} a_2a_3 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ -\frac{\alpha\epsilon a_4-\alpha^2a_3a_7+\alpha^2\epsilon a_5+\epsilon a_3-\epsilon a_2a_3}{\alpha^3} & 0 & a_3 & a_4 & a_5 & 0 \\ -\frac{\epsilon(\alpha a_4+a_3-a_2a_3)}{\alpha^2} & 0 & 0 & a_3 & a_4 & 0 \\ \frac{\epsilon a_3(a_2-1)}{\alpha} & 0 & 0 & 0 & a_3 & 0 \\ a_6 & a_7 & -\frac{a_1\beta}{a_2} & a_8 & a_9 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,28}$	$f_{23}^1=1, f_{16}^1=2, f_{26}^2=1, f_{26}^3=1$ $, f_{36}^3=1, f_{46}^4=\beta, f_{46}^5=1, f_{56}^5=\beta$	$\begin{pmatrix} a_1^2 & 0 & 0 & 0 & 0 & 0 \\ a_1a_6+a_2a_6-a_1a_7 & a_1 & a_2 & 0 & 0 & 0 \\ a_1a_6 & 0 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & a_4 & 0 \\ 0 & 0 & 0 & 0 & a_3 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,29}$	$f_{23}^1=1, f_{16}^1=2, f_{26}^2=1, f_{26}^3=1, f_{36}^3=1$ $, f_{46}^4=2, f_{46}^5=1, f_{56}^1=1, f_{56}^5=2$	$\begin{pmatrix} a_2^2 & 0 & 0 & 0 & 0 & 0 \\ a_1a_6-a_2a_7+a_2a_6 & a_2 & a_1 & 0 & 0 & 0 \\ a_2a_6 & 0 & a_2 & 0 & 0 & 0 \\ a_3 & 0 & 0 & a_2^2 & a_4 & 0 \\ a_4 & 0 & 0 & 0 & a_2^2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,30}$	$f^1_{23}=1, f^3_{26}=1, f^4_{46}=1$ $, f^5_{46}=1, f^5_{56}=1$	$\begin{pmatrix} a_2^2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 \\ a_2a_7 & 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_6 & 0 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,31}$	$f^1_{23}=1, f^1_{16}=2, f^2_{26}=1, f^3_{26}=1, f^3_{36}=1$ $, f^4_{36}=1, f^4_{46}=1, f^5_{46}=1, f^5_{56}=1$	$\begin{pmatrix} a_1^2 & 0 & 0 & 0 & 0 & 0 \\ a_1a_6+a_2a_6-a_1a_7 & a_1 & a_2 & a_3 & a_4 & 0 \\ a_1a_6 & 0 & a_1 & a_2 & a_3 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & 0 \\ 0 & 0 & 0 & 0 & a_1 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,32}$ $2\alpha+h>\gamma$ $,\epsilon h=0$	$f^1_{23}=1, f^1_{16}=2\alpha, f^2_{26}=\alpha, f^3_{26}=1, f^2_{36}=-1$ $, f^3_{36}=\alpha, f^1_{46}=\epsilon, f^4_{46}=2\alpha+h, f^5_{56}=\gamma$	$\begin{pmatrix} a_1^2+a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6+a_2a_7+\alpha a_2a_6-\alpha a_1a_7}{1+\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{a_1a_7-a_2a_6+\alpha a_1a_6+\alpha a_2a_7}{1+\alpha^2} & -a_2 & a_1 & 0 & 0 & 0 \\ \frac{\epsilon(a_3-a_1^2-a_2^2)}{h} & 0 & 0 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,33}$ $\beta\leq 2\alpha$	$f^1_{23}=1, f^1_{16}=2\alpha, f^2_{26}=\alpha, f^3_{26}=1, f^2_{36}=-1$ $, f^3_{36}=\alpha, f^4_{46}=\beta, f^1_{56}=1, f^5_{56}=2\alpha$	$\begin{pmatrix} a_1^2+a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6+a_2a_7+\alpha a_2a_6-\alpha a_1a_7}{1+\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{a_1a_7-a_2a_6+\alpha a_1a_6+\alpha a_2a_7}{1+\alpha^2} & -a_2 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & 0 \\ a_4 & 0 & 0 & 0 & a_1^2+a_2^2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,34}$ $\epsilon h=0$	$f^1_{23}=1, f^1_{16}=2\alpha, f^2_{26}=\alpha, f^3_{26}=1, f^2_{36}=-1, f^3_{36}=\alpha$ $, f^4_{46}=2\alpha+h, f^5_{46}=1, f^1_{56}=\epsilon, f^5_{56}=2\alpha+h$	$\begin{pmatrix} a_1^2+a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_2a_6-a_1a_7-\alpha a_1a_6-\alpha a_2a_7}{1+\alpha^2} & a_2 & -a_1 & 0 & 0 & 0 \\ \frac{a_1a_6+a_2a_7+\alpha a_2a_6-\alpha a_1a_7}{1+\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{\epsilon(ha_4-a_3+a_1^2+a_2^2)}{h^2} & 0 & 0 & a_3 & a_4 & 0 \\ \frac{\epsilon(a_3-a_1^2-a_2^2)}{h} & 0 & 0 & 0 & a_3 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,35}$ $\alpha^2+\beta^2\neq 0$	$f^1_{23}=1, f^1_{16}=\alpha+\beta, f^2_{26}=\alpha, f^3_{36}=\beta$ $, f^4_{46}=\gamma, f^5_{46}=1, f^4_{56}=-1, f^5_{56}=\gamma$	$\begin{pmatrix} a_2a_4 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_5 & a_6 & 0 \\ & 0 & 0 & -a_6 & a_5 & 0 \\ a_7 & \frac{a_3\alpha}{a_4} & -\frac{a_1\beta}{a_2} & a_8 & a_9 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,36}$	$f_{23}^1=1, f_{16}^1=2\alpha, f_{26}^2=\alpha, f_{26}^3=1, f_{36}^3=\alpha$ $, f_{46}^4=\gamma, f_{46}^5=1, f_{56}^4=-1, f_{56}^5=\gamma$	$\begin{pmatrix} a_2^2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 \\ \frac{a_2a_7}{\alpha} & 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & 0 \\ 0 & 0 & 0 & -a_5 & a_4 & 0 \\ a_6 & a_7 & \frac{a_2a_7+\alpha a_3a_7-\alpha^2a_1}{a_2\alpha} & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,37}$ $s\neq 0$	$f_{23}^1=1, f_{16}^1=2\alpha, f_{26}^2=\alpha, f_{26}^3=1, f_{36}^2=-1$ $, f_{36}^3=\alpha, f_{46}^4=\beta, f_{46}^5=s, f_{56}^4=-s, f_{56}^5=\beta$	$\begin{pmatrix} a_1^2+a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6+a_2a_7+\alpha a_2a_6-\alpha a_1a_7}{1+\alpha^2} & a_1 & a_2 & 0 & 0 & 0 \\ \frac{a_1a_7-a_2a_6+\alpha a_1a_6+\alpha a_2a_7}{1+\alpha^2} & -a_2 & a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_4 & -a_3 & 0 \\ 0 & 0 & 0 & a_3 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,38}$	$f_{23}^1=1, f_{16}^1=2\alpha, f_{26}^2=\alpha, f_{26}^3=1, f_{26}^4=1, f_{36}^2=-1$ $, f_{36}^3=\alpha, f_{36}^5=1, f_{46}^4=\alpha, f_{46}^5=1, f_{56}^4=-1, f_{56}^5=\alpha$	$\begin{pmatrix} a_1^2+a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1a_6+a_2a_7+\alpha a_2a_6-\alpha a_1a_7}{1+\alpha^2} & a_1 & a_2 & a_3 & a_4 & 0 \\ \frac{a_1a_7-a_2a_6+\alpha a_1a_6+\alpha a_2a_7}{1+\alpha^2} & -a_2 & a_1 & -a_4 & a_3 & 0 \\ 0 & 0 & 0 & a_1 & a_2 & 0 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,39}$ $\gamma\neq 0$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1+h, f_{26}^2=h+2$ $, f_{36}^3=\gamma, f_{46}^4=h, f_{56}^5=1$	$\begin{pmatrix} a_2a_5 & -a_2a_5a_8 & 0 & 0 & 0 & 0 \\ 0 & a_2a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ -a_2a_8 & \frac{a_2a_8^2}{2} & 0 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & a_5 & 0 \\ \frac{a_3a_8+ha_4+a_4}{a_5} & a_6 & a_7 & \frac{ha_3}{a_5} & a_8 & 1 \end{pmatrix}$
$g_{6,40}$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1+h, f_{26}^2=h+2$ $, f_{36}^2=1, f_{36}^3=h+2, f_{46}^4=h, f_{56}^5=1$	$\begin{pmatrix} a_2a_5 & -a_2a_5a_8 & 0 & 0 & 0 & 0 \\ 0 & a_2a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & a_2a_5^2 & 0 & 0 \\ -a_2a_8 & \frac{a_2a_8^2}{2} & 0 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & a_5 & 0 \\ \frac{a_3a_8+ha_4+a_4}{a_5} & a_6 & a_7 & \frac{ha_3}{a_5} & a_8 & 1 \end{pmatrix}$
$g_{6,41}$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1+h, f_{26}^2=h+2$ $, f_{36}^3=h, f_{46}^3=1, f_{46}^4=h, f_{56}^5=1$	$\begin{pmatrix} a_2a_5 & -a_2a_5a_8 & 0 & 0 & 0 & 0 \\ 0 & a_2a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_2 & 0 & 0 & 0 \\ -a_2a_8 & \frac{a_2a_8^2}{2} & a_1 & a_2 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & a_5 & 0 \\ \frac{a_3a_8+ha_4+a_4}{a_5} & a_6 & a_7 & \frac{ha_3}{a_5} & a_8 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,42}$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1+h, f_{26}^2=h+2$ $, f_{36}^3=1, f_{46}^4=h, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_1a_5 & -a_1a_5a_8 & 0 & 0 & 0 & 0 \\ 0 & a_1a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ -a_1a_8 & \frac{a_1a_8^2}{2} & 0 & a_1 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & a_5 & 0 \\ \frac{a_2a_8+ha_3+a_3}{a_5} & a_6 & a_7 & \frac{ha_2}{a_5} & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,43}$	$f_{15}^2=1, f_{45}^1=1, f_{26}^2=1, f_{36}^2=1$ $, f_{36}^3=1, f_{46}^4=-1, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} 1 & -a_8 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & a_2-a_3a_4-a_3a_7a_8 & a_3 & 0 & 0 & 0 \\ -\frac{a_8}{a_3} & \frac{a_8^2}{2a_3} & 0 & \frac{1}{a_3} & 0 & 0 \\ -a_3a_7 & a_1 & a_2 & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,44}$ $\gamma \neq 0$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=2, f_{26}^2=3$ $, f_{36}^3=\gamma, f_{46}^4=1, f_{56}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & -a_3^2a_8 & 0 & 0 & 0 & 0 \\ 0 & a_3^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ -a_3a_8 & \frac{a_3a_8^2}{2} & 0 & a_3 & 0 & 0 \\ a_3a_7-a_2a_8-a_3a_8 & \frac{2a_3a_4-2a_3a_7a_8+2a_2a_8^2+3a_3a_8^2}{4} & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,45}$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=2, f_{26}^2=3, f_{36}^2=1$ $, f_{36}^3=3, f_{46}^4=1, f_{56}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & -a_3^2a_8 & 0 & 0 & 0 & 0 \\ 0 & a_3^3 & 0 & 0 & 0 & 0 \\ 0 & a_1 & a_3^3 & 0 & 0 & 0 \\ -a_3a_8 & \frac{a_3a_8^2}{2} & 0 & a_3 & 0 & 0 \\ a_3a_7-a_2a_8-a_3a_8 & \frac{2a_3a_4-2a_3a_7a_8+2a_2a_8^2+3a_3a_8^2}{4} & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,46}$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=2, f_{26}^2=3, f_{36}^3=1$ $, f_{46}^3=1, f_{46}^4=1, f_{56}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & -a_3^2a_8 & 0 & 0 & 0 & 0 \\ 0 & a_3^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 \\ -a_3a_8 & \frac{a_3a_8^2}{2} & a_2 & a_3 & 0 & 0 \\ a_3a_7-a_2a_8-a_3a_8 & \frac{2a_3a_4-2a_3a_7a_8+2a_2a_8^2+3a_3a_8^2}{4} & a_1 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,47}$ $\gamma \neq 0$ $\epsilon = \pm 1$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1, f_{26}^2=1$ $, f_{36}^3=\gamma, f_{46}^2=\epsilon, f_{46}^4=1$	$\begin{pmatrix} a_4 & a_2 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 0 & 0 \\ a_8 & a_5 & 0 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,47}$ $\gamma \neq 0$ $\epsilon = 0$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1, f_{26}^2 = 1$ $, f_{36}^3 = \gamma, f_{46}^4 = 1$	$\begin{pmatrix} a_4a_5 & a_2a_5 & 0 & 0 & 0 & 0 \\ 0 & a_4a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 0 & 0 \\ a_5a_9 & a_5a_6 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,48}$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1, f_{26}^2 = 1$ $, f_{36}^2 = 1, f_{36}^3 = 1, f_{46}^4 = 1$	$\begin{pmatrix} a_4a_5 & a_2a_5 & 0 & 0 & 0 & 0 \\ 0 & a_4a_5^2 & 0 & 0 & 0 & 0 \\ 0 & a_1 & a_4a_5^2 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 0 & 0 \\ a_5a_9 & a_5a_6 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,49}$ $\epsilon = \pm 1$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1, f_{26}^2 = 1$ $, f_{46}^2 = \epsilon, f_{46}^4 = 1, f_{56}^3 = 1$	$\begin{pmatrix} a_3 & a_1 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & a_3 & 0 & 0 \\ a_8 & a_5 & a_4 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,49}$ $\epsilon = 0$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1$ $, f_{26}^2 = 1, f_{46}^4 = 1, f_{56}^3 = 1$	$\begin{pmatrix} a_3a_5 & a_1a_5 & 0 & 0 & 0 & 0 \\ 0 & a_3a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & a_3 & 0 & 0 \\ a_5a_9 & a_5a_6 & a_4 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,50}$ $\epsilon = 1$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1, f_{36}^3 = 1$ $, f_{26}^2 = 1, f_{36}^2 = 1, f_{46}^3 = 1, f_{46}^4 = 1$	$\begin{pmatrix} a_4 & a_1 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_8 & a_5 & 0 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,50}$ $\epsilon = 0$	$f_{15}^2 = 1, f_{45}^1 = 1, f_{16}^1 = 1, f_{36}^3 = 1$ $, f_{26}^2 = 1, f_{46}^3 = 1, f_{46}^4 = 1$	$\begin{pmatrix} a_4a_5 & a_1a_5 & 0 & 0 & 0 & 0 \\ 0 & a_4a_5^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_5a_9 & a_5a_6 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,50}$ $\epsilon=-1$	$f_{15}^2=1, f_{45}^1=1, f_{16}^1=1, f_{36}^3=1$ $, f_{26}^2=1, f_{36}^2=-1, f_{46}^3=1, f_{46}^4=1$	$\begin{pmatrix} a_4 & a_1 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & -a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_8 & a_5 & 0 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$g_{6,51}$ $\epsilon=1$	$f_{15}^2=1, f_{45}^1=1$ $, f_{36}^3=1, f_{46}^2=1$	$\begin{pmatrix} a_4 & a_2 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 0 & 0 \\ a_5 & a_6 & 0 & a_7 & 1 & 0 \\ a_7 & a_8 & a_9 & 0 & 0 & 1 \end{pmatrix}$
$g_{6,51}$ $\epsilon=-1$	$f_{15}^2=1, f_{45}^1=1$ $, f_{36}^3=1, f_{46}^2=-1$	$\begin{pmatrix} a_4 & a_2 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 0 & 0 \\ a_5 & a_6 & 0 & -a_7 & 1 & 0 \\ a_7 & a_8 & a_9 & 0 & 0 & 1 \end{pmatrix}$
$g_{6,52}$ $\epsilon=1$	$f_{15}^2=1, f_{45}^1=1, f_{36}^3=1$ $, f_{46}^2=1, f_{56}^4=1$	$\begin{pmatrix} 1 & -a_8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ -a_8 & a_4-a_5 & 0 & 1 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$g_{6,52}$ $\epsilon=0$	$f_{15}^2=1, f_{45}^1=1$ $, f_{36}^3=1, f_{56}^4=1$	$\begin{pmatrix} a_5^2 & -a_5^2a_9 & 0 & 0 & 0 & 0 \\ 0 & a_5^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ -a_5a_9 & -a_5a_6 & 0 & a_5 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$
$g_{6,52}$ $\epsilon=-1$	$f_{15}^2=1, f_{45}^1=1, f_{36}^3=1$ $, f_{46}^2=-1, f_{56}^4=1$	$\begin{pmatrix} 1 & -a_8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_1 & 0 & 0 & 0 \\ -a_8 & -a_4-a_5 & 0 & 1 & 0 & 0 \\ a_2 & a_3 & 0 & a_4 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,53}$	$f^1_{35}=1, f^2_{45}=1, f^3_{36}=1$ $, f^4_{46}=1, f^5_{56}=-1$	$\begin{pmatrix} a_1a_5 & a_2a_5 & 0 & 0 & 0 & 0 \\ a_3a_5 & a_4a_5 & 0 & 0 & 0 & 0 \\ a_1a_{10} & a_2a_{10} & a_1 & a_2 & 0 & 0 \\ a_3a_{10} & a_4a_{10} & a_3 & a_4 & 0 & 0 \\ a_5a_8 & a_5a_9 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,54}$	$f^1_{35}=1, f^2_{45}=1, f^1_{16}=1, f^2_{26}=\lambda$ $, f^3_{36}=1-\gamma, f^4_{46}=\lambda-\gamma, f^5_{56}=\gamma$	$\begin{pmatrix} a_1a_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3a_6 & 0 & 0 & 0 & 0 \\ \frac{a_1a_2}{a_3} & 0 & a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & a_3 & 0 & 0 \\ a_4 & a_5 & 0 & 0 & a_6 & 0 \\ a_7 & a_8 & \frac{a_4(1-\gamma)}{a_6} & \frac{a_5(\lambda-\gamma)}{a_6} & -\frac{a_2\gamma}{a_3} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,55}$	$f^1_{35}=1, f^2_{45}=1, f^1_{16}=1, f^2_{26}=1+\gamma$ $, f^3_{36}=1-\gamma, f^1_{46}=1, f^4_{46}=1, f^5_{56}=\gamma$	$\begin{pmatrix} a_1a_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1a_5^2 & 0 & 0 & 0 & 0 \\ \frac{a_3}{a_5} & 0 & a_1 & 0 & 0 & 0 \\ a_2 & a_3 & 0 & a_1a_5 & 0 & 0 \\ a_4 & a_5a_8 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & \frac{a_4(1-\gamma)}{a_5} & a_8 & -\frac{a_3\gamma}{a_1a_5} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,56}$	$f^1_{35}=1, f^2_{45}=1, f^1_{16}=1, f^2_{26}=1-\gamma$ $, f^2_{36}=1, f^3_{36}=1-\gamma, f^4_{46}=1-2\gamma, f^5_{56}=\gamma$	$\begin{pmatrix} a_3a_6^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3a_6 & 0 & 0 & 0 & 0 \\ a_2a_6 & a_1 & a_3a_6 & 0 & 0 & 0 \\ 0 & a_2 & 0 & a_3 & 0 & 0 \\ a_4 & a_5 & 0 & 0 & a_6 & 0 \\ a_7 & a_8 & \frac{a_4(1-\gamma)}{a_6} & \frac{a_5(1-2\gamma)}{a_6} & -\frac{a_2\gamma}{a_3} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,57}$	$f^1_{35}=1, f^2_{45}=1, f^1_{16}=1, f^2_{26}=2\gamma$ $, f^3_{36}=1-\gamma, f^4_{46}=\gamma, f^4_{56}=1, f^5_{56}=\gamma$	$\begin{pmatrix} a_2a_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_6^2 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & 0 & 0 & 0 \\ 0 & \frac{a_1a_6}{a_2} & 0 & a_6 & 0 & 0 \\ a_3 & a_4 & 0 & a_5 & a_6 & 0 \\ a_7 & a_8 & \frac{a_3(1-\gamma)}{a_6} & -\frac{\gamma a_1a_5+a_1a_6-\gamma a_2a_4}{a_2a_6} & -\frac{a_1\gamma}{a_2} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,58}$ $\omega=1$	$f^1_{35}=1, f^2_{45}=1, f^1_{16}=3, f^2_{26}=2, f^2_{36}=1$ $, f^3_{36}=2, f^4_{46}=1, f^4_{56}=1, f^5_{56}=1$	$\begin{pmatrix} a_3^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3^2 & 0 & 0 & 0 & 0 \\ -a_3^2a_8 & a_1 & a_3^2 & 0 & 0 & 0 \\ 0 & -a_3a_8 & 0 & a_3 & 0 & 0 \\ \frac{a_3a_6}{2} & a_3a_7-a_2a_8-a_3a_8 & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,58}$ $\omega=0$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=3, f_{26}^2=2, f_{36}^2=1$ $, f_{36}^3=2, f_{46}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_2a_4^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2a_4 & 0 & 0 & 0 & 0 \\ -a_2a_4a_9 & a_1 & a_2a_4 & 0 & 0 & 0 \\ 0 & -a_2a_9 & 0 & a_2 & 0 & 0 \\ \frac{a_4a_7}{2} & a_4a_8-a_3a_9 & 0 & a_3 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,59}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=1, f_{36}^3=1$ $, f_{46}^2=1, f_{56}^4=h$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 1 & 0 & 0 \\ a_7 & a_4 & 0 & ha_3+a_8 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$g_{6,60}$ $\omega=1$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=1, f_{26}^2=2$ $, f_{46}^1=1, f_{46}^4=1, f_{56}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3^2 & 0 & 0 & 0 & 0 \\ -a_8 & 0 & 1 & 0 & 0 & 0 \\ a_2-a_3a_6 & -a_3a_8 & 0 & a_3 & 0 & 0 \\ a_1 & a_3a_7-a_2a_8-a_3a_8 & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,60}$ $\omega=0$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=1, f_{26}^2=2$ $, f_{46}^1=1, f_{46}^4=1, f_{56}^5=1$	$\begin{pmatrix} a_1a_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1a_4^2 & 0 & 0 & 0 & 0 \\ -a_1a_9 & 0 & a_1 & 0 & 0 & 0 \\ a_2 & -a_1a_4a_9 & 0 & a_1a_4 & 0 & 0 \\ a_3 & a_4a_8-a_4a_7a_9 & 0 & a_4a_7 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,61}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=2, f_{26}^2=2\lambda$ $, f_{36}^3=1, f_{46}^4=2\lambda-1, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1a_3 & 0 & 0 & 0 & 0 \\ -a_3a_8 & 0 & a_3 & 0 & 0 & 0 \\ 0 & -a_1a_8 & 0 & a_1 & 0 & 0 \\ a_3a_6-a_2a_8-a_3a_8 & \frac{a_3a_7}{2\lambda-1} & a_2 & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,62}$ $\omega=1$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=2, f_{26}^2=1$ $, f_{36}^2=1, f_{36}^3=1, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ -a_3a_8 & -a_3a_7+a_2 & a_3 & 0 & 0 & 0 \\ 0 & -a_8 & 0 & 1 & 0 & 0 \\ a_3a_6-a_2a_8-a_3a_8 & a_1 & a_2 & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,62}$ $\omega=0$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=2, f_{26}^2=1$ $, f_{36}^2=1, f_{36}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_2a_4^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_2a_4 & 0 & 0 & 0 & 0 \\ -a_2a_4a_9 & a_1 & a_2a_4 & 0 & 0 & 0 \\ 0 & -a_2a_9 & 0 & a_2 & 0 & 0 \\ a_4a_7-a_4a_8a_9 & a_3 & a_4a_8 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$
$g_{6,63}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=1, f_{26}^2=\lambda$ $, f_{36}^3=1, f_{46}^2=1, f_{46}^4=\lambda$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_4 & 0 & 0 & 0 & 0 \\ a_1 & 0 & a_2 & 0 & 0 & 0 \\ 0 & a_3 & 0 & a_4 & 0 & 0 \\ a_8 & a_5 & 0 & 0 & 1 & 0 \\ a_6 & a_7 & a_8 & \lambda a_5 & 0 & 1 \end{pmatrix}$
$g_{6,64}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=1, f_{26}^2=1$ $, f_{36}^2=h, f_{36}^3=1, f_{46}^1=1, f_{46}^4=1$	$\begin{pmatrix} a_3 & ha_6 & 0 & 0 & 0 & 0 \\ a_6 & a_3 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & ha_6 & 0 & 0 \\ a_4 & a_5 & a_6 & a_3 & 0 & 0 \\ a_9 & a_{10} & 0 & 0 & 1 & 0 \\ a_7 & a_8 & a_9 & a_{10} & 0 & 1 \end{pmatrix}$
$g_{6,65}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=\lambda, f_{16}^2=1$ $, f_{36}^4=1, f_{26}^2=\lambda, f_{36}^3=\lambda-\gamma$ $, f_{46}^4=\lambda-\gamma, f_{56}^5=\gamma$	$\begin{pmatrix} a_2a_5 & a_3a_5 & 0 & 0 & 0 & 0 \\ 0 & a_2a_5 & 0 & 0 & 0 & 0 \\ a_1 & \frac{a_1a_3}{a_2} & a_2 & a_3 & 0 & 0 \\ 0 & a_1 & 0 & a_2 & 0 & 0 \\ a_5a_8-\lambda a_4+\gamma a_4 & a_4 & 0 & 0 & a_5 & 0 \\ a_6 & a_7 & \frac{\lambda a_5a_8+2\lambda\gamma a_4-\gamma a_5a_8-\lambda^2a_4-\gamma^2a_4}{a_5} & a_8 & -\frac{a_1\gamma}{a_2} & 1 \end{pmatrix}$
$g_{6,66}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=2, f_{16}^2=1$ $, f_{26}^2=2, f_{36}^3=1, f_{36}^4=1$ $, f_{46}^4=1, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & a_1a_3 & 0 & 0 & 0 & 0 \\ 0 & a_3^2 & 0 & 0 & 0 & 0 \\ -a_3a_8 & -a_1a_8 & a_3 & a_1 & 0 & 0 \\ 0 & -a_3a_8 & 0 & a_3 & 0 & 0 \\ a_3a_6-a_1a_8-a_3a_8 & a_3a_7+a_3a_8-a_3a_6-a_2a_8 & a_1 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,67}$	$f_{35}^1=1, f_{45}^2=1, f_{16}^1=2, f_{16}^2=1$ $, f_{26}^2=2, f_{36}^3=1, f_{36}^4=1$ $, f_{46}^4=1, f_{56}^4=h, f_{56}^5=1$	$\begin{pmatrix} a_1a_4 & a_2a_4 & 0 & 0 & 0 & 0 \\ 0 & a_1a_4 & 0 & 0 & 0 & 0 \\ -a_1a_9 & -a_2a_9 & a_1 & a_2 & 0 & 0 \\ 0 & -a_1a_9 & 0 & a_1 & 0 & 0 \\ a_4a_7+ha_4a_9-ha_1a_9 & a_4a_8-a_3a_9-a_4a_7-ha_4a_9 & h(a_1-a_4) & a_3 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,68}$	$f_{35}^1 = 1, f_{45}^2 = 1, f_{16}^1 = 1$ $, f_{16}^2 = 1, f_{26}^2 = 1, f_{36}^3 = 1$ $, f_{36}^4 = 1, f_{46}^1 = c, f_{46}^4 = 1$	$\begin{pmatrix} a_3 & a_4 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ ca_4 & a_1 & 0 & a_3 & 0 & 0 \\ a_8 - a_5 & a_5 & 0 & 0 & 1 & 0 \\ a_6 & a_7 & a_8 - a_5 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,69}$	$f_{35}^1 = 1, f_{45}^2 = 1, f_{16}^1 = 1$ $, f_{16}^2 = 1, f_{26}^2 = 1, f_{36}^3 = 1$ $, f_{36}^4 = 1, f_{46}^2 = 1, f_{46}^4 = 1$	$\begin{pmatrix} a_3a_6 & a_4a_6 & 0 & 0 & 0 & 0 \\ 0 & a_3a_6 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ \frac{a_3(a_6-1)}{2} & \frac{a_4a_6+a_4+2a_1}{2} & 0 & a_3 & 0 & 0 \\ a_6a_9 - a_5 & a_5 & 0 & 0 & a_6 & 0 \\ a_7 & a_8 & \frac{a_6a_9-a_5}{a_6} & a_9 & \frac{(a_6-1)}{2} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,70}$	$f_{35}^1 = 1, f_{45}^2 = 1, f_{16}^1 = p, f_{16}^2 = 1$ $, f_{26}^1 = -1, f_{26}^2 = p, f_{36}^4 = 1$ $, f_{36}^3 = p - \gamma, f_{46}^3 = -1$ $, f_{46}^4 = p - \gamma, f_{56}^5 = \gamma$	$\begin{pmatrix} a_1a_4 & a_2a_4 & 0 & 0 & 0 & 0 \\ -a_2a_4 & a_1a_4 & 0 & 0 & 0 & 0 \\ -\frac{a_1a_8}{\gamma} & -\frac{a_2a_8}{\gamma} & a_1 & a_2 & 0 & 0 \\ \frac{a_2a_8}{\gamma} & -\frac{a_1a_8}{\gamma} & -a_2 & a_1 & 0 & 0 \\ a_4a_7 - pa_3 + \gamma a_3 & a_3 & 0 & 0 & a_4 & 0 \\ a_5 & a_6 & \frac{pa_4a_7+2p\gamma a_3-\gamma a_4a_7-p^2a_3-\gamma^2a_3-a_3}{a_4} & a_7 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,71}$	$f_{25}^1 = 1, f_{35}^2 = 1, f_{26}^2 = h + 2, f_{45}^3 = 1$ $, f_{36}^3 = h + 1, f_{16}^1 = h + 3$ $, f_{46}^4 = h, f_{56}^5 = 1$	$\begin{pmatrix} a_1a_5^3 & 0 & 0 & 0 & 0 & 0 \\ -a_1a_7a_5^2 & a_1a_5^2 & 0 & 0 & 0 & 0 \\ \frac{a_1a_5a_7^2}{2} & -a_1a_7a_5 & a_1a_5 & 0 & 0 & 0 \\ -\frac{a_1a_7^3}{6} & \frac{a_1a_7^2}{2} & -a_1a_7 & a_1 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & a_5 & 0 \\ a_6 & \frac{a_3a_7+ha_2+2a_2}{a_5} & \frac{a_4a_7+ha_3+a_3}{a_5} & \frac{ha_4}{a_5} & a_7 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,72}$	$f_{25}^1 = 1, f_{35}^2 = 1, f_{45}^3 = 1$ $, f_{16}^1 = 4, f_{26}^2 = 3, f_{36}^3 = 2$ $, f_{46}^4 = 1, f_{56}^4 = 1, f_{56}^5 = 1$	$\begin{pmatrix} a_2^4 & 0 & 0 & 0 & 0 & 0 \\ -a_7a_2^3 & a_2^3 & 0 & 0 & 0 & 0 \\ \frac{a_2^2a_7^2}{2} & -a_7a_2^2 & a_2^2 & 0 & 0 & 0 \\ -\frac{a_7^3a_2}{2} & \frac{a_7^2a_2}{2} & -a_2a_7 & a_2 & 0 & 0 \\ \frac{12a_2a_4-6a_7a_2a_5+6a_2a_6a_7^2-6a_1a_7^3-11a_2a_7^3}{36} & \frac{2a_2a_5-2a_7a_2a_6+2a_1a_7^2+3a_2a_7^2}{4} & a_2a_6 - a_1a_7 - a_2a_7 & a_1 & a_2 & 0 \\ a_3 & a_4 & a_5 & a_6 & a_7 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,73}$ $\epsilon=1$	$f_{25}^1=1, f_{35}^2=1, f_{45}^3=1, f_{16}^1=1, f_{26}^2=1$ $, f_{36}^1=1, f_{36}^3=1, f_{46}^2=1, f_{46}^4=1$	$\begin{pmatrix} a_4 & 0 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_6-a_8 & a_7 & a_8 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,73}$ $\epsilon=-1$	$f_{25}^1=1, f_{35}^2=1, f_{45}^3=1, f_{16}^1=1, f_{26}^2=1$ $, f_{36}^1=-1, f_{36}^3=1, f_{46}^2=-1, f_{46}^4=1$	$\begin{pmatrix} a_4 & 0 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_6+a_8 & a_7 & a_8 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,74}$	$f_{25}^1=1, f_{35}^2=1, f_{45}^3=1, f_{16}^1=1$ $, f_{26}^2=1, f_{36}^3=1, f_{46}^4=1$	$\begin{pmatrix} a_4a_5^3 & 0 & 0 & 0 & 0 & 0 \\ a_3a_5^2 & a_4a_5^2 & 0 & 0 & 0 & 0 \\ a_2a_5 & a_3a_5 & a_4a_5 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_5a_7 & a_5a_8 & a_5a_9 & 0 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,75}$	$f_{25}^1=1, f_{35}^2=1, f_{45}^3=1, f_{16}^1=1$ $, f_{26}^2=1, f_{36}^3=1, f_{46}^1=1, f_{46}^4=1$	$\begin{pmatrix} a_4 & 0 & 0 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & a_4 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 & 0 & 0 \\ a_6 & a_7 & a_8 & 0 & 1 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,76}$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=2h+1$ $, f_{26}^2=h+1, f_{36}^3=h+2, f_{46}^4=1, f_{56}^5=h$	$\begin{pmatrix} a_2a_4^2 & 0 & 0 & 0 & 0 & 0 \\ a_1a_4 & a_2a_4 & -a_2a_4a_7 & 0 & 0 & 0 \\ 0 & 0 & a_2^2a_4 & 0 & 0 & 0 \\ \frac{a_1^2}{2a_2} & a_1 & \frac{a_2a_3-a_1a_4a_7}{a_4} & a_2 & 0 & 0 \\ a_3 & a_4a_7 & -\frac{a_4a_7^2}{2} & 0 & a_4 & 0 \\ a_5 & \frac{a_2a_3+ha_2a_3-ha_1a_4a_7}{a_2a_4} & a_6 & a_7 & -\frac{ha_1}{a_2} & 1 \end{pmatrix}$
$\mathfrak{g}_{6,77}$ $\epsilon=\pm1$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=1$ $, f_{26}^2=1, f_{36}^3=2, f_{46}^1=\epsilon, f_{46}^4=1$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ a_2 & a_3 & -a_3a_7 & 0 & 0 & 0 \\ 0 & 0 & a_3^2 & 0 & 0 & 0 \\ a_1 & a_2 & a_3a_5-a_2a_7 & a_3 & 0 & 0 \\ a_5 & a_7 & -\frac{a_7^2}{2} & 0 & 1 & 0 \\ a_4 & a_5 & a_6 & a_7 & 0 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$\mathfrak{g}_{6,78}$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=-1$ $, f_{36}^3=1, f_{46}^3=1, f_{46}^4=1, f_{56}^5=-1$	$\begin{pmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ a_7 & 1 & -a_6 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 \\ \frac{a_7^2}{2a_3} & \frac{a_7}{a_3} & a_1 & \frac{1}{a_3} & 0 & 0 \\ a_2 & a_3a_6 & -\frac{a_3a_6^2}{2} & 0 & a_3 & 0 \\ a_4 & a_6a_7 & a_5 & a_6 & a_7 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,79}$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=3$ $, f_{26}^2=2, f_{36}^1=1, f_{36}^3=3$ $, f_{46}^4=1, f_{46}^5=1, f_{56}^5=1$	$\begin{pmatrix} a_1^3 & 0 & 0 & 0 & 0 & 0 \\ (a_6-a_7)a_1^2 & a_1^2 & -a_6a_1^2 & 0 & 0 & 0 \\ a_2a_1^2 & 0 & a_1^3 & 0 & 0 & 0 \\ \frac{2a_2a_4-2a_2a_6a_7+2a_1a_7^2-4a_1a_6a_7+a_2a_6^2+2a_1a_6^2}{4} & a_1a_6+a_2a_6-a_1a_7 & \frac{2a_1a_4-2a_2a_6^2+2a_1a_6a_7-3a_1a_6^2}{4} & a_1 & a_2 & 0 \\ \frac{2a_1a_4+a_1a_6^2-2a_1a_6a_7}{4} & a_1a_6 & -\frac{a_1a_6^2}{2} & 0 & a_1 & 0 \\ a_3 & a_4 & a_5 & a_6 & a_7 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,80}$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=2$ $, f_{26}^2=1, f_{36}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_1a_4^2 & 0 & 0 & 0 & 0 & 0 \\ -a_1a_4a_8 & a_1a_4 & -a_1a_2 & 0 & 0 & 0 \\ 0 & 0 & a_1^2a_4 & 0 & 0 & 0 \\ \frac{a_1a_8^2}{2} & -a_1a_8 & a_1a_6 & a_1 & 0 & 0 \\ a_4a_6-a_2a_8 & a_2 & a_3 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & 0 & a_8 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,81}$ $\epsilon=\pm1$	$f_{24}^3=1, f_{25}^1=1, f_{45}^2=1, f_{16}^1=2$ $, f_{26}^2=1, f_{36}^3=1, f_{56}^3=\epsilon, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & 0 & 0 & 0 & 0 & 0 \\ -a_3a_7 & a_3 & -a_1 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 \\ \frac{a_7^2}{2} & -a_7 & a_5 & 1 & 0 & 0 \\ a_3a_5-a_1a_7 & a_1 & a_2 & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & 0 & a_7 & 1 \end{pmatrix}$
$\mathfrak{g}_{6,82}$ $\alpha=2$	$f_{24}^1=1, f_{35}^1=1, f_{16}^1=2$ $, f_{26}^2=\lambda+1, f_{36}^3=\lambda_1+1$ $, f_{46}^4=1-\lambda, f_{56}^5=1-\lambda_1$	$\begin{pmatrix} a_3a_7 & 0 & 0 & 0 & 0 & 0 \\ a_1 & \frac{a_3a_7}{a_5} & 0 & 0 & 0 & 0 \\ a_2 & 0 & a_3 & 0 & 0 & 0 \\ a_4 & 0 & 0 & a_5 & 0 & 0 \\ a_6 & 0 & 0 & 0 & a_7 & 0 \\ a_8 & \frac{a_4(\lambda+1)}{a_5} & \frac{a_6(\lambda_1+1)}{a_7} & \frac{a_1a_5(\lambda-1)}{a_3a_7} & \frac{a_2(\lambda_1-1)}{a_3} & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,82}$ $\alpha = 0$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^2 = \lambda$ $, f_{36}^3 = \lambda_1, f_{46}^4 = -\lambda, f_{56}^5 = -\lambda_1$	$\begin{pmatrix} a_3a_7 & 0 & 0 & 0 & 0 & 0 \\ a_1 & \frac{a_3a_7}{a_5} & 0 & 0 & 0 & 0 \\ a_2 & 0 & a_3 & 0 & 0 & 0 \\ a_4 & 0 & 0 & a_5 & 0 & 0 \\ a_6 & 0 & 0 & 0 & a_7 & 0 \\ a_8 & \frac{a_4\lambda}{a_5} & \frac{a_6\lambda_1}{a_7} & \frac{a_1a_5\lambda}{a_3a_7} & \frac{a_2\lambda_1}{a_3} & 1 \end{pmatrix}$
$g_{6,83}$ $\alpha \neq 0$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{16}^1 = \alpha$ $, f_{26}^2 = \frac{\alpha}{2} + \lambda, f_{26}^3 = 1$ $, f_{36}^3 = \frac{\alpha}{2} + \lambda, f_{46}^4 = \frac{\alpha}{2} - \lambda$ $, f_{56}^4 = -1, f_{56}^5 = \frac{\alpha}{2} - \lambda$	$\begin{pmatrix} a_2a_5 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & -\frac{a_2a_4}{a_5} & 0 & 0 & 0 \\ \frac{2a_2a_8}{2\lambda-\alpha} & 0 & a_2 & 0 & 0 & 0 \\ \frac{2a_5a_7}{2\lambda+\alpha} & 0 & 0 & a_5 & 0 & 0 \\ a_3 & 0 & 0 & a_4 & a_5 & 0 \\ a_6 & a_7 & \frac{-2\alpha a_4a_7+4\alpha\lambda a_3+\alpha^2a_3-4\lambda a_4a_7+4a_3\lambda^2+4a_5a_7}{2a_5(\alpha+2\lambda)} & \frac{-2\alpha a_2a_4a_8-4\alpha\lambda a_1a_5+4\lambda^2a_1a_5+\alpha^2a_1a_5+4\lambda a_2a_4a_8+4a_2a_5a_8}{2a_2a_5(2\lambda-\alpha)} & a_8 & 1 \end{pmatrix}$
$g_{6,83}$ $\alpha = 0$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^2 = \lambda$ $, f_{56}^4 = -1, f_{36}^3 = \lambda, f_{46}^4 = -\lambda$ $, f_{56}^5 = -\lambda, f_{26}^3 = 1$	$\begin{pmatrix} a_1a_3 & 0 & 0 & 0 & 0 & 0 \\ -\frac{a_1(a_3a_8-\lambda a_3a_7+\lambda a_2a_8)}{\lambda^2a_3} & a_1 & -\frac{a_1a_2}{a_3} & 0 & 0 & 0 \\ \frac{a_1a_8}{\lambda} & 0 & a_1 & 0 & 0 & 0 \\ \frac{a_3a_5}{\lambda} & 0 & 0 & a_3 & 0 & 0 \\ \frac{\lambda a_2a_5-a_3a_5+\lambda a_3a_6}{\lambda^2} & 0 & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,84}$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^2 = 1$ $, f_{46}^4 = -1, f_{56}^3 = 1$	$\begin{pmatrix} a_4^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_4^2a_8}{a_1} & \frac{a_4^2}{a_1} & 0 & 0 & 0 & 0 \\ -a_4a_7 & 0 & a_4 & 0 & 0 & 0 \\ a_1a_6 & 0 & 0 & a_1 & 0 & 0 \\ a_2 & 0 & a_3 & 0 & a_4 & 0 \\ a_5 & a_6 & a_7 & a_8 & 0 & 1 \end{pmatrix}$
$g_{6,85}$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^2 = 1 + \lambda$ $, f_{16}^1 = 2, f_{36}^3 = 1, f_{56}^5 = 1$ $, f_{46}^4 = 1 - \lambda, f_{56}^3 = 1$	$\begin{pmatrix} a_5^2 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & 0 & 0 & 0 & 0 \\ -a_5a_8 & 0 & a_5 & 0 & 0 & 0 \\ a_3 & 0 & 0 & \frac{a_5^2}{a_2} & 0 & 0 \\ a_5a_7 - a_4a_8 - a_5a_8 & 0 & a_4 & 0 & a_5 & 0 \\ a_6 & \frac{a_2a_3(\lambda+1)}{a_5^2} & a_7 & \frac{a_1(\lambda-1)}{a_2} & a_8 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,86}$	$f_{24}^1=1, f_{35}^1=1, f_{16}^1=2, f_{26}^2=1$ $, f_{26}^3=1, f_{36}^3=1, f_{46}^4=1, f_{56}^4=-1, f_{56}^5=1$	$\begin{pmatrix} a_1a_5 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_2a_5a_7-a_1a_5a_9-a_1a_5a_{10}+a_1a_4a_{10}}{a_5} & a_1 & -\frac{a_1a_4}{a_5} & a_2 & 0 & 0 \\ -a_1a_{10} & 0 & a_1 & 0 & 0 & 0 \\ a_5a_7 & 0 & 0 & a_5 & 0 & 0 \\ a_4a_7-a_5a_7+a_5a_8-a_3a_{10} & 0 & a_3 & a_4 & a_5 & 0 \\ a_6 & a_7 & a_8 & a_9 & a_{10} & 1 \end{pmatrix}$
$g_{6,87}$	$f_{24}^1=1, f_{35}^1=1, f_{16}^1=2, f_{26}^2=1, f_{26}^5=1$ $, f_{36}^3=1, f_{36}^4=1, f_{46}^4=1, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_1^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{2a_1a_2a_5+2a_1a_3a_6-2a_1^2a_7-a_3^2a_8+2a_1^2a_5+a_3^2a_5-2a_1a_3a_8+2a_1^2a_6-2a_1^2a_8+2a_1a_3a_5}{2a_1} & a_1 & \frac{a_3^2}{2a_1} & a_2 & a_3 & 0 \\ a_1a_5-a_1a_8+a_3a_5 & 0 & a_1 & a_3 & 0 & 0 \\ a_1a_5 & 0 & 0 & a_1 & 0 & 0 \\ \frac{2a_1^2a_5+a_3^2a_5-2a_1a_3a_8+2a_1^2a_6-2a_1^2a_8+2a_1a_3a_5}{2a_1} & 0 & a_3 & \frac{a_3^2}{2a_1} & a_1 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,88}$ $\alpha \neq 0$	$f_{24}^1=1, f_{35}^1=1, f_{16}^1=\alpha, f_{26}^2=\frac{\alpha}{2}+\mu_0, f_{26}^3=\nu_0$ $, f_{36}^2=-\nu_0, f_{36}^3=\frac{\alpha}{2}+\mu_0, f_{46}^4=\frac{\alpha}{2}-\mu_0$ $, f_{46}^5=\nu_0, f_{56}^4=-\nu_0, f_{56}^5=\frac{\alpha}{2}-\mu_0$	$\begin{pmatrix} \frac{a_1(a_2^2+a_3^2)}{a_3} & 0 & 0 & 0 & 0 & 0 \\ \frac{2a_1(2\mu_0a_3a_7-2\mu_0a_2a_8-2\nu_0a_2a_7-2\nu_0a_3a_8-\alpha a_3a_7+\alpha a_2a_8)}{a_3(4\mu_0^2-4\alpha\mu_0+4\nu_0^2+\alpha^2)} & a_1 & -\frac{a_1a_2}{a_3} & 0 & 0 & 0 \\ -\frac{2a_1(-2\mu_0a_2a_7-2\mu_0a_3a_8-2\nu_0a_3a_7+2\nu_0a_2a_8+\alpha a_2a_7+\alpha a_3a_8)}{a_3(4\mu_0^2-4\alpha\mu_0+4\nu_0^2+\alpha^2)} & \frac{a_1a_2}{a_3} & a_1 & 0 & 0 & 0 \\ -\frac{2(-2\mu_0a_3a_5+2\mu_0a_2a_6-2\nu_0a_2a_5-2\nu_0a_3a_6-\alpha a_3a_5+\alpha a_2a_6)}{4\mu_0^2+4\alpha\mu_0+4\nu_0^2+\alpha^2} & 0 & 0 & a_3 & -a_2 & 0 \\ \frac{2(2\mu_0a_2a_5+2\mu_0a_3a_6-2\nu_0a_3a_5+2\nu_0a_2a_6+\alpha a_2a_5+\alpha a_3a_6)}{4\mu_0^2+4\alpha\mu_0+4\nu_0^2+\alpha^2} & 0 & 0 & a_2 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,88}$ $\alpha = 0$	$f_{24}^1=1, f_{35}^1=1, f_{26}^2=\mu_0, f_{26}^3=\nu_0$ $, f_{46}^4=-\mu_0, f_{36}^2=-\nu_0, f_{36}^3=+\mu_0$ $, f_{46}^5=\nu_0, f_{56}^4=-\nu_0, f_{56}^5=-\mu_0$	$\begin{pmatrix} \frac{a_1(a_2^2+a_3^2)}{a_2} & 0 & 0 & 0 & 0 & 0 \\ \frac{a_1(-\mu_0a_3a_8+\mu_0a_2a_7-\nu_0a_3a_7-\nu_0a_2a_8)}{a_2(\mu_0^2+\nu_0^2)} & a_1 & -\frac{a_1a_3}{a_2} & 0 & 0 & 0 \\ \frac{a_1(-\nu_0a_3a_8+\nu_0a_2a_7+\mu_0a_3a_7+\mu_0a_2a_8)}{a_2(\mu_0^2+\nu_0^2)} & \frac{a_1a_3}{a_2} & a_1 & 0 & 0 & 0 \\ \frac{\nu_0a_3a_5+\nu_0a_2a_6-\mu_0a_3a_6+\mu_0a_2a_5}{\mu_0^2+\nu_0^2} & 0 & 0 & a_2 & -a_3 & 0 \\ \frac{\mu_0a_3a_5+\mu_0a_2a_6+\nu_0a_3a_6-\nu_0a_2a_5}{\mu_0^2+\nu_0^2} & 0 & 0 & a_3 & a_2 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,89}$ $\alpha \neq 0$	$f_{24}^1=1, f_{35}^1=1, f_{16}^1=\alpha, f_{26}^2=\frac{\alpha}{2}+s, f_{36}^5=\nu_0$ $, f_{36}^3=\frac{\alpha}{2}, f_{46}^4=\frac{\alpha}{2}-s, f_{56}^3=-\nu_0, f_{56}^5=\frac{\alpha}{2}$	$\begin{pmatrix} a_3^2+a_4^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{2a_7(a_3^2+a_4^2)}{a_2(2s-\alpha)} & \frac{a_3^2+a_4^2}{a_2} & 0 & 0 & 0 & 0 \\ -\frac{2(\alpha a_3a_6+\alpha a_4a_8+2\nu_0a_3a_8-2\nu_0a_4a_6)}{4\nu_0^2+\alpha^2} & 0 & a_4 & 0 & -a_3 & 0 \\ a_1 & 0 & 0 & a_2 & 0 & 0 \\ -\frac{2(\alpha a_3a_8-\alpha a_4a_6-2\nu_0a_3a_6-2\nu_0a_4a_8)}{4\nu_0^2+\alpha^2} & 0 & a_3 & 0 & a_4 & 0 \\ a_5 & \frac{a_1(2s+\alpha)}{2a_2} & a_6 & a_7 & a_8 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,89}$ $\alpha=0$	$f^1_{24}=1, f^1_{35}=1, f^2_{26}=s, f^5_{36}=\nu_0$ $, f^4_{46}=-s, f^3_{56}=-\nu_0$	$\begin{pmatrix} a_3^2+a_4^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_7(a_3^2+a_4^2)}{a_2 s} & \frac{a_3^2+a_4^2}{a_2} & 0 & 0 & 0 & 0 \\ \frac{a_4a_6-a_3a_8}{\nu_0} & 0 & a_4 & 0 & -a_3 & 0 \\ a_1 & 0 & 0 & a_2 & 0 & 0 \\ \frac{a_3a_6+a_4a_8}{\nu_0} & 0 & a_3 & 0 & a_4 & 0 \\ a_5 & \frac{a_1 s}{a_2} & a_6 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,90}$ $\alpha\neq 0$	$f^1_{24}=1, f^1_{35}=1, f^1_{16}=\alpha, f^2_{26}=\frac{\alpha}{2}$ $, f^4_{26}=1, f^3_{36}=\frac{\alpha}{2}, f^5_{36}=\nu_0, f^2_{46}=1$ $, f^4_{46}=\frac{\alpha}{2}, f^3_{56}=-\nu_0, f^5_{56}=\frac{\alpha}{2}$	$\begin{pmatrix} a_1^2-a_2^2 & 0 & 0 & 0 & 0 & 0 \\ \frac{2(2a_1a_5-2a_2a_7+\alpha a_2a_5-\alpha a_1a_7)}{\alpha^2-4} & a_1 & 0 & a_2 & 0 & 0 \\ -\frac{2(r\alpha a_6+\alpha a_3a_8-2\nu_0a_3a_6+2r\nu_0a_8)}{\alpha^2+4\nu_0^2} & 0 & a_3 & 0 & -r & 0 \\ \frac{2(2a_2a_5-2a_1a_7+\alpha a_1a_5-\alpha a_2a_7)}{\alpha^2-4} & a_2 & 0 & a_1 & 0 & 0 \\ -\frac{2(r\alpha a_8-\alpha a_3a_6-2\nu_0a_3a_8-2r\nu_0a_6)}{\alpha^2+4\nu_0^2} & 0 & r & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$ $r=Rootof(z^2+a_2^2+a_3^2-a_1^2)$
$g_{6,90}$ $\alpha=0$ $,\nu_0\neq 1$	$f^1_{24}=1, f^1_{35}=1, f^4_{26}=1, f^5_{36}=\nu_0$ $, f^2_{46}=1, f^3_{56}=-\nu_0$	$\begin{pmatrix} a_4^2-a_3^2 & 0 & 0 & 0 & 0 & 0 \\ a_3a_8-a_4a_7 & a_4 & 0 & a_3 & 0 & 0 \\ a_1 & 0 & r & 0 & a_2 & 0 \\ a_4a_8-a_3a_7 & a_3 & 0 & a_4 & 0 & 0 \\ a_5 & 0 & -a_2 & 0 & r & 0 \\ a_6 & a_7 & \frac{\nu_0(a_1r-a_2a_5)}{a_4^2-a_3^2} & a_8 & \frac{\nu_0(a_5r+a_1a_2)}{a_4^2-a_3^2} & 1 \end{pmatrix}$ $r=Rootof(z^2+a_2^2+a_3^2-a_4^2)$
$g_{6,91}$	$f^1_{24}=1, f^1_{35}=1, f^4_{26}=1, f^5_{36}=1$ $, f^2_{46}=1, f^3_{56}=-1$	$\begin{pmatrix} a_1^2-a_2^2 & 0 & 0 & 0 & 0 & 0 \\ a_2a_7-a_1a_5 & a_1 & 0 & a_2 & 0 & 0 \\ a_3a_6+ra_8 & 0 & a_3 & 0 & r & 0 \\ a_1a_7-a_2a_5 & a_2 & 0 & a_1 & 0 & 0 \\ a_3a_8-ra_6 & 0 & -r & 0 & a_3 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$ $r=Rootof(z^2+a_3^2+a_2^2-a_1^2)$
$g_{6,92}$ $\alpha\neq 0$	$f^1_{24}=1, f^1_{35}=1, f^1_{16}=\alpha, f^2_{26}=\frac{\alpha}{2}, f^3_{26}=\nu_0$ $, f^2_{36}=-\mu_0, f^3_{36}=\frac{\alpha}{2}, f^5_{46}=\mu_0$ $, f^4_{46}=\frac{\alpha}{2}, f^4_{56}=-\nu_0, f^5_{56}=\frac{\alpha}{2}$	*See the appendix*
$g_{6,92}$ $\alpha=0$	$f^1_{24}=1, f^1_{35}=1, f^3_{26}=\nu_0, f^2_{36}=-\mu_0$ $, f^5_{46}=\mu_0, f^4_{56}=-\nu_0$	*See the appendix*

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g^*_{6,92}$ $p \neq 0$	$f^5_{24}=1, f^5_{13}=1, f^1_{16}=p$ $, f^3_{16}=1, f^2_{26}=p, f^4_{26}=1$ $, f^1_{36}=-1, f^3_{36}=p, f^2_{46}=-1$ $, f^4_{46}=p, f^5_{56}=2p$	$\begin{pmatrix} r & 0 & a_1 & 0 & \frac{pa_1a_4+ra_4+a_1a_6-pra_6}{1+p^2} & 0 \\ 0 & a_2 & 0 & a_3 & \frac{a_3a_7+pa_3a_5-pa_2a_7+a_2a_5}{1+p^2} & 0 \\ -a_1 & 0 & r & 0 & \frac{-a_1a_4+ra_6+pra_4+pa_1a_6}{1+p^2} & 0 \\ 0 & -a_3 & 0 & a_2 & \frac{pa_2a_5+pa_3a_7+a_2a_7-a_3a_5}{1+p^2} & 0 \\ 0 & 0 & 0 & 0 & a_2^2+a_3^2 & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$ $r = Rootof(z^2+a_1^2-a_2^2-a_3^2)$
$g^*_{6,92}$ $p = 0$	$f^5_{24}=1, f^5_{13}=1$ $, f^3_{16}=1, f^4_{26}=1$ $, f^1_{36}=-1, f^2_{46}=-1$	$\begin{pmatrix} r & a_1 & 0 & -a_3 & a_1a_6-a_3a_8+ra_5 & 0 \\ -\frac{a_1a_2+a_3a_4}{r} & a_2 & \frac{a_1a_4-a_2a_3}{r} & -a_4 & \frac{-a_3a_4a_5-a_1a_2a_5+ra_2a_6+a_1a_4a_7-a_2a_3a_7-ra_4a_8}{r} & 0 \\ 0 & a_3 & r & a_1 & a_3a_6+a_1a_8+ra_7 & 0 \\ \frac{a_2a_3-a_1a_4}{r} & a_4 & -\frac{a_1a_2+a_3a_4}{r} & a_2 & \frac{-a_1a_4a_5+a_2a_3a_5+ra_4a_6-a_3a_4a_7-a_1a_2a_7+ra_2a_8}{r} & 0 \\ 0 & 0 & 0 & 0 & a_1^2+a_2^2+a_3^2+a_4^2 & 0 \\ a_5 & a_6 & a_7 & a_8 & a_9 & 1 \end{pmatrix}$ $r = Rootof(z^2-a_2^2-a_4^2)$
$g_{6,93}$ $\alpha \neq 0$	$f^1_{24}=1, f^1_{35}=1, f^1_{16}=\alpha$ $, f^2_{26}=\frac{\alpha}{2}, f^4_{26}=1, f^5_{26}=\nu_0$ $, f^4_{36}=\nu_0, f^3_{36}=\frac{\alpha}{2}, f^2_{46}=1$ $, f^3_{46}=-\nu_0, f^4_{46}=\frac{\alpha}{2}$ $, f^2_{56}=-\nu_0, f^5_{56}=\frac{\alpha}{2}$	*See the appendix*
$g_{6,93}$ $\alpha = 0$	$f^1_{24}=1, f^1_{35}=1$ $, f^4_{26}=1, f^5_{26}=\nu_0$ $, f^4_{36}=\nu_0, f^2_{46}=1$ $, f^3_{46}=-\nu_0, f^2_{56}=-\nu_0$	$\begin{pmatrix} -a_2^2+2a_2^2\nu_0^2+2\nu_0^2a_1a_2+a_3^2-2a_3^2\nu_0^2+2a_3\nu_0^2r & 0 & 0 & 0 & 0 & 0 \\ \frac{-\nu_0a_3a_5+\nu_0a_2a_7+a_6r-a_1a_8+a_5\nu_0r+a_1a_7\nu_0}{\nu_0} & a_3 & \nu_0(r-a_3) & a_2 & \nu_0(a_1+a_2) & 0 \\ \frac{\nu_0^2a_2a_8+\nu_0^2a_1a_8+\nu_0^2a_3a_6-a_6\nu_0^2r+a_6r-a_1a_8+a_5\nu_0r+a_1a_7\nu_0}{\nu_0^2} & \nu_0(a_3-r) & r & \nu_0(a_1+a_2) & a_1 & 0 \\ -\frac{\nu_0a_2a_5-\nu_0a_3a_7+\nu_0a_1a_5+\nu_0a_7r+a_1a_6-ra_8}{\nu_0} & a_2 & -\nu_0(a_1+a_2) & a_3 & \nu_0(a_3-r) & 0 \\ -\frac{-\nu_0a_1a_5-\nu_0a_7r-a_1a_6+a_8r+a_2a_6\nu_0^2+a_1a_6\nu_0^2+a_3a_8\nu_0^2-a_8r\nu_0^2}{\nu_0^2} & -\nu_0(a_1+a_2) & a_1 & -\nu_0(a_3-r) & r & 0 \\ a_4 & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$ $r = Rootof(z^2+a_2^2-a_1^2-a_3^2)$
$g_{6,94}$	$f^1_{34}=1, f^1_{25}=1, f^2_{35}=1$ $, f^1_{16}=\lambda+2, f^2_{26}=\lambda+1$ $, f^3_{36}=\lambda, f^4_{46}=2, f^5_{56}=1$	$\begin{pmatrix} a_1a_4^2 & 0 & 0 & 0 & 0 & 0 \\ -a_1a_4a_7 & a_1a_4 & 0 & 0 & 0 & 0 \\ \frac{a_1a_7^2-a_1a_6}{2} & -a_1a_7 & a_1 & 0 & 0 & 0 \\ a_3a_4 & 0 & 0 & a_4^2 & 0 & 0 \\ a_2 & a_3 & 0 & 0 & a_4 & 0 \\ a_5 & \frac{a_3a_7+\lambda a_2+a_2}{a_4} & \frac{\lambda a_3}{a_4} & a_6 & a_7 & 1 \end{pmatrix}$

TABLE 1. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$g_{6,95}$	$f_{34}^1=1, f_{25}^1=1, f_{35}^2=1, f_{16}^1=2$ $, f_{26}^2=1, f_{46}^1=1, f_{46}^4=2, f_{56}^5=1$	$\begin{pmatrix} a_3^2 & 0 & 0 & 0 & 0 & 0 \\ -a_3a_8 & a_3 & 0 & 0 & 0 & 0 \\ \frac{a_8^2-a_7}{2} & -a_8 & 1 & 0 & 0 & 0 \\ a_1 & 0 & 0 & a_3^2 & 0 & 0 \\ a_3a_5-a_2a_8 & a_2 & 0 & 0 & a_3 & 0 \\ a_4 & a_5 & 0 & a_7 & a_8 & 1 \end{pmatrix}$
$g_{6,96}$	$f_{34}^1=1, f_{25}^1=1, f_{35}^2=1, f_{16}^1=3, f_{26}^2=2$ $, f_{36}^3=1, f_{46}^2=1, f_{46}^4=2, f_{56}^3=1, f_{56}^5=1$	$\begin{pmatrix} a_2^3 & 0 & 0 & 0 & 0 & 0 \\ -a_2^2a_7 & a_2^2 & 0 & 0 & 0 & 0 \\ \frac{a_7^2a_2-a_6a_2}{2} & -a_2a_7 & a_2 & 0 & 0 & 0 \\ a_2^2a_5-a_1a_2a_7-a_2^2a_7 & a_1a_2 & 0 & a_2^2 & 0 & 0 \\ \frac{2a_2a_4-2a_1a_6-a_2a_6-2a_2a_5a_7+2a_1a_7^2+3a_2a_7^2}{4} & a_2a_5-a_1a_7-a_2a_7 & a_1 & 0 & a_2 & 0 \\ a_3 & a_4 & a_5 & a_6 & a_7 & 1 \end{pmatrix}$
$g_{6,97}$	$f_{34}^1=1, f_{25}^1=1, f_{35}^2=1, f_{16}^1=4$ $, f_{26}^2=3, f_{36}^3=2, f_{36}^4=1, f_{46}^4=2, f_{56}^5=1$	$\begin{pmatrix} a_2^4 & 0 & 0 & 0 & 0 & 0 \\ -a_2^3a_7 & a_2^3 & 0 & 0 & 0 & 0 \\ \frac{2a_1a_5-2a_2^2a_6+2a_2^2a_7^2+a_2^2a_5}{4} & -a_2^2a_7 & a_2^2 & a_1 & 0 & 0 \\ \frac{a_2^2a_5}{2} & 0 & 0 & a_2^2 & 0 & 0 \\ \frac{2a_2a_4-a_2a_5a_7}{6} & \frac{a_2a_5}{2} & 0 & 0 & a_2 & 0 \\ a_3 & a_4 & a_5 & a_6 & a_7 & 1 \end{pmatrix}$
$g_{6,98}$	$f_{34}^1=1, f_{25}^1=1, f_{35}^2=1, f_{16}^1=1, f_{26}^1=h$ $, f_{26}^2=1, f_{36}^3=1, f_{56}^4=h$	$\begin{pmatrix} a_2 & 0 & 0 & 0 & 0 & 0 \\ \frac{a_2(a_3h-a_7)}{h} & a_2 & 0 & 0 & 0 & 0 \\ a_1 & -\frac{a_2a_7}{h} & a_2 & 0 & 0 & 0 \\ a_6 & 0 & 0 & 1 & 0 & 0 \\ a_5+a_3a_6 & a_6 & 0 & a_3 & 1 & 0 \\ a_4 & a_5 & a_6 & a_7 & 0 & 1 \end{pmatrix}$
$g_{6,99}$	$f_{34}^1=1, f_{25}^1=1, f_{35}^2=1, f_{45}^3=1, f_{16}^1=5$ $, f_{26}^2=4, f_{36}^3=3, f_{46}^4=2, f_{56}^5=1$	$\begin{pmatrix} a_1^5 & 0 & 0 & 0 & 0 & 0 \\ -a_1^4a_6 & a_1^4 & 0 & 0 & 0 & 0 \\ \frac{a_1^3a_6^2-a_1^3a_5}{2} & -a_1^3a_6 & a_1^3 & 0 & 0 & 0 \\ \frac{2a_1^2a_4+2a_1^2a_5a_6-a_1^2a_6^3}{6} & \frac{a_1^2a_6^2}{2} & -a_1^2a_6 & a_1^2 & 0 & 0 \\ \frac{6a_1a_3-3a_1a_5^2-2a_1a_4a_6+a_1a_5a_6^2}{24} & \frac{2a_1a_4-a_1a_5a_6}{6} & \frac{a_1a_5}{2} & 0 & a_1 & 0 \\ a_2 & a_3 & a_4 & a_5 & a_6 & 1 \end{pmatrix}$

TABLE 2: The automorphism groups of six dimensional solvable real Lie algebras with nilradical 4

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,1}^{\alpha\beta\gamma\delta}$ $\alpha\beta \neq 0$ $\gamma^2+\delta^2 \neq 0$	$f_{13}^3=\alpha, f_{14}^4=\gamma, f_{16}^6=1, f_{23}^3=\beta$ $, f_{24}^4=\delta, f_{25}^5=1$	$\begin{pmatrix} 1 & 0 & \frac{a_2\alpha}{\beta} & \frac{a_3\gamma}{\delta} & 0 & a_1 \\ 0 & 1 & a_2 & a_3 & a_4 & 0 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_8 \end{pmatrix}$
$N_{6,2}^{\alpha\beta\gamma}$ $\alpha^2+\beta^2 \neq 0$	$f_{13}^3=\alpha, f_{14}^4=1, f_{15}^6=1, f_{23}^3=\beta$ $, f_{24}^4=\gamma, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & \frac{a_3\alpha}{\beta} & a_1 & 0 & a_2 \\ 0 & 1 & a_3 & \gamma a_1 & a_2 & a_4 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,3}^{\alpha}$	$f_{13}^3=1, f_{14}^4=1, f_{15}^6=1, f_{23}^3=\alpha$ $, f_{23}^4=1, f_{24}^4=\alpha, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & -\alpha a_1+a_2 & a_1 & 0 & a_3 \\ 0 & 1 & -\alpha^2 a_1+\alpha a_2 & a_2 & a_3 & a_4 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,4}^{\alpha\beta}$ $\alpha \neq 0$	$f_{13}^3=1, f_{14}^4=1, f_{15}^6=1, f_{23}^4=1$ $, f_{24}^3=-1, f_{25}^5=\alpha, f_{25}^6=\beta, f_{26}^6=\alpha$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & 0 & a_3 \\ 0 & 1 & -a_2 & a_1 & \alpha a_3 & a_4 \\ 0 & 0 & a_6 & -a_5 & 0 & 0 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,5}^{\alpha\beta}$ $\alpha\beta \neq 0$	$f_{13}^3=\alpha, f_{15}^5=1, f_{15}^6=1, f_{16}^6=1$ $, f_{23}^3=\beta, f_{24}^4=1$	$\begin{pmatrix} 1 & 0 & \frac{\alpha a_3}{\beta} & 0 & a_1 & a_2 \\ 0 & 1 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,6}^{\alpha\beta}$ $\alpha^2+\beta^2 \neq 0$	$f_{13}^3=\alpha, f_{14}^4=\alpha, f_{15}^5=1, f_{15}^6=1, f_{16}^6=1$ $, f_{23}^3=1, f_{23}^4=1, f_{24}^4=1, f_{25}^6=\beta$	$\begin{pmatrix} 1 & 0 & \alpha a_3 & \alpha(a_4-a_3) & a_1 & a_2 \\ 0 & 1 & a_3 & a_4 & 0 & \beta a_1 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,7}^{\alpha\beta\gamma}$ $\alpha^2+\beta^2\neq 0$	$f_{13}^3=\alpha, f_{14}^4=\alpha, f_{15}^5=1, f_{15}^6=1, f_{16}^6=1$ $, f_{23}^3=\gamma, f_{23}^4=1, f_{24}^3=-1, f_{24}^4=\gamma, f_{25}^6=\beta$	$\begin{pmatrix} 1 & 0 & \frac{\alpha(\gamma a_3+a_4)}{1+\gamma^2} & \frac{\alpha(\gamma a_4-a_3)}{1+\gamma^2} & a_1 & a_2 \\ 0 & 1 & a_3 & a_4 & 0 & \beta a_1 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & -a_6 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,8}$	$f_{13}^3=1, f_{14}^6=1, f_{24}^4=1, f_{25}^5=1$ $, f_{25}^6=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & a_2 & a_3 & a_4 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & a_7 \\ 0 & 0 & 0 & 0 & a_6 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}$
$N_{6,9}^{\alpha}$	$f_{13}^3=1, f_{14}^6=1, f_{24}^4=1, f_{24}^5=1$ $, f_{25}^5=1, f_{25}^6=\alpha, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & a_2 & a_3 & a_4 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & a_6 & \alpha a_7 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}$
$N_{6,10}^{\alpha\beta}$	$f_{13}^3=\alpha, f_{14}^4=1, f_{14}^6=\beta, f_{15}^5=1$ $, f_{16}^6=1, f_{23}^3=1, f_{24}^5=1, f_{25}^6=1$	$\begin{pmatrix} 1 & 0 & \alpha a_4 & a_1 & a_2 & a_3 \\ 0 & 1 & a_4 & 0 & a_1 & a_2 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & a_6 & a_7 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}$
$N_{6,11}^{\alpha}$	$f_{13}^4=1, f_{15}^5=1, f_{15}^6=1, f_{16}^6=1$ $, f_{23}^3=1, f_{24}^4=1, f_{25}^5=\alpha, f_{26}^6=\alpha$	$\begin{pmatrix} 1 & 0 & 0 & a_3 & a_1 & a_2 \\ 0 & 1 & a_3 & a_4 & \alpha a_1 & \alpha(a_2-a_1) \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,12}^{\alpha\beta}$	$f_{13}^3=1, f_{13}^4=1, f_{14}^4=1, f_{15}^5=1, f_{15}^6=1$ $, f_{16}^6=1, f_{23}^4=\alpha, f_{23}^5=1, f_{25}^6=\alpha, f_{26}^4=-1$ $, f_{23}^6=-\beta, f_{24}^6=1, f_{25}^3=-1, f_{25}^4=\beta$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & -a_3 & -\beta a_3+\alpha a_1-a_3-a_4 \\ 0 & 1 & a_3 & a_4 & a_1 & -a_1-\alpha a_3-\beta a_1+a_2 \\ 0 & 0 & a_5 & a_6 & a_7 & a_8 \\ 0 & 0 & 0 & a_5 & 0 & a_7 \\ 0 & 0 & -a_7 & -a_8 & a_5 & a_6 \\ 0 & 0 & 0 & -a_7 & 0 & a_5 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,13}^{\alpha\beta\gamma\delta}$ $\alpha^2+\beta^2\neq 0$ $\gamma^2+\delta^2\neq 0$	$f_{13}^3=\alpha, f_{14}^4=\gamma, f_{15}^6=1, f_{16}^5=-1$ $, f_{23}^3=\beta, f_{24}^4=\delta, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & \frac{a_3\gamma}{\delta} & -a_4 & a_2 \\ 0 & 1 & \frac{a_1\beta}{\alpha} & a_3 & a_2 & a_4 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,14}^{\alpha\beta\gamma}$ $\alpha\beta\neq 0$	$f_{13}^3=\alpha, f_{15}^5=\gamma, f_{15}^6=1, f_{16}^5=-1$ $, f_{16}^6=\gamma, f_{23}^3=\beta, f_{24}^4=1$	$\begin{pmatrix} 1 & 0 & \frac{a_3\alpha}{\beta} & 0 & a_1 & a_2 \\ 0 & 1 & a_3 & a_4 & 0 & 0 \\ 0 & 0 & a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,15}^{\alpha\beta\gamma\delta}$ $\beta\neq 0$	$f_{13}^3=1, f_{14}^4=1, f_{15}^5=\alpha, f_{15}^6=\beta$ $, f_{16}^5=-\beta, f_{16}^6=\alpha, f_{23}^3=\gamma, f_{23}^4=1$ $, f_{24}^3=-1, f_{24}^4=\gamma, f_{25}^5=\delta, f_{26}^6=\delta$	$\begin{pmatrix} 1 & 0 & a_1 & \gamma a_1-a_3 & -\frac{-a_2\alpha\delta+a_4(\alpha^2+\beta^2)}{\delta\beta} & a_2 \\ 0 & 1 & a_3 & \gamma^2 a_1-\gamma a_3+a_1 & -\frac{-\delta a_2+\alpha a_4}{\beta} & a_4 \\ 0 & 0 & a_6 & -a_5 & 0 & 0 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_8 & -a_7 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \end{pmatrix}$
$N_{6,16}^{\alpha\beta}$	$f_{13}^4=1, f_{15}^5=\alpha, f_{15}^6=1, f_{16}^5=-1, f_{16}^6=\alpha$ $, f_{23}^3=1, f_{24}^4=1, f_{25}^5=\beta, f_{26}^6=\beta$	$\begin{pmatrix} 1 & 0 & 0 & a_2 & -\frac{a_4+\alpha^2 a_4-\alpha\beta a_1}{\beta} & a_1 \\ 0 & 1 & a_2 & a_3 & \beta a_1-\alpha a_4 & a_4 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,17}^{\alpha}$	$f_{13}^3=\alpha, f_{13}^4=1, f_{14}^4=\alpha, f_{15}^6=1$ $, f_{16}^5=-1, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & -a_4 & a_3 \\ 0 & 1 & 0 & 0 & a_3 & a_4 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,18}^{\alpha\beta\gamma}$ $\beta\neq 0$	$f_{13}^4=1, f_{14}^3=-1, f_{15}^5=\alpha, f_{15}^6=\beta$ $, f_{16}^5=-\beta, f_{16}^6=\alpha, f_{23}^3=1, f_{24}^4=1$ $, f_{25}^5=\gamma, f_{26}^6=\gamma$	$\begin{pmatrix} 1 & 0 & -a_3 & a_2 & \frac{a_1\alpha\gamma-a_4(\alpha^2+\beta^2)}{\gamma\beta} & a_1 \\ 0 & 1 & a_2 & a_3 & \frac{\gamma a_1-\alpha a_4}{\beta} & a_4 \\ 0 & 0 & a_6 & -a_5 & 0 & 0 \\ 0 & 0 & a_5 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_8 & -a_7 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,19}$	$f_{13}^4=1, f_{13}^5=1, f_{14}^3=-1, f_{14}^6=1, f_{15}^6=1$ $, f_{16}^5=-1, f_{23}^3=1, f_{24}^4=1, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & -a_3 & a_1+a_4 & a_1 & a_2 \\ 0 & 1 & a_1+a_4 & a_3 & a_2-a_3 & a_4 \\ 0 & 0 & a_5 & -a_8 & a_7 & -a_6 \\ 0 & 0 & a_8 & a_5 & a_6 & a_7 \\ 0 & 0 & 0 & 0 & a_5 & -a_8 \\ 0 & 0 & 0 & 0 & a_8 & a_5 \end{pmatrix}$
$N_{6,20}^{\alpha\beta}$ $\alpha^2+\beta^2\neq 0$	$f_{14}^4=\alpha, f_{16}^6=1, f_{24}^4=\beta, f_{25}^5=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & \frac{\alpha a_4}{\beta} & 0 & a_2 \\ 0 & 1 & a_3 & a_4 & a_5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_8 \end{pmatrix}$
$N_{6,21}^{\alpha}$	$f_{14}^4=1, f_{15}^6=1, f_{24}^4=\alpha, f_{25}^5=1, f_{26}^6=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & 0 & a_4 \\ 0 & 1 & a_3 & \alpha a_2 & a_4 & a_5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,22}^{\alpha\epsilon}$ $\alpha^2+\epsilon^2\neq 0, \epsilon=1$	$f_{13}^3=1, f_{15}^6=1, f_{23}^3=\alpha, f_{24}^4=1, f_{12}^5=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & a_2 & a_3 \\ 0 & 1 & \alpha a_1 & a_4 & a_5 & a_6 \\ 0 & 0 & a_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a_5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
$N_{6,22}^{\alpha\epsilon}$ $\alpha^2+\epsilon^2\neq 0, \epsilon=0$	$f_{13}^3=1, f_{15}^6=1, f_{23}^3=\alpha, f_{24}^4=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & a_2 & a_3 \\ 0 & 1 & \alpha a_1 & a_4 & 0 & a_5 \\ 0 & 0 & a_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_8 & a_9 \\ 0 & 0 & 0 & 0 & 0 & a_8 \end{pmatrix}$
$N_{6,23}^{\alpha\epsilon}$ $\epsilon=1$	$f_{13}^3=1, f_{14}^4=1, f_{15}^6=1, f_{23}^4=1, f_{24}^3=-1, f_{25}^6=\alpha, f_{12}^5=1$	$\begin{pmatrix} 1 & 0 & a_4 & a_1 & a_2 & a_3 \\ 0 & 1 & -a_1 & a_4 & a_5 & a_6 \\ 0 & 0 & a_8 & -a_7 & 0 & 0 \\ 0 & 0 & a_7 & a_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a_5-\alpha a_2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
$N_{6,23}^{\alpha\epsilon}$ $\epsilon=0$	$f_{13}^3=1, f_{14}^4=1, f_{15}^6=1, f_{23}^4=1, f_{24}^3=-1, f_{25}^6=\alpha$	$\begin{pmatrix} 1 & 0 & a_4 & a_1 & a_2 & a_3 \\ 0 & 1 & -a_1 & a_4 & \alpha a_2 & a_5 \\ 0 & 0 & a_7 & -a_6 & 0 & 0 \\ 0 & 0 & a_6 & a_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_8 & a_9 \\ 0 & 0 & 0 & 0 & 0 & a_8 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,24}$	$f_{15}^5=1, f_{15}^6=1, f_{16}^6=1$ $, f_{24}^4=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & a_2 & a_3 \\ 0 & 1 & a_4 & a_5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,25}^{\alpha\beta}$ $\alpha^2+\beta^2\neq 0$	$f_{14}^4=\alpha, f_{15}^6=1, f_{16}^5=-1, f_{23}^4=\beta$ $, f_{25}^5=1, f_{26}^6=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & \frac{\alpha a_3}{\beta} & a_1 & -a_5 & a_4 \\ 0 & 1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & -a_6 \\ 0 & 0 & 0 & 0 & a_6 & a_7 \end{pmatrix}$
$N_{6,26}^{\alpha}$	$f_{15}^5=\alpha, f_{15}^6=1, f_{16}^5=-1$ $, f_{16}^6=\alpha, f_{24}^4=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & 0 & a_2 & a_3 \\ 0 & 1 & a_4 & a_5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,27}^{\epsilon}$ $\epsilon=1$	$f_{13}^4=1, f_{15}^6=1, f_{16}^5=-1$ $, f_{25}^5=1, f_{26}^6=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & -a_6 & a_5 \\ 0 & 1 & a_3 & a_4 & a_5 & a_6 \\ 0 & 0 & 1 & a_3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_7 & a_8 \\ 0 & 0 & 0 & 0 & -a_8 & a_7 \end{pmatrix}$
$N_{6,27}^{\epsilon}$ $\epsilon=0$	$f_{13}^4=1, f_{15}^6=1, f_{16}^5=-1$ $, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & -a_5 & a_3 \\ 0 & 1 & 0 & a_4 & a_3 & a_5 \\ 0 & 0 & a_6 & a_7 & 0 & 0 \\ 0 & 0 & 0 & a_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_8 & a_9 \\ 0 & 0 & 0 & 0 & -a_9 & a_8 \end{pmatrix}$
$N_{6,28}$	$f_{46}^3=1, f_{56}^4=1, f_{13}^3=1, f_{15}^5=-1$ $, f_{16}^6=1, f_{24}^4=1, f_{25}^5=2, f_{26}^6=-1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2a_3 & a_2 & a_3 \\ 0 & 1 & a_3a_4+a_2a_3^2 & a_4 & -2a_2 & -a_3 \\ 0 & 0 & a_5a_6^2 & 0 & 0 & 0 \\ 0 & 0 & a_3a_5a_6 & a_5a_6 & 0 & 0 \\ 0 & 0 & \frac{a_5a_3^2}{2} & a_3a_5 & a_5 & 0 \\ 0 & 0 & -a_2a_3a_6-a_4a_6 & a_2a_6 & 0 & a_6 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,29}^{\alpha\beta}$ $\alpha^2+\beta^2\neq 0$	$f_{45}^3=1, f_{13}^3=1, f_{14}^4=1, f_{16}^6=\alpha$ $, f_{23}^3=1, f_{25}^5=1, f_{26}^6=\beta$	$\begin{pmatrix} 1 & 0 & a_2+a_1a_3 & a_1 & 0 & \frac{\alpha a_4}{\beta} \\ 0 & 1 & a_2 & 0 & a_3 & a_4 \\ 0 & 0 & a_5a_6 & 0 & 0 & 0 \\ 0 & 0 & a_3a_5 & a_5 & 0 & 0 \\ 0 & 0 & -a_1a_6 & 0 & a_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,30}^{\alpha}$	$f_{45}^3=1, f_{13}^3=2, f_{14}^4=1, f_{15}^5=1$ $, f_{16}^6=\alpha, f_{24}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_3 & a_2 & \alpha a_4 \\ 0 & 1 & -\frac{a_3^2}{2} & 0 & a_3 & a_4 \\ 0 & 0 & a_5^2 & 0 & 0 & 0 \\ 0 & 0 & a_2a_5-a_3a_6 & a_5 & a_6 & 0 \\ 0 & 0 & -a_3a_5 & 0 & a_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,31}$	$f_{45}^3=1, f_{14}^4=1, f_{15}^5=-1, f_{23}^3=1$ $, f_{25}^5=1, f_{26}^3=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & -a_1a_2 & a_1 & a_2 & 0 \\ 0 & 1 & a_3 & 0 & -a_2 & a_4 \\ 0 & 0 & a_5a_6 & 0 & 0 & 0 \\ 0 & 0 & -a_2a_5 & a_5 & 0 & 0 \\ 0 & 0 & -a_1a_6 & 0 & a_6 & 0 \\ 0 & 0 & a_7 & 0 & 0 & a_5a_6 \end{pmatrix}$
$N_{6,32}^{\alpha}$	$f_{45}^3=1, f_{14}^4=1, f_{15}^5=-1, f_{16}^3=1, f_{23}^3=1$ $, f_{24}^4=\alpha, f_{25}^5=1-\alpha, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & a_3 & 0 \\ 0 & 1 & a_4 & \alpha a_2 & a_3(1-\alpha) & a_1+a_2a_3 \\ 0 & 0 & a_5a_6 & 0 & 0 & 0 \\ 0 & 0 & -a_3a_5 & a_5 & 0 & 0 \\ 0 & 0 & -a_2a_6 & 0 & a_6 & 0 \\ 0 & 0 & a_7 & 0 & 0 & a_5a_6 \end{pmatrix}$
$N_{6,33}$	$f_{45}^3=1, f_{13}^3=1, f_{14}^4=1, f_{23}^3=1$ $, f_{25}^5=1, f_{25}^6=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_2+a_1a_3 & a_1 & 0 & 0 \\ 0 & 1 & a_2 & 0 & a_3 & a_4 \\ 0 & 0 & a_5a_6 & 0 & 0 & 0 \\ 0 & 0 & a_3a_5 & a_5 & 0 & 0 \\ 0 & 0 & -a_1a_6 & 0 & a_6 & a_7 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}$
$N_{6,34}^{\alpha}$	$f_{45}^3=1, f_{13}^3=1, f_{14}^4=1, f_{15}^6=1$ $, f_{23}^3=1+\alpha, f_{24}^4=\alpha, f_{25}^5=1, f_{26}^6=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & 0 & a_3 \\ 0 & 1 & a_1+\alpha a_1-a_2a_3 & \alpha a_2 & a_3 & a_4 \\ 0 & 0 & a_5a_6 & 0 & 0 & 0 \\ 0 & 0 & a_3a_5 & a_5 & 0 & 0 \\ 0 & 0 & -a_2a_6 & 0 & a_6 & a_7 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{pmatrix}$

TABLE 2. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$N_{6,35}^{\alpha\beta}$ $\alpha^2+\beta^2\neq 0$	$f_{45}^3=1, f_{14}^5=1, f_{15}^4=-1, f_{16}^6=\alpha, f_{23}^3=2$ $, f_{24}^4=1, f_{25}^5=1, f_{26}^6=\beta$	$\begin{pmatrix} 1 & 0 & -\frac{a_2^2+a_3^2}{2} & -a_3 & a_2 & \frac{\alpha a_4}{\beta} \\ 0 & 1 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & a_5^2+a_6^2 & 0 & 0 & 0 \\ 0 & 0 & a_3a_6+a_2a_5 & a_6 & -a_5 & 0 \\ 0 & 0 & a_3a_5-a_2a_6 & a_5 & a_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$
$N_{6,36}$	$f_{45}^3=1, f_{14}^5=1, f_{15}^4=-1, f_{23}^3=2$ $, f_{24}^4=1, f_{25}^5=1, f_{26}^3=1, f_{26}^6=2$	$\begin{pmatrix} 1 & 0 & -\frac{a_2^2+a_3^2}{2} & -a_3 & a_2 & 0 \\ 0 & 1 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & a_5^2+a_6^2 & 0 & 0 & 0 \\ 0 & 0 & a_3a_6+a_2a_5 & a_6 & -a_5 & 0 \\ 0 & 0 & a_3a_5-a_2a_6 & a_5 & a_6 & 0 \\ 0 & 0 & a_7 & 0 & 0 & a_5^2+a_6^2 \end{pmatrix}$
$N_{6,37}^{\alpha}$	$f_{45}^3=1, f_{14}^5=1, f_{15}^4=-1, f_{16}^3=1$ $, f_{23}^3=2, f_{24}^4=1, f_{24}^5=\alpha$ $, f_{25}^4=-\alpha, f_{25}^5=1, f_{26}^6=2$	$\begin{pmatrix} 1 & 0 & \frac{2\alpha a_1a_3+a_4-a_3^2-a_1^2-\alpha^2a_1^2}{2} & a_1 & a_3-\alpha a_1 & 0 \\ 0 & 1 & a_2 & a_3 & \alpha a_3-\alpha^2a_1-a_1 & a_4 \\ 0 & 0 & a_5^2+a_6^2 & 0 & 0 & 0 \\ 0 & 0 & \alpha a_1a_6-a_3a_6-a_1a_5 & a_5 & a_6 & 0 \\ 0 & 0 & \alpha a_1a_5-a_3a_5+a_1a_6 & -a_6 & a_5 & 0 \\ 0 & 0 & a_7 & 0 & 0 & a_5^2+a_6^2 \end{pmatrix}$
$N_{6,38}$	$f_{45}^3=1, f_{13}^3=1, f_{14}^4=1$ $, f_{23}^3=1, f_{25}^5=1, f_{12}^6=1$	$\begin{pmatrix} 1 & 0 & a_1a_4+a_3 & a_1 & 0 & a_2 \\ 0 & 1 & a_3 & 0 & a_4 & a_5 \\ 0 & 0 & a_6a_7 & 0 & 0 & 0 \\ 0 & 0 & a_4a_6 & a_6 & 0 & 0 \\ 0 & 0 & -a_1a_7 & 0 & a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
$N_{6,39}$	$f_{45}^3=1, f_{14}^5=1, f_{15}^4=-1, f_{23}^3=2$ $, f_{24}^4=1, f_{25}^5=1, f_{12}^6=1$	$\begin{pmatrix} 1 & 0 & -\frac{a_3^2+a_4^2}{2} & -a_4 & a_3 & a_1 \\ 0 & 1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 0 & a_6^2+a_7^2 & 0 & 0 & 0 \\ 0 & 0 & a_4a_7+a_3a_6 & a_7 & -a_6 & 0 \\ 0 & 0 & a_4a_6-a_3a_7 & a_6 & a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$
$N_{6,40}$	$f_{45}^3=1, f_{14}^5=1, f_{15}^4=-1$ $, f_{26}^6=1, f_{12}^3=1$	$\begin{pmatrix} 1 & 0 & a_1 & a_2 & a_3 & 0 \\ 0 & 1 & a_4 & 0 & 0 & a_5 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & a_3a_6-a_2r & r & -a_6 & 0 \\ 0 & 0 & -a_3r-a_2a_6 & a_6 & r & 0 \\ 0 & 0 & 0 & 0 & 0 & a_7 \end{pmatrix}$ $r = Rootof(z^2+a_6^2-1)$

TABLE 3: The automorphism groups of six dimensional nilpotent real Lie algebras

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$A_{6,1}$	$f_{12}^3=1, f_{13}^4=1, f_{15}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & a_1a_7 & a_1a_8 & 0 & a_1a_{10} \\ 0 & 0 & 0 & a_1^2a_7 & 0 & 0 \\ 0 & 0 & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & 0 & 0 & a_1a_{12} & 0 & a_1a_{14} \end{pmatrix}$
$A_{6,2}$	$f_{12}^3=1, f_{13}^4=1, f_{14}^5=1, f_{15}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & a_1a_7 & a_1a_8 & a_1a_9 & a_1a_{10} \\ 0 & 0 & 0 & a_1^2a_7 & a_1^2a_8 & a_1^2a_9 \\ 0 & 0 & 0 & 0 & a_1^3a_7 & a_1^3a_8 \\ 0 & 0 & 0 & 0 & 0 & a_1^4a_7 \end{pmatrix}$
$A_{6,3}$	$f_{12}^6=1, f_{13}^4=1, f_{23}^5=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ 0 & 0 & 0 & a_1a_{15}-a_3a_{13} & a_2a_{15}-a_3a_{14} & a_1a_{14}-a_2a_{13} \\ 0 & 0 & 0 & a_7a_{15}-a_9a_{13} & a_8a_{15}-a_9a_{14} & a_7a_{14}-a_8a_{13} \\ 0 & 0 & 0 & a_1a_9-a_3a_7 & a_2a_9-a_3a_8 & a_1a_8-a_2a_7 \end{pmatrix}$
$A_{6,4}$	$f_{12}^5=1, f_{13}^6=1, f_{24}^6=1$	$\begin{pmatrix} a_1 & -\frac{a_1a_{14}}{a_{15}} & a_2 & a_3 & a_4 & a_5 \\ -\frac{a_1a_{11}}{a_{15}} & \frac{a_1a_{10}}{a_{15}} & a_6 & a_7 & a_8 & a_9 \\ 0 & 0 & a_{10} & a_{11} & a_{12} & a_{13} \\ 0 & 0 & a_{14} & a_{15} & a_{16} & a_{17} \\ 0 & 0 & 0 & 0 & -\frac{a_1^2(a_{11}a_{14}-a_{10}a_{15})}{a_{15}^2} & \frac{a_1(a_2a_{11}+a_6a_{15}-a_3a_{10}-a_7a_{14})}{a_{15}} \\ 0 & 0 & 0 & 0 & 0 & -\frac{a_1(a_{11}a_{14}-a_{10}a_{15})}{a_{15}} \end{pmatrix}$
$A_{6,5}^a$ $a\neq 0$	$f_{13}^5=1, f_{14}^6=1, f_{23}^6=a, f_{24}^5=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ aa_2 & a_1 & a_4 & aa_3 & a_7 & a_8 \\ aa_{12} & a_{11} & a_{14} & aa_{13} & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & 0 & 0 & 0 & aa_2a_{13}+a_1a_{14}-a_4a_{11}-aa_3a_{12} & a(a_2a_{14}+a_1a_{13}-a_4a_{12}-a_3a_{11}) \\ 0 & 0 & 0 & 0 & a_2a_{14}+a_1a_{13}-a_4a_{12}-a_3a_{11} & aa_2a_{13}+a_1a_{14}-a_4a_{11}-aa_3a_{12} \end{pmatrix}$

TABLE 3. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$A_{6,6}$	$f_{12}^6=1, f_{13}^4=1, f_{14}^5=1, f_{23}^5=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_1^2 & 0 & a_7 & a_8 & a_9 \\ 0 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & 0 & 0 & a_1a_{11} & a_2a_{11}+a_1a_{12}-a_3a_{10} & a_1a_{10} \\ 0 & 0 & 0 & 0 & a_1^2a_{11} & 0 \\ 0 & 0 & 0 & 0 & a_1a_7-a_1^2a_3 & a_1^3 \end{pmatrix}$
$A_{6,7}$	$f_{13}^4=1, f_{14}^5=1, f_{23}^6=1$	$\begin{pmatrix} a_1 & a_2 & 0 & a_3 & a_4 & a_5 \\ 0 & a_6 & 0 & 0 & a_7 & a_8 \\ 0 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 & a_1a_{10} & a_1a_{11} & a_2a_{10} \\ 0 & 0 & 0 & 0 & a_1^2a_{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_6a_{10} \end{pmatrix}$
$A_{6,8}$	$f_{12}^3=1, f_{12}^5=1, f_{13}^4=1, f_{25}^6=1$	$\begin{pmatrix} a_1 & 0 & a_2 & a_3 & a_4 & a_5 \\ 0 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & a_1a_6 & a_1a_7-a_{11} & 0 & -a_4a_6-a_{12} \\ 0 & 0 & 0 & a_1^2a_6 & 0 & 0 \\ 0 & 0 & 0 & a_{11} & a_1a_6 & a_{12} \\ 0 & 0 & 0 & 0 & 0 & a_1a_6^2 \end{pmatrix}$
$A_{6,9}$	$f_{12}^3=1, f_{13}^4=1, f_{15}^6=1, f_{23}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & a_1a_7 & a_1a_8 & 0 & a_2a_8+a_1a_{10}-a_3a_7 \\ 0 & 0 & 0 & a_1^2a_7 & 0 & a_1a_2a_7 \\ 0 & 0 & 0 & a_{12} & a_7^2 & a_{13} \\ 0 & 0 & 0 & 0 & 0 & a_1a_7^2 \end{pmatrix}$
$A_{6,10}^a$ $a\neq 0$	$f_{12}^3=1, f_{13}^5=1, f_{14}^6=1$ $, f_{23}^6=a, f_{24}^5=1$	$\begin{pmatrix} a_6 & a_1 & a_2 & a_3 & a_4 & a_5 \\ aa_1 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & a_6^2-aa_1^2 & 0 & a_1a_8+a_6a_7-a_3a_6-aa_1a_2 & a_6a_8+aa_1a_7-aa_2a_6-aa_1a_3 \\ 0 & 0 & 0 & a_6^2-aa_1^2 & a_{11} & a_{12} \\ 0 & 0 & 0 & 0 & a_6(a_6^2-aa_1^2) & aa_1(a_6^2-aa_1^2) \\ 0 & 0 & 0 & 0 & a_1(a_6^2-aa_1^2) & a_6(a_6^2-aa_1^2) \end{pmatrix}$
$A_{6,11}$	$f_{12}^3=1, f_{13}^4=1, f_{14}^5=1, f_{23}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & a_1a_7 & a_1a_8 & a_1a_9 & a_2a_8-a_3a_7 \\ 0 & 0 & 0 & a_1^2a_7 & a_1^2a_8 & a_1a_2a_7 \\ 0 & 0 & 0 & 0 & a_1^3a_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1a_7^2 \end{pmatrix}$

TABLE 3. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$A_{6,12}$	$f_{13}^4=1, f_{14}^6=1, f_{25}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & 0 & \frac{a_5a_7-a_2a_8}{a_1} & a_8 & a_9 \\ 0 & 0 & \frac{a_7a_{13}-a_8a_{12}}{a_1^2} & a_{10} & 0 & a_{11} \\ 0 & 0 & 0 & \frac{a_7a_{13}-a_8a_{12}}{a_1} & 0 & a_1a_{10} \\ 0 & a_{12} & 0 & \frac{a_5a_{12}-a_2a_{13}}{a_1} & a_{13} & a_{14} \\ 0 & 0 & 0 & 0 & 0 & a_7a_{13}-a_8a_{12} \end{pmatrix}$
$A_{6,13}$	$f_{12}^5=1, f_{13}^4=1, f_{14}^6=1, f_{25}^6=1$	$\begin{pmatrix} \frac{a_6^2}{a_{10}} & a_1 & a_2 & a_3 & a_4 & a_5 \\ 0 & a_6 & -\frac{a_1a_{10}}{a_6} & a_7 & a_8 & a_9 \\ 0 & 0 & a_{10} & a_{11} & 0 & a_{12} \\ 0 & 0 & 0 & a_6^2 & 0 & \frac{a_{11}a_6^2}{a_{10}} \\ 0 & 0 & 0 & -a_1a_6 & \frac{a_6^3}{a_{10}} & \frac{a_6^2a_7+a_1a_8a_{10}-a_4a_6a_{10}}{a_{10}} \\ 0 & 0 & 0 & 0 & 0 & \frac{a_6^4}{a_{10}} \end{pmatrix}$
$A_{6,14}^a$ $a\neq 0$	$f_{13}^4=1, f_{14}^6=1, f_{23}^5=1, f_{25}^6=a$	$\begin{pmatrix} a_5 & a_1 & 0 & a_2 & a_3 & a_4 \\ -aa_1 & a_5 & 0 & -\frac{a(a_1a_2+a_1a_6-a_3a_5)}{a_5} & a_6 & a_7 \\ 0 & 0 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & 0 & a_5a_8 & a_1a_8 & a_5a_9+aa_1a_{10} \\ 0 & 0 & 0 & -aa_1a_8 & a_5a_8 & a(a_5a_{10}-a_1a_9) \\ 0 & 0 & 0 & 0 & 0 & a_5^2a_8+aa_1^2a_8 \end{pmatrix}$
$A_{6,15}$	$f_{12}^3=1, f_{12}^5=1, f_{13}^4=1$ $, f_{14}^6=1, f_{25}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_1^2 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & a_1^3 & a_1a_7+a_1^2a_2 & 0 & a_1a_8+a_2a_9-a_1^2a_5-a_{11} \\ 0 & 0 & 0 & a_1^4 & 0 & a_1^2a_7+a_1^3a_2 \\ 0 & 0 & 0 & -a_1^2a_2 & a_1^3 & a_{11} \\ 0 & 0 & 0 & 0 & 0 & a_1^5 \end{pmatrix}$
$A_{6,16}$	$f_{13}^4=1, f_{14}^5=1, f_{15}^6=1$ $, f_{23}^5=1, f_{24}^6=1$	$\begin{pmatrix} a_1 & 0 & a_2 & a_3 & a_4 & a_5 \\ 0 & a_1^2 & 0 & a_1a_2 & a_1a_3 & a_6 \\ 0 & 0 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & 0 & a_1a_7 & a_1a_8 & a_1a_9 \\ 0 & 0 & 0 & 0 & a_1^2a_7 & a_1^2a_8 \\ 0 & 0 & 0 & 0 & 0 & a_1^3a_7 \end{pmatrix}$
$A_{6,17}$	$f_{12}^3=1, f_{13}^4=1, f_{14}^6=1, f_{25}^6=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ 0 & 0 & a_1a_7 & a_1a_8 & 0 & a_1a_9+a_2a_{10}-a_5a_7 \\ 0 & 0 & 0 & a_1^2a_7 & 0 & a_1^2a_8 \\ 0 & 0 & 0 & -a_1^2a_2 & a_1^3 & a_{12} \\ 0 & 0 & 0 & 0 & 0 & a_1^3a_7 \end{pmatrix}$

TABLE 3. (Continued)

Lie Algebra	Non Vanishing Structure Constants	Automorphism group
$A^a_{6,18}$ $a \neq 0$	$f^3_{12}=1, f^4_{13}=1, f^6_{14}=1$ $, f^5_{23}=1, f^6_{25}=a$	$\begin{pmatrix} a_6 & a_1 & a_2 & a_3 & a_4 & a_5 \\ -aa_1 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & a_6^2+aa_1^2 & a_6a_7+aa_1a_2 & a_1a_7-a_2a_6 & aa_1a_3+aa_1a_9+a_6a_8-aa_4a_6 \\ 0 & 0 & 0 & a_6(a_6^2+aa_1^2) & a_1(a_6^2+aa_1^2) & a_7(a_6^2+aa_1^2) \\ 0 & 0 & 0 & -aa_1(a_6^2+aa_1^2) & a_6(a_6^2+aa_1^2) & -aa_2(a_6^2+aa_1^2) \\ 0 & 0 & 0 & 0 & 0 & (a_6^2+aa_1^2)^2 \end{pmatrix}$
$A_{6,19}$	$f^3_{12}=1, f^4_{13}=1, f^5_{14}=1$ $, f^6_{15}=1, f^6_{23}=1$	$\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 0 & a_1^3 & a_7 & a_8 & a_9 & a_{10} \\ 0 & 0 & a_1^4 & a_1a_7 & a_1a_8 & a_2a_7+a_1a_9-a_1^3a_3 \\ 0 & 0 & 0 & a_1^5 & a_1^2a_7 & a_1^4a_2+a_1^2a_8 \\ 0 & 0 & 0 & 0 & a_1^6 & a_1^3a_7 \\ 0 & 0 & 0 & 0 & 0 & a_1^7 \end{pmatrix}$
$A_{6,20}$	$f^3_{12}=1, f^4_{13}=1, f^5_{14}=1$ $, f^6_{15}=1, f^5_{23}=1, f^6_{24}=1$	$\begin{pmatrix} a_1 & 0 & a_2 & a_3 & a_4 & a_5 \\ 0 & a_1^2 & a_6 & a_7 & a_8 & a_9 \\ 0 & 0 & a_1^3 & a_1a_6 & a_1a_7-a_1^2a_2 & a_1a_8-a_1^2a_3 \\ 0 & 0 & 0 & a_1^4 & a_1^2a_6 & a_1^2a_7-a_1^3a_2 \\ 0 & 0 & 0 & 0 & a_1^5 & a_1^3a_6 \\ 0 & 0 & 0 & 0 & 0 & a_1^6 \end{pmatrix}$
$A_{6,21}$	$f^3_{12}=1, f^6_{15}=1, f^4_{23}=1$ $, f^5_{24}=1, f^6_{34}=1$	$\begin{pmatrix} a_1 & 0 & a_2 & -\frac{a_2^2}{2a_1} & a_3 & a_4 \\ 0 & a_5 & a_6 & a_7 & a_8 & a_9 \\ 0 & 0 & a_1a_5 & -a_2a_5 & \frac{a_2^2a_5}{2a_1} & \frac{2a_1^2a_8+2a_1a_2a_7+a_2^2a_6}{2a_1} \\ 0 & 0 & 0 & a_1a_5^2 & -a_2a_5^2 & -a_1a_5a_7-a_2a_5a_6 \\ 0 & 0 & 0 & 0 & a_1a_5^3 & a_1a_6a_5^2 \\ 0 & 0 & 0 & 0 & 0 & a_1^2a_5^3 \end{pmatrix}$
$A_{6,22}$	$f^3_{12}=1, f^5_{13}=1, f^6_{15}=1$ $, f^4_{23}=1, f^5_{24}=1, f^6_{34}=1$	$\begin{pmatrix} a_4^2 & 0 & a_1 & -\frac{a_1^2}{2a_4^2} & a_2 & a_3 \\ 0 & a_4 & a_5 & a_6 & a_7 & a_8 \\ 0 & 0 & a_4^3 & -a_1a_4 & \frac{a_1^2+2a_4^3a_5}{2a_4} & \frac{a_1^2a_5+2a_1a_6a_4^2+2a_4^4a_7}{2a_4^2} \\ 0 & 0 & 0 & a_4^4 & -a_1a_4^2 & -a_1a_4a_5-a_4^3a_6 \\ 0 & 0 & 0 & 0 & a_4^5 & a_4^4a_5 \\ 0 & 0 & 0 & 0 & 0 & a_4^7 \end{pmatrix}$

TABLE 4: ad-invariant metric on all six dimensional solvable real Lie algebras

Lie Algebra	Non Vanishing Structure Constants	ad-invariant metric
$g_{6,23}$ $\alpha=0,\epsilon=0$ $,h=0$	$f_{23}^{1}=1, f_{26}^{3}=1, f_{36}^{4}=1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & g_2 \\ 0 & g_1 & 0 & g_2 & g_3 & g_4 \\ 0 & 0 & -g_2 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 & 0 \\ 0 & g_3 & 0 & 0 & g_5 & g_6 \\ g_2 & g_4 & 0 & 0 & g_6 & g_7 \end{pmatrix}$
$g_{6,82}$ $\alpha=0$	$f_{24}^{1}=1, f_{35}^{1}=1, f_{26}^{2}=\lambda, f_{36}^{3}=\lambda_1$ $, f_{46}^{4}=-\lambda, f_{56}^{5}=-\lambda_1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\lambda_1 g_1 \\ 0 & 0 & 0 & \frac{\lambda_1 g_1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & 0 \\ 0 & \frac{\lambda_1 g_1}{\lambda} & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ -\lambda_1 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,83}$ $\alpha=0$	$f_{24}^{1}=1, f_{35}^{1}=1, f_{26}^{2}=\lambda, f_{36}^{3}=\lambda$ $, f_{46}^{4}=-\lambda, f_{56}^{4}=-1, f_{56}^{5}=-\lambda, f_{26}^{3}=1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \lambda^2 g_1 \\ 0 & 0 & 0 & -\lambda g_1 & g_1 & 0 \\ 0 & 0 & 0 & 0 & -\lambda g_1 & 0 \\ 0 & -\lambda g_1 & 0 & 0 & 0 & 0 \\ 0 & g_1 & -\lambda g_1 & 0 & 0 & 0 \\ \lambda^2 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,88}$ $\alpha=0$	$f_{24}^{1}=1, f_{35}^{1}=1, f_{26}^{2}=\mu_0$ $, f_{26}^{3}=\nu_0, f_{36}^{2}=-\nu_0, f_{36}^{3}=+\mu_0$ $, f_{46}^{4}=-\mu_0, f_{46}^{5}=\nu_0, f_{56}^{4}=-\nu_0, f_{56}^{5}=-\mu_0$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{g_1(\mu_0^2+\nu_0^2)}{\mu_0} \\ 0 & 0 & 0 & g_1 & -\frac{g_1\nu_0}{\mu_0} & 0 \\ 0 & 0 & 0 & \frac{g_1\nu_0}{\mu_0} & g_1 & 0 \\ 0 & g_1 & \frac{g_1\nu_0}{\mu_0} & 0 & 0 & 0 \\ 0 & -\frac{g_1\nu_0}{\mu_0} & g_1 & 0 & 0 & 0 \\ -\frac{g_1(\mu_0^2+\nu_0^2)}{\mu_0} & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,89}$ $\alpha=0$	$f_{24}^{1}=1, f_{35}^{1}=1, f_{26}^{2}=s, f_{36}^{5}=\nu_0$ $, f_{46}^{4}=-s, f_{56}^{3}=-\nu_0$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\nu_0 g_1 \\ 0 & 0 & 0 & \frac{\nu_0 g_1}{s} & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & \frac{\nu_0 g_1}{s} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & 0 \\ -\nu_0 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$

TABLE 4. (Continued)

Lie Algebra	Non Vanishing Structure Constants	ad-invariant metric
$g_{6,90}$ $\alpha = 0$ $\nu_0 \neq 1$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^4 = 1, f_{36}^5 = \nu_0$ $, f_{46}^2 = 1, f_{56}^3 = -\nu_0$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\nu_0 g_1 \\ 0 & -\nu_0 g_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \nu_0 g_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & 0 \\ -\nu_0 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,91}$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^4 = 1, f_{36}^5 = 1$ $, f_{46}^2 = 1, f_{56}^3 = -1$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -g_1 \\ 0 & -g_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & 0 \\ -g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,92}$ $\alpha = 0$	$f_{24}^1 = 1, f_{35}^1 = 1, f_{26}^3 = \nu_0, f_{36}^2 = -\mu_0$ $, f_{46}^5 = \mu_0, f_{56}^4 = -\nu_0$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -\nu_0 g_1 \\ 0 & 0 & 0 & 0 & -\frac{\nu_0 g_1}{\mu_0} & 0 \\ 0 & 0 & 0 & g_1 & 0 & 0 \\ 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & -\frac{\nu_0 g_1}{\mu_0} & 0 & 0 & 0 & 0 \\ -\nu_0 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$g_{6,92}^*$ $p = 0$	$f_{24}^5 = 1, f_{13}^5 = 1, f_{16}^3 = 1$ $, f_{26}^4 = 1, f_{36}^1 = -1, f_{46}^2 = -1$	$\begin{pmatrix} -g_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_1 \\ 0 & 0 & 0 & 0 & g_1 & g_2 \end{pmatrix}$
$g_{6,93}$ $\alpha = 0$	$f_{24}^1 = 1, f_{35}^1 = 1$ $, f_{26}^4 = 1, f_{26}^5 = \nu_0$ $, f_{36}^4 = \nu_0, f_{46}^2 = 1$ $, f_{46}^3 = -\nu_0, f_{56}^2 = -\nu_0$	$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \nu_0^2 g_1 \\ 0 & 0 & -\nu_0 g_1 & 0 & 0 & 0 \\ 0 & -\nu_0 g_1 & -g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\nu_0 g_1 & 0 \\ 0 & 0 & 0 & -\nu_0 g_1 & g_1 & 0 \\ \nu_0^2 g_1 & 0 & 0 & 0 & 0 & g_2 \end{pmatrix}$
$A_{6,3}$	$f_{12}^6 = 1, f_{13}^4 = 1, f_{23}^5 = 1$	$\begin{pmatrix} g_1 & g_2 & g_3 & 0 & g_7 & 0 \\ g_2 & g_4 & g_5 & -g_7 & 0 & 0 \\ g_3 & g_5 & g_6 & 0 & 0 & g_7 \\ 0 & -g_7 & 0 & 0 & 0 & 0 \\ g_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_7 & 0 & 0 & 0 \end{pmatrix}$

A Appendix

We give the automorphisms group for $g_{6,92}(\alpha \neq 0)$, $g_{6,92}(\alpha = 0)$,and $g_{6,93}(\alpha \neq 0)$ Lie Algebras.

$g_{6,92}(\alpha \neq 0)$

$$\begin{pmatrix}
\frac{-a_2a_3a_5^2\nu_0^2-a_2a_3a_4^2\mu_0\nu_0+a_1a_5^3\mu_0\nu_0+a_1a_5a_4^2\mu_0^2}{a_5^2\mu_0\nu_0} & 0 & 0 & 0 & 0 & 0 \\
\frac{2(2a_2a_4a_7\mu_0\nu_0+2a_2a_5a_8\mu_0\nu_0-2a_1a_4a_9\mu_0^2-2a_1a_5a_{10}\mu_0\nu_0+a_2a_5a_7\alpha\nu_0-a_2a_4a_8\alpha\mu_0-a_1a_5a_9\alpha\mu_0+a_1a_4a_{10}\alpha\mu_0)}{a_5\mu_0(\alpha^2+4\mu_0\nu_0)} & a_1 & -\frac{a_1a_4}{a_5} & \frac{a_2\nu_0}{\mu_0} & -\frac{a_2a_4}{a_5} & 0 \\
\frac{2(a_2a_4a_7\alpha\nu_0+a_2a_5a_8\alpha\nu_0-a_1a_4a_9\alpha\mu_0-a_1a_5a_{10}\alpha\nu_0-2a_2a_5a_7\alpha\nu_0^2+2a_2a_4a_8\mu_0\nu_0+2a_1a_5a_9\mu_0\nu_0-2a_1a_4a_{10}\mu_0\nu_0)}{a_5\nu_0(\alpha^2+4\mu_0\nu_0)} & \frac{\mu_0a_1a_4}{\nu_0a_5} & a_1 & \frac{a_2a_4}{a_5} & a_2 & 0 \\
\frac{2(a_5^2a_7\alpha\nu_0-a_4a_5a_8\alpha\mu_0-a_3a_5a_9\alpha\nu_0+a_3a_4a_{10}\alpha\nu_0+2a_4a_5a_7\mu_0\nu_0+2a_5^2a_8\mu_0\nu_0-2a_3a_4a_9\mu_0\nu_0-2a_3a_5a_{10}\nu_0^2)}{a_5\nu_0(\alpha^2+4\mu_0\nu_0)} & a_3 & -\frac{a_3a_4}{a_5} & a_5 & -\frac{\mu_0a_4}{\nu_0} & 0 \\
\frac{2(-2a_5^2a_7\mu_0\nu_0+2a_4a_5a_8\mu_0^2+2a_3a_5a_9\mu_0\nu_0-2a_3a_4a_{10}\mu_0\nu_0+a_4a_5a_7\alpha\mu_0+a_5^2a_8\mu_0\alpha-a_3a_4a_9\mu_0\alpha-a_3a_5a_{10}\nu_0\alpha)}{a_5\mu_0(\alpha^2+4\mu_0\nu_0)} & \frac{a_3a_4}{a_5} & \frac{a_3\nu_0}{\mu_0} & a_4 & a_5 & 0 \\
a_6 & a_7 & a_8 & a_9 & a_{10} & 1
\end{pmatrix}$$

$g_{6,92}(\alpha = 0)$

$$\begin{pmatrix}
\frac{a_2(a_3^2\nu_0+a_6^2\mu_0)}{a_3\nu_0} & 0 & 0 & 0 & 0 & 0 \\
a_1 & a_2 & -\frac{a_2a_6}{a_3} & 0 & 0 & 0 \\
\frac{a_2a_9a_6^2\mu_0+a_1a_3a_6\mu_0\nu_0+a_2a_9a_3^2\nu_0}{a_3^2\nu_0^2} & \frac{\mu_0a_2a_6}{\nu_0a_3} & a_2 & 0 & 0 & 0 \\
A & \frac{a_5\mu_0}{\nu_0} & -\frac{\mu_0a_5a_6}{\nu_0a_3} & a_3 & -\frac{\mu_0a_6}{\nu_0} & 0 \\
a_4 & \frac{a_5a_6\mu_0}{a_3\nu_0} & a_5 & a_6 & a_3 & 0 \\
a_7 & \frac{\mu_0(a_2a_6a_8a_3^2\nu_0+a_2a_5a_9a_3^2\nu_0+a_2a_5a_9a_6^2\mu_0+a_1a_3a_5a_6\mu_0\nu_0-a_2a_4a_3^2\nu_0^2)}{a_3^3\nu_0^2a_2} & a_8 & a_9 & -\frac{\mu_0(a_2a_6a_9+\nu_0a_1a_3)}{a_2a_3\nu_0} & 1
\end{pmatrix}$$

$$\text{A}=\frac{a_2a_8a_3^4\nu_0^2+a_2a_5a_6a_9a_3^2\mu_0\nu_0+a_1a_5a_3^3\mu_0\nu_0^2+a_2a_5a_9a_6^3\mu_0^2+a_1a_3a_5a_6^2\mu_0^2\nu_0+a_2a_8a_3^2a_6^2\mu_0\nu_0-a_2a_4a_6a_3^2\mu_0\nu_0^2}{a_3^3a_2\nu_0^3}$$

$g_{6,93}(\alpha \neq 0)$

$$\begin{pmatrix} A & 0 & 0 & 0 & 0 & 0 \\ B & r & \nu_0(a_3-r) & a_1 & \nu_0(a_1+a_2) & 0 \\ C & \nu_0(r-a_3) & a_3 & \nu_0(a_1+a_2) & a_2 & 0 \\ D & a_1 & -\nu_0(a_1+a_2) & r & \nu_0(r-a_3) & 0 \\ E & -\nu_0(a_1+a_2) & a_2 & \nu_0(a_3-r) & a_3 & 0 \\ F & a_5 & a_6 & a_7 & a_8 & 1 \end{pmatrix}$$

A=$2a_1a_2\nu_0^2+2a_2^2\nu_0^2-a_2^2-2a_3^2\nu_0^2+2r\nu_0^2a_3+a_3^2$

B=$2(\alpha^3a_1a_5+\alpha^3\nu_0a_2a_6-\alpha^3ra_7+2\alpha^2ra_5+8\nu_0^4a_3a_5+8\nu_0^3a_3a_6+8\nu_0^4a_1a_7+8\nu_0^4a_2a_7-8\nu_0^3a_2a_8-2\alpha^2a_1a_7-8\nu_0^4ra_5+\alpha^3\nu_0a_1a_6-\alpha^3\nu_0a_3a_8+\alpha^3\nu_0ra_8-4\alpha\nu_0a_2a_6-4\alpha\nu_0^2a_2a_5+4\alpha\nu_0^3a_1a_6+4\alpha\nu_0^3a_2a_6-4\alpha\nu_0^3a_3a_8+2\alpha^2\nu_0^2a_3a_5+2\alpha^2\nu_0^2a_1a_7+2\alpha^2\nu_0^2a_2a_7+2\alpha^2\nu_0a_1a_8-4\alpha\nu_0^2a_3a_7+4\alpha\nu_0a_3a_8+4\alpha\nu_0^3ra_8-2\alpha^2\nu_0^2ra_5+2\alpha^2\nu_0ra_6)/(8\alpha^2\nu_0^2+16\nu_0^4+\alpha^4-4\alpha^2)$

C=$2(\nu_0\alpha^3a_1a_5+\alpha^3\nu_0a_2a_5+\alpha^3\nu_0a_3a_7-\alpha^3\nu_0ra_7-4\alpha\nu_0a_2a_5-4\alpha\nu_0a_3a_7+4\alpha\nu_0^2a_1a_6+8\nu_0^2\alpha a_2a_6-8\nu_0^2\alpha a_3a_8-2\alpha^2\nu_0^2a_3a_6+2\alpha^2\nu_0^2a_2a_8+4\nu_0^3\alpha a_2a_5+2\alpha^2\nu_0^2a_1a_8+4\alpha\nu_0^3a_3a_7+4\alpha\nu_0^3a_1a_5+2\alpha^2\nu_0^2ra_6-4\alpha\nu_0^3ra_7+4\alpha\nu_0^2ra_8+2\alpha^2\nu_0ra_5+\alpha^3a_2a_6-\alpha^3a_3a_8+4\alpha a_3a_8+8\nu_0^4ra_6-8\nu_0^4a_3a_6+8\nu_0^4a_2a_8+8\nu_0^4a_1a_8-4\alpha a_2a_6+8\nu_0^3a_3a_5+8\nu_0^2a_3a_6+8\nu_0^3a_2a_7-8\nu_0^2a_2a_8-2\alpha^2\nu_0a_1a_7)/(8\alpha^2\nu_0^2+16\nu_0^4+\alpha^4-4\alpha^2)$

D=$2(\nu_0\alpha^3a_1a_8+\alpha^3\nu_0a_2a_8-\alpha^3\nu_0a_3a_6+\alpha^3\nu_0ra_6+2\alpha^2\nu_0a_1a_6-2\alpha^2\nu_0^2a_1a_5-2\alpha^2\nu_0^2a_2a_5-4\nu_0^3\alpha a_3a_6-2\nu_0^2\alpha^2a_3a_7+4\alpha\nu_0^2a_2a_7+4\alpha\nu_0^2a_3a_5+4\nu_0^3\alpha a_1a_8+4\nu_0^3\alpha a_2a_8-4\alpha\nu_0a_2a_8+4\alpha\nu_0a_3a_6+2\alpha^2\nu_0ra_8+2\alpha^2\nu_0^2ra_7+4\alpha\nu_0^3ra_6+\alpha^3ra_5-\alpha^3a_1a_7-8\nu_0^4a_3a_7-8\nu_0^4a_2a_5-8\nu_0^3a_2a_6+8\nu_0^3a_3a_8+2\alpha^2a_1a_5-8\nu_0^4a_1a_5-2\alpha^2ra_7+8\nu_0^4ra_7)/(8\alpha^2\nu_0^2+16\nu_0^4+\alpha^4-4\alpha^2)$

E=$2(\nu_0\alpha^3a_3a_5+\alpha^3\nu_0a_1a_7+\alpha^3\nu_0a_2a_7-\alpha^3\nu_0ra_5+4\alpha\nu_0^3a_3a_5-4\alpha\nu_0^2a_1a_8-4\alpha\nu_0a_3a_5+4\alpha\nu_0^3a_1a_7+4\alpha\nu_0^3a_2a_7-4\alpha\nu_0a_2a_7-2\alpha^2\nu_0^2a_2a_6-2\alpha^2\nu_0^2a_1a_6+2\nu_0^2\alpha^2a_3a_8+2\alpha^2\nu_0ra_7-2\alpha^2\nu_0a_1a_5+8\alpha\nu_0^2a_3a_6-8\alpha\nu_0^2a_2a_8-4\alpha\nu_0^3ra_5-2\alpha^2\nu_0^2ra_8-4\alpha\nu_0^2ra_6+\alpha^3a_3a_6-\alpha^3a_2a_8-4\alpha a_3a_6-8\nu_0^4a_1a_6-8\nu_0^4ra_8+8\nu_0^2a_2a_6-8\nu_0^2a_3a_8+4\alpha a_2a_8-8\nu_0^4a_2a_6+8\nu_0^4a_3a_8+8\nu_0^3a_2a_5+8\nu_0^3a_3a_7)/(8\alpha^2\nu_0^2+16\nu_0^4+\alpha^4-4\alpha^2)$

F=a_4

$r = Rootof(z^2+a_2^2-a_1^2-a_3^2)$

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