Adv. Theor. Appl. Mech., Vol. 3, 2010, no. 6, 253 - 261

A New Technique of Laminated Composites Homogenization

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Abstract

The homogenization technique of the laminated composite is presented in this paper. The behaviour law of each layer expressed in the laminated coordination axis is used.
The homogenization technique is based on the partially inversed and reversed behaviour law technique. This allows the determination of the rigidity tensor eventually resulting in the classification of laminate's behaviour.

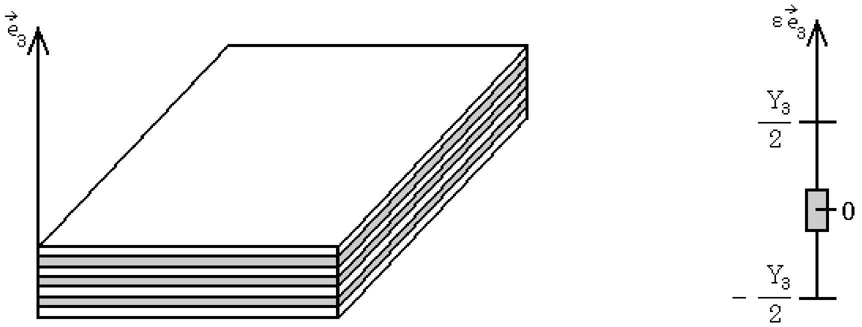
Keywords: Laminated, composites, homogenization, behaviour law, monoclinic

1 Introduction

The variety in composite materials with different shapes are continuously replaced the conventional materials. This reality has caused an increasing interest in the modeling of composites. Several approaches are adopted to predict the behaviour of the composites using developing a model, which can used to determine the rigidity tensor of this type of materials in practical situations **[2]**.
The composite laminates are obtained by superposition of the various layers of the materials with the same or different nature **[3].** Generally, the layers are identical from both the material and the thickness stand points. Each layer is formed by submerging the fibres in the matrix resin material. The layers generally are orthotropic (i.e. with principal properties in orthogonal direction to each layer) or transversely isotropic (with isotropic properties in the transverse plane), with the laminate then exhibiting anisotropy (with variable direction of principal properties), orthotropic, or quasi-isotropic properties.

2 Description of problem

We assume a laminated material composed of several homogenous and elastic layers periodically stacked and perfectly pasted in $\vec{e}_3$ direction (Figure 1, a).



(a) (b)

Figure 1 : (a) Periodic structure laminate, (b) Basic cell y

The modelling will be undertaken on the rectangular prism as a basic element in the $\vec{e}_3$ direction and the other directions (i.e. $\vec{e}_1$ and $\vec{e}_2$) will not be taken into account.
The axes coordination centre is made on the centre of the basic cell charaterizing by the dimensions of $Y = \left]-\frac{1}{2},\frac{1}{2}\right[\times \left]-\frac{1}{2},\frac{1}{2}\right[\times y$ and $y = \left]-\frac{Y_3}{2},\frac{Y_3}{2}\right[$ and having the coordination of (o,y_1,y_2,y_3) **[6][7]** (Figure1, b).
The elasticity coefficients at a microscopic scale A(y) are only the function

of Y_3 and they are periodic on y and constant per part. The local coefficient of the heterogeneous volume in a macroscopic scale, $A^{\varepsilon}(x)$, is given by $A^{\varepsilon}(x) = A\left(x_3/\varepsilon\right)$ and is εy-periodic **[9].** The ratio of the basic cell thickness, Y_3, and the laminate thickness is called as parameter ε and is assumed to be small.
The hooks <.> indicate the average of a size on the one-dimensional basic cell y.
We consider:

$$< f\left(y_3\right) >_Y = \frac{1}{Y_3} \int_{-\frac{Y_3}{2}}^{\frac{Y_3}{2}} f\left(y_3\right) dy_3 \tag{1}$$

The displacement vector u and the stress tensor are the periodic functions with unspecified period in y_1 and y_2. They are only the function of the scalar variable, y_3, considered as the solution for the following equilibrium problem, Eq.(2), written for a one-dimensional (y-dimension) basic cell **[5] [1] [4] [10].**

$$\begin{cases} \dfrac{d}{dy_3}\left(\sigma_{i3}(y_3)\right) = 0 \quad \text{in } Y, \quad i \in \{1,2,3\} & (2-1) \\ \sigma(y_3) = A(y_3)e(y_3) \quad \text{in } Y & (2-2) \\ e(y_3) = E + \dfrac{1}{2}\left[\nabla_y\left(u(y_3)\right) + \left(\nabla_y\left(u(y_3)\right)\right)^T\right] \quad \text{in } Y & (2-3) \\ y_3 \to \sigma(y_3), y_3 \to u(y_3) \quad Y-\text{periodical} & (2-4) \end{cases} \tag{2}$$

where E is the second order symmetrical tensor, which is independent of y.
The fourth order tensor $M(y_3)$ is introduced to relate the stress and E tensors as:

$$\sigma\left(y_3\right) = M\left(y_3\right)E. \tag{3}$$

The homogenous behaviour law is a relationship obtained by making an average over the eq. (3), **[5]**.

$$< \sigma(y_3) >_Y = < M(y_3) >_Y < e(y_3) >_Y \tag{4}$$

The homogenous coefficients Q are given by eq. (5):

$$Q = < M(y_3) >_Y \tag{5}$$

3 Problem solution

In order to solve this problem, in first step, we proposed a partially inversed behaviour law for each layer of laminated material as follows:

$$A = \left\{\sigma_{11}, \sigma_{22}, \sigma_{12}, e_{13}, e_{23}, e_{33}\right\} \tag{6}$$

$$B = \left\{\sigma_{13}, \sigma_{23}, \sigma_{33}, e_{11}, e_{22}, e_{12}\right\}$$

where the components of the tensor A are only the function of y_3 while those of tensor B are independent of y_3. The two tensors, A and B, are related by y_3 -dependent fourth-order tensor (K):

$$A = K : B \tag{7}$$

Making an average over the Eq.(7) results in:

$$< A >_y = < K : B >_y = < K >_y : < B >_y \tag{8}$$

The second step is to re-inverse the resulting partially inversed behaviour law relationship, Eq.(8), which allows to determine the homogenous components corresponding to the stress tensor average ($<\sigma(y_3)>$) and strain tensor average

($<e(y_3)>$) as well:

$$< \sigma(y_3) >_Y = Q < e(y_3) >_Y \tag{9}$$

The procedure is schematically illustrated in the figure 2.

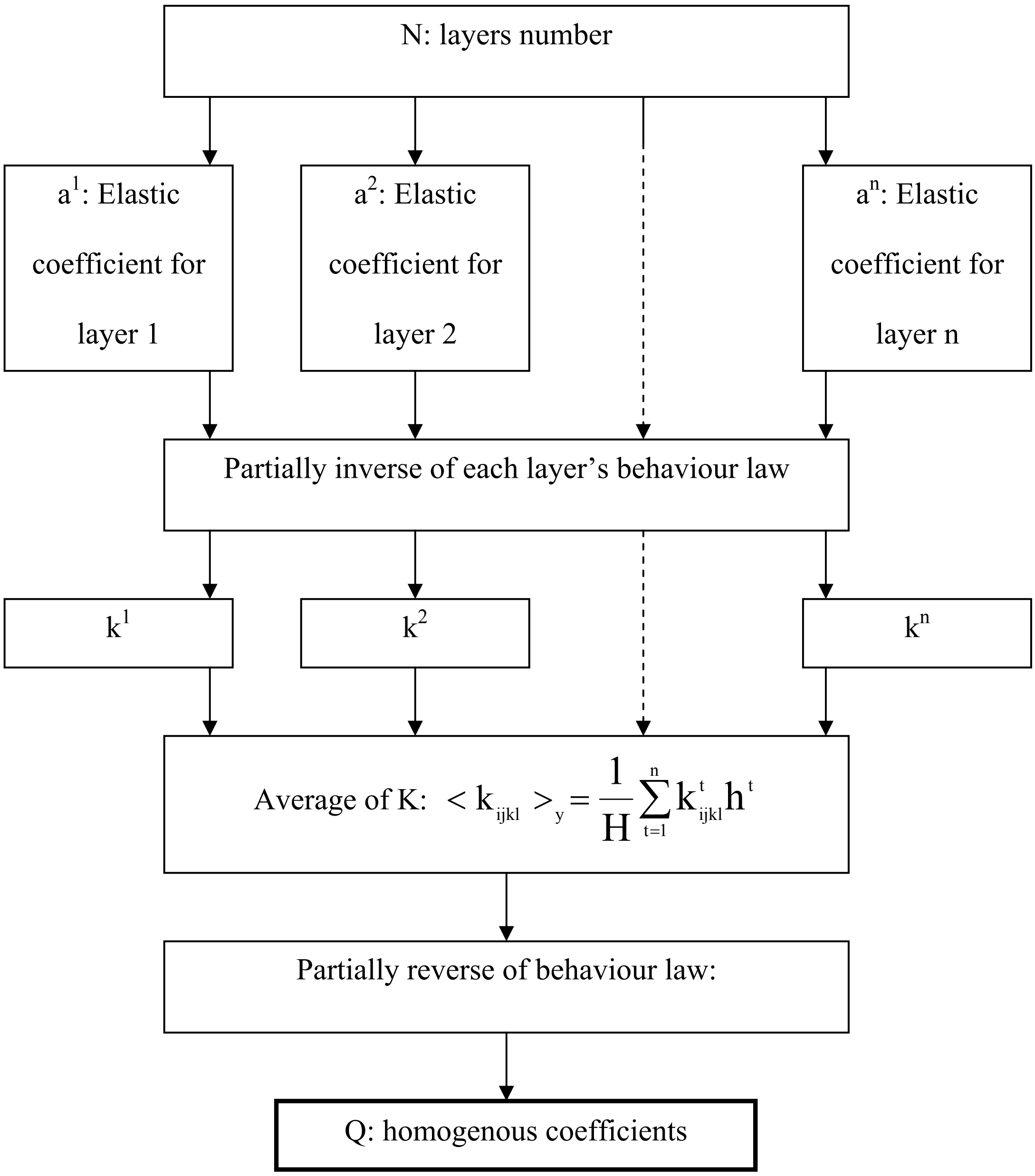


Figure 2: the procedure used to solve the problem

4 Application

We consider a laminated material with periodical structure along the direction $\vec{e}_3$. The layers are assumed elastic, homogeneous, and $\vec{e}_3$ monoclinic **[8]**.
The behaviour law for each layer is written as follows:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} a_{1111} & a_{1122} & a_{1133} & 0 & 0 & a_{1112} \\ & a_{2222} & a_{2233} & 0 & 0 & a_{2212} \\ & & a_{3333} & 0 & 0 & a_{3312} \\ & \text{Sym} & & a_{2323} & a_{2313} & 0 \\ & & & & a_{1313} & 0 \\ & & & & & a_{1212} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{13} \\ 2e_{12} \end{pmatrix} \tag{10}$$

Using the partially inversed behaviour law, Eq.(10), the fourth order tensor K is obtained.

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ e_{13} \\ e_{23} \\ e_{33} \end{pmatrix} = K \begin{pmatrix} \sigma_{13} \\ \sigma_{23} \\ \sigma_{33} \\ e_{11} \\ e_{22} \\ e_{12} \end{pmatrix} \tag{11}$$

where the components of tensor K for each layer exhibited as:

$$K = \begin{pmatrix} k_{1113} & k_{1123} & k_{1133} & k_{1111} & k_{1122} & 2k_{1112} \\ k_{2213} & k_{2223} & k_{2233} & k_{2211} & k_{2222} & 2k_{2212} \\ k_{1213} & k_{1223} & k_{1233} & k_{1211} & k_{1222} & 2k_{1212} \\ 2k_{1313} & 2k_{1323} & 2k_{1333} & 2k_{1311} & 2k_{1322} & 2k_{1312} \\ 2k_{2313} & 2k_{2323} & 2k_{2333} & 2k_{2311} & 2k_{2322} & 2k_{2312} \\ k_{3313} & k_{3323} & k_{3333} & k_{3311} & k_{3322} & 2k_{3312} \end{pmatrix} \tag{12}$$

and are given in Eq.(13)

$$\begin{cases} \left.\begin{aligned} & k_{\alpha\beta\gamma\delta}(y_3) = a_{\alpha\beta\gamma\delta} - \frac{a_{\alpha\beta33}}{a_{3333}} a_{33\gamma\delta} \\ & k_{\alpha\beta33}(y_3) = \frac{a_{\alpha\beta33}}{a_{3333}} \\ & k_{33\alpha\beta}(y_3) = -\frac{a_{33\alpha\beta}}{a_{3333}} \end{aligned}\right\} \text{for } (\alpha,\beta,\gamma,\delta) \in \{1,2\} \\ k_{3333}(y_3) = \frac{1}{a_{3333}} \\ k_{1313}(y_3) = \frac{a_{2323}}{\Delta}, \quad k_{2323}(y_3) = \frac{a_{1313}}{\Delta} \\ k_{1323}(y_3) = k_{2313}(y_3) = -\frac{a_{2313}}{\Delta}, \quad \text{with} \quad \Delta = 4\left(a_{2323}a_{1313} - a_{2313}^2\right) \end{cases} \tag{13}$$

The other components are zero.
Using the relationships (13), the tensor K of each layer becomes (14):

$$K = \begin{pmatrix} 0 & 0 & k_{1133} & k_{1111} & k_{1122} & 2k_{1112} \\ 0 & 0 & k_{2233} & k_{2211} & k_{2222} & 2k_{2212} \\ 0 & 0 & k_{1233} & k_{1211} & k_{1222} & 2k_{1212} \\ 2k_{1313} & 2k_{1323} & 0 & 0 & 0 & 0 \\ 2k_{2313} & 2k_{2323} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{3333} & k_{3311} & k_{3322} & 2k_{3312} \end{pmatrix} \tag{14}$$

Recalling that
$A = K : B \Rightarrow \langle A \rangle = \langle K \rangle : \langle B \rangle = \langle K \rangle : B$ and that the components of tensor B are independent of y_3. Replacing K tensor from Eq.(14) and the average values of A and B will give:

$$\begin{pmatrix} <\sigma_{11}> \\ <\sigma_{22}> \\ <\sigma_{12}> \\ <e_{13}> \\ <e_{23}> \\ <e_{33}> \end{pmatrix} = \begin{pmatrix} 0 & 0 & <k_{1133}> & <k_{1111}> & <k_{1122}> & 2<k_{1112}> \\ 0 & 0 & <k_{2233}> & <k_{2211}> & <k_{2222}> & 2<k_{2212}> \\ 0 & 0 & <k_{1233}> & <k_{1211}> & <k_{1222}> & 2<k_{1212}> \\ 2<k_{1313}> & 2<k_{1323}> & 0 & 0 & 0 & 0 \\ 2<k_{2313}> & 2<k_{2323}> & 0 & 0 & 0 & 0 \\ 0 & 0 & <k_{3333}> & <k_{3311}> & <k_{3322}> & 2<k_{3312}> \end{pmatrix} \begin{pmatrix} <\sigma_{13}> \\ <\sigma_{23}> \\ <\sigma_{33}> \\ <e_{11}> \\ <e_{22}> \\ <e_{12}> \end{pmatrix} \tag{15}$$

The average of each component is calculated as follows:

$$< k_{ijkl} >_y = \frac{1}{H}\sum_{t=1}^{n} k_{ijkl}^{t} h^{t} \qquad (16)$$

Where H is the laminate thickness, n is the number of layer in each laminate, k_{ijkl}^{t} is the fourth order tensor component corresponding to layer t and h^t is the layer thickness.

In order to determine the homogenous behaviour law, we proceed by re-inversing the resulting partially inversed behaviour law relationship, Eq.(15), which allows to determine the fourth order tensor Q indicating the homogenous components:

$$\left.\begin{aligned} q_{\alpha\beta\gamma\delta}(y_3) &= < k_{\alpha\beta\gamma\delta} > - \frac{< k_{\alpha\beta 33} >}{< k_{3333} >} < k_{33\gamma\delta} > \\ q_{\alpha\beta 33}(y_3) &= \frac{< k_{\alpha\beta 33} >}{< k_{3333} >} \\ q_{33\alpha\beta}(y_3) &= -\frac{< k_{33\alpha\beta} >}{< k_{3333} >} \end{aligned}\right\} \quad (\alpha,\beta,\gamma,\delta) \in \{1,2\}$$

$$q_{3333}(y_3) = \frac{1}{< k_{3333} >}$$

$$q_{1313}(y_3) = \frac{< k_{2323} >}{D}, \quad q_{2323}(y_3) = \frac{< k_{1313} >}{D}$$

$$q_{1323}(y_3) = q_{2313}(y_3) = -\frac{< k_{2313} >}{D} \qquad (17)$$

Where $D = 4\left(< k_{2323} >< k_{1313} > - < k_{1323} >< k_{2313} >\right)$

The values of the tensor Q components given in Eq.(17) obviously shows that tensor Q is symmetrical:

$$\begin{pmatrix} < \sigma_{11} > \\ < \sigma_{22} > \\ < \sigma_{33} > \\ < \sigma_{23} > \\ < \sigma_{13} > \\ < \sigma_{12} > \end{pmatrix} = \begin{pmatrix} q_{1111} & q_{1122} & q_{1133} & 0 & 0 & q_{1112} \\ & q_{2222} & q_{2233} & 0 & 0 & q_{2212} \\ & & q_{3333} & 0 & 0 & q_{3312} \\ & \text{Sym} & & q_{2323} & q_{2313} & 0 \\ & & & & q_{1313} & 0 \\ & & & & & q_{1212} \end{pmatrix} \begin{pmatrix} < e_{11} > \\ < e_{22} > \\ < e_{33} > \\ 2 < e_{23} > \\ 2 < e_{13} > \\ 2 < e_{12} > \end{pmatrix} \qquad (18)$$

It is possible to show that the homogenized material is also monoclinic with respect to Y_3-axis.

4 Conclusion

The present approach allowed to determine the behavior law of the laminated material using the behaviour law of each layer and thickness.

The results obtained are in perfect agreement with the literature because the behaviour law of laminate is transversely isotropic if the behaviour law of all layers is isotropic and it is monoclinic if at least one layer is monoclinic.

The present work was concluded by an numerical application using our approach whose permitting the tensor homogenized coefficients from the data of the layer so the coefficient independent of the rigidity tensor and thickness.

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Received: October, 2009