

Magnetotransport and spin dynamics in an electron gas formed at oxide interfaces

Chenglong Jia and Jamal Berakdar

Institut für Physik, Martin-Luther Universität Halle-Wittenberg, 06099 Halle (Saale), Germany

We investigate the spin-dependent transport properties of a two-dimensional electron gas formed at oxides' interface in the presence of a magnetic field. We consider several scenarios for the oxides' properties, including oxides with co-linear or spiral magnetic and ferroelectric order. For spiral multiferroic oxides, the magnetoelectric coupling and the topology of the localized magnetic moments introduce additional, electric field controlled spin-orbit coupling that affects the magneto-oscillation of the current. An interplay of this spin-orbit coupling, the exchange field, and of the applied magnetic field results in a quantum, gate-controlled spin and charge Hall conductance.

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I. INTRODUCTION

The transport properties of a semiconductor-based two-dimensional electron gas (2DEG) by external electric and magnetic fields is at the heart of mesoscopic and spintronic research with a wide range of applications. A new impetus has been the recent discovery of 2DEG formed at the interface of insulating oxides^{1,2}. This 2DEG can be laterally confined and patterned to achieve new functionalities such as oxide-based field effect transistors^{3,4}. In view of the remarkable phenomena observed in the conventional 2DEG under a magnetic field, including the Shubnikov-de Hass (SdH) effect^{5,6}, and the spin/charge Hall effect⁷ in the presence of spin-orbit interactions (SOI)^{8,9}, it is timely to consider the properties of oxide-based 2DEG in a magnetic field. Generally, the relatively large effective mass m^* together with a high carrier concentration N_e at the oxide interfaces (for instance $m^*/m_e \approx 3.2$ with m_e being the free electron mass and $N_e \approx 5 - 9 \times 10^{13} \text{cm}^{-2}$ at the $\text{LaAlO}_3/\text{SrTiO}_3$ interface¹⁰) imply a strong magnetic field for the SdH oscillation and for the quantized Hall conductance to be observable. In addition, oxide interfaces have a multitude of inherent properties such as multiferroicity and strong electronic correlations^{11,12}. It is our aim here to inspect how such properties affect the magnetotransport in 2DEG. One of the exciting results is that, utilizing 2DEG formed at multiferroic oxides (e.g., RMnO_3 ^{11,13}) the magnetotransport can be modulated by both a magnetic field and also with a small transverse electric field ($\sim 1 \text{kV/cm}$). The topology of the local helical magnetic moments in multiferroics results in an induced effective SOI³ that is electrically¹⁴ and/or magnetically¹⁵ tunable. Consequently, we find a large magnetic and electric field-dependence of the quantum oscillations in the longitudinal conductance, as well as a spin/charge Hall conductance. Furthermore, the cases of collinear order and/or weak on-site correlations are also considered.

II. THEORETICAL FORMALISM

We start from the general case by considering a 2DEG formed at the intersurface of a spiral multiferroic oxide (e.g., RMnO_3) with a polar oxide (cf. Fig.1). In a homogeneous, static magnetic field $\mathbf{B} = (0, 0, B_z)$ the single-particle 2DEG Hamiltonian reads

$$H = h_k + h_{ex}, \quad \text{with} \quad (1)$$

$$h_k = \frac{1}{2m^*} [\mathbf{P} + e\mathbf{A}_B]^2, \quad h_{ex} = U \mathbf{n}_r \cdot \boldsymbol{\sigma} + g\mu_B \mathbf{B} \cdot \boldsymbol{\sigma}.$$

Here g is the gyromagnetic constant, μ_B is the Bohr magneton, and $\boldsymbol{\sigma}$ are the Pauli matrices. $U \mathbf{n}_r$ is the exchange field determined by Coulomb repulsion and Hund's rule coupling at the oxides ionic sites, and can be described by an effective unit-vector field \mathbf{n}_r and a strength U . A useful parameterizations is

$$\mathbf{n}_r = (\sin \theta_r, 0, \cos \theta_r)$$

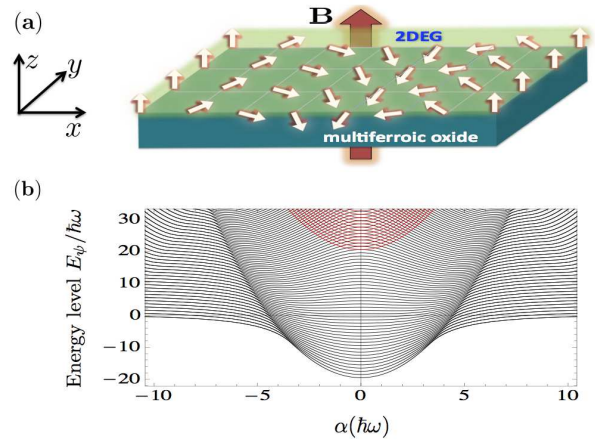


FIG. 1. (a) Schematic view of 2DEG at the interface of a multiferroic oxide with a polar oxide and in the presence of a magnetic field \mathbf{B} . The spiral ($x - z$) plane is perpendicular to 2DEG. (b) The 2DEG energy levels as functions of the scaled spin-orbit interaction α with $\Delta/\hbar\omega = 20$. The spin-down (gray) and the spin-up (black) subbands are separated by the exchange energy.

with

$$\theta_r = \mathbf{q}_m \cdot \mathbf{r}$$

and $\mathbf{q}_m = (q, 0, 0)$ is the spin wave vector of the spiral^{13,14}.

$$\mathbf{A}_B = (-B_z y, 0, 0)$$

in the Landau gauge. The case of a collinear order and/or weak electronic correlations follow as special cases, as discussed below. Upon a unitary gauge transformation

$$U_g = \exp(-i\theta_r \sigma_y / 2)$$

the local quantization axis becomes aligned with \mathbf{n}_r on the expense of introducing the topological vector potential

$$\mathbf{A}_g = -i\hbar U_g^\dagger \nabla U_g = (-\hbar q \sigma_y / 2, 0, 0).$$

The transformed kinetic energy³ reads

$$\tilde{h}_k = \frac{1}{2m^*} [\tilde{\mathbf{P}} + e\mathbf{A}_B + \mathbf{A}_g]^2$$

(hereafter, transformed quantities are marked by a tilde). \mathbf{A}_g depends only on the geometry of the local magnetization at the oxide and acts as a helicity q and momentum-dependent effective SOI $\sim q\tilde{P}_x\tilde{\sigma}_y$. The transformed term \tilde{h}_{ex} turns however diagonal, $\tilde{h}_{ex} = \Delta\tilde{\sigma}_z$ (after omitting a small periodic modulation on the spins¹⁶). The value of Δ depends on the system under study:

i) For a strong local correlation compared to the external Zeeman field we find $\Delta \approx U$. This is the case of spiral multiferroics.

ii) The case of a collinear $\mathbf{n}(\mathbf{r})$ (no ferroelectric order) corresponds to $q \rightarrow 0$, and hence $\mathbf{A}_g \rightarrow 0$.

iii) For a weak local correlation field $U \rightarrow 0$ and/or under a strong magnetic field we find $\Delta \approx g\mu_B B_z$, i.e. the conventional 2DEG in a magnetic field.

The cases *i*-*iii*) correspond to different combinations of the oxides at which interface the 2DEG is formed. We will focus here on *i*) and *ii*). Predictions for the case *iii*) are readily deduced, the physics however in this case resembles the well-known situation in a semi-conductor-based 2DEG (without SOI).

For the transformed system we infer

$$[\tilde{P}_x, \tilde{h}_k + \tilde{h}_{ex}] = 0.$$

Introducing the bosonic operator

$$a = [\tilde{P}_x - eB_z y - i\tilde{P}_y] / \sqrt{2m\hbar\omega} \quad (2)$$

and scaling all energies in unit of the cyclotron energy

$$\hbar\omega = \hbar e B_z / m^*,$$

we obtain the dimensionless harmonic Hamiltonian

$$\bar{H} = (a^\dagger a + 1/2) - \alpha\tilde{\sigma}_y(a^\dagger + a) + \alpha^2 + \bar{\Delta}\tilde{\sigma}_z \quad (3)$$

where

$$\bar{\Delta} = \Delta / \hbar\omega, \quad \alpha = \frac{q l_B}{2\sqrt{2}\bar{a}}, \quad l_B = \sqrt{\hbar / e B_z}.$$

\bar{a} is the lattice constant, and l_B is the magnetic length.

In the limit of a collinear spin order, the eigenstates of the Hamiltonian are the Landau levels,

$$|n, s\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle \otimes |s\rangle$$

with the energy

$$\epsilon_{ns} = (n + 1/2) + s\bar{\Delta} + \alpha^2, \quad n = 0, 1, 2, \dots, \quad s = \pm.$$

However, in most cases, the state $|n, s\rangle$ is coupled to $|n \pm 1, \bar{s}\rangle$ via SOI. The case of a weak exchange (local correlation) field follows for $\bar{\Delta} = 0$ (cf. [16]). Expanding the eigenstate

$$|\psi\rangle = \sum_{n\sigma} C_n^s |n, s\rangle$$

in the Landau space, the secular equation

$$\bar{H}|\psi\rangle = E_\psi|\psi\rangle$$

leads to the following matrix form,

$$E_\psi \begin{bmatrix} \dots & C_{n-1}^{\bar{s}} & C_n^s & C_{n+1}^{\bar{s}} & \dots \end{bmatrix}^T = \quad (4)$$

$$\begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \epsilon_{n-1, \bar{s}} & i\alpha\bar{s}\sqrt{n} & 0 & \dots \\ \dots & i\alpha s\sqrt{n} & \epsilon_{n, s} & i\alpha s\sqrt{n+1} & \dots \\ \dots & 0 & i\alpha\bar{s}\sqrt{n+1} & \epsilon_{n+1, \bar{s}} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} \dots \\ C_{n-1}^{\bar{s}} \\ C_n^s \\ C_{n+1}^{\bar{s}} \\ \dots \end{bmatrix}.$$

The Hamiltonian in the Landau space reduces then to two independent, infinitely dimensional tridiagonal matrices with reference to two groups¹⁷ $|n, (-1)^n\rangle$ and $|n, -(-1)^n\rangle$. The energy spectrum can be obtained numerically by truncating the matrix dimensions while including a sufficient number of Landau levels (up to 1000 energy levels were taken during the calculations of the transport coefficients).

Fig.1b shows the energy levels as a function of the SOI α with $\bar{\Delta} = 20$. Two spin subbands (parallel or antiparallel to the exchange field) are shown. Hence, the electronic structure varies strongly with α which is tunable by a small transverse electric field due to the magnetoelectric coupling¹⁴. The high density of states around $\alpha \approx 5$ follows from a perturbation considerations:

Let SOI be smaller than the exchange splitting, $\alpha \ll \bar{\Delta}$. Up to a second order in α the two energy branches read

$$E_{ns}(\alpha) = \epsilon_{ns} + s \frac{(n+1)\alpha^2}{2\bar{\Delta} - s} + s \frac{n\alpha^2}{2\bar{\Delta} + s}. \quad (5)$$

At critical value

$$\alpha_c = [(4\bar{\Delta}^2 - 1)/4\bar{\Delta}]^{1/2},$$

we find infinitely degenerate states, $E_{n, s=-1}(\alpha_c) \equiv 0$. For large α and/or n , the degeneracy is lifted and distributed around α_c , e.g. $\alpha_c = 4.47$ with a given $\bar{\Delta} = 20$.

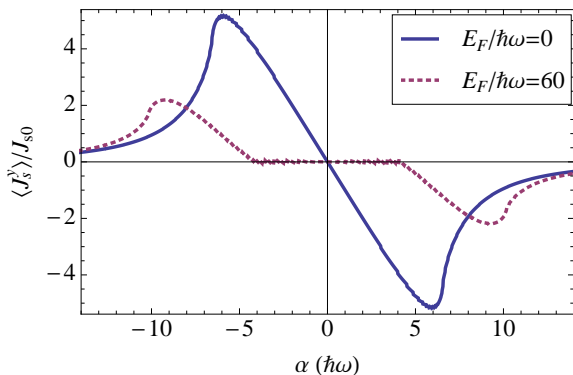


FIG. 2. The persistent spin current $\langle \hat{J}_s^y \rangle$ in units of $J_{s0} = \sqrt{\hbar\omega}/2m$ vs. SOI strength α with $\Delta/\hbar\omega = 43$. Solid line: $E_F < \Delta$, only the spin-down subband intersects the Fermi level. Dotted line: $E_F > \Delta$, both spin subbands are occupied when $|\alpha| \lesssim 4$.

III. PERSISTENT SPIN CURRENT

As pointed out in Ref.[3] without a magnetic field, a nonzero spin current $\langle \hat{J}_s^y \rangle$ is generated when only the spin-down subband is below the Fermi level E_F . With a magnetic field B_z the spin current is

$$\langle \hat{J}_s^y \rangle = 1/\nu \sum_{\psi} \langle \psi | \hat{J}_s^y | \psi \rangle f(E_{\psi}), \quad (6)$$

where ν is the filling factor and f is the Fermi distribution function. The spin current operator \hat{J}_s^y for the y component of the spin is defined as

$$\hat{J}_s^y = \frac{1}{2}(\tilde{\sigma}_y \hat{v}_x + \hat{v}_x \tilde{\sigma}_y), \quad (7)$$

$$\hat{v}_x = \frac{[x, \hat{H}]}{i\hbar} = \sqrt{\frac{\hbar\omega}{2m}}[(a^\dagger + a) - 2\alpha\tilde{\sigma}_y]. \quad (8)$$

Note, $\tilde{\sigma}_y = \sigma_y$ because $[\sigma_y, U_g] \equiv 0$. An overall α -dependence of the amplitude of spin current is related to the electron density through the filling factor (Fig.2). In the case of $E_F/\hbar\omega = 60$, two subbands are both occupied only when $|\alpha| \lesssim 4$. It is then numerically confirmed that the persistent spin current vanishes when two spin subbands are intersected by the Fermi level³. No charge current is generated, i.e. $\langle \hat{J}_{cx} \rangle = \langle -e\hat{v}_x \rangle = 0$ in the absence of an in-plane electric field. The spin polarization vanishes as well $\langle \sigma_y \rangle = 0$ under a normal magnetic field; only a pure, electrically tunable persistent spin current exists.

IV. LONGITUDINAL CONDUCTIVITY

The topological SOI is exactly analogous to the semiconductor-based 2DEG with the Rashba and Dresselhaus spin-orbit couplings being equal in strength.

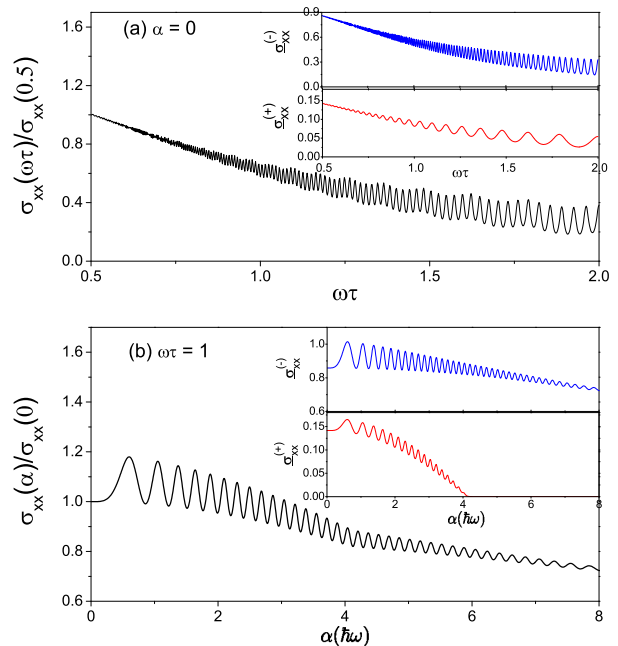


FIG. 3. The longitudinal conductivity σ_{xx} as functions of (a) magnetic field $\omega\tau$ and (b) SOI strength α at a fixed Fermi energy $E_F\tau/\hbar = 60$. The exchange energy is $\Delta\tau/\hbar = 43$. Insets show the contributions from the different subband, $\sigma_{xx}^{(\pm)}$ is re-scaled with respect to the total conductivity σ_{xx} at (a) $\omega\tau = 0.5$ and (b) $\alpha = 0$.

Therefore, the spin beats in the magneto-oscillation at the helimagnetic interface are completely suppressed¹⁸. However, the oscillatory magnetoresistance contains at least two components: SdH oscillations of the spin-down and spin-up subbands, respectively¹⁹. $\Delta \gg 1$ means a large energy separation between the two subbands, $\sim 2\langle \tilde{\sigma}_z \rangle \bar{\Delta}$, which results in negligible off-diagonal terms of the Green function²⁰. The longitudinal conductivity is then given by $\sigma_{xx} = \sigma_{xx}^{(-)} + \sigma_{xx}^{(+)}$ with

$$\sigma_{xx}^{(\pm)} = \frac{m\omega}{4\pi^2} \sum \frac{\langle \psi_{\pm} | \hat{J}_{cx} | \psi'_{\pm} \rangle \langle \psi'_{\pm} | \hat{J}_{cx} | \psi_{\pm} \rangle}{(E_{\psi_{\pm}} - \mu_{\pm} + i\hbar/\tau_{\pm})(E_{\psi'_{\pm}} - \mu_{\pm} + i\hbar/\tau_{\pm})}$$

where $\langle \psi_{\pm} | \hat{J}_{cx} | \psi'_{\pm} \rangle$ is the matrix elements of the charge current density operator. μ_{\pm} is the energy distance between the Fermi energy and the bottom of the spin-up(down) subband. τ_{\pm} is the total relaxation time including the intrasubband and intersubband scattering at a short-range random potential. Experiments on semiconductor-based 2DEG²¹ suggest a Lorentzian shape as an excellent fit for the disorder-induced broadening of Landau levels; we assume this to hold in our case, i.e. $\tau_{\pm} = \tau$. Fig.3a demonstrates the dependence of σ_{xx} on the magnetic field for the following parameters: $E_F\tau/\hbar = 60$, $\Delta\tau/\hbar = 43$, and $\alpha = 0$. The spectrum of σ_{xx} consists of two harmonics: The oscillating part of the partial conductivities $\sigma_{xx}^{(\pm)}$ is described well analytically by $\cos(2\pi\frac{\mu_{\pm}}{\hbar\omega} + \pi)$. The magnitudes are proportional to

the carrier concentrations in the spin-up and spin-down subbands, respectively. On the other hand, assuming a fixed Fermi energy the carrier concentration varies with the electrically tunable spin-orbit interaction parameter α , the quantum oscillation of the longitudinal conductance σ_{xx} is caused then by a crossing of the chemical potential and the energy levels, see also in Fig.3b. The harmonics stem from a change of the energy distance μ_{\pm} , which induces an equivalent frequency of $\sigma_{xx}^{(\pm)}$. As $\alpha > 4$, only the low subband is occupied however, $\sigma_{xx}^{(+)} \rightarrow 0$.

When accounting explicitly for intersubband transitions, the total relaxation time τ_{\pm} reads²²

$$\tau_{-} = \tau \left[1 + \left(1 - \frac{\tau}{\tau'} \right) F_{-} + \frac{\tau}{\tau'} F_{+} \right] \quad (9)$$

$$\tau_{+} = \tau \left[1 + \left(1 - \frac{\tau}{\tau'} \right) F_{+} + \frac{\tau}{\tau'} F_{-} \right] \quad (10)$$

with the times τ and τ' for intra- and inter-subband scattering at a short-range random potential, respectively. F_{\pm} is oscillatory as inferred from

$$F_{\pm} = 2 \cos \left(2\pi \frac{\mu_{\pm}}{\hbar\omega} + \pi \right) \exp \left(-\frac{\pi}{\omega\tau} \right) \quad (11)$$

to the first order of the parameter $\exp(-\frac{\pi}{\omega\tau})$. Fig.4 presents the dependencies of the σ_{xx} on the magnetic field under the condition that only the lower subband is below the Fermi level: $E_F\tau/\hbar = 0$ and $\Delta\tau/\hbar = 43$. A high-frequency oscillation component emerges (termed

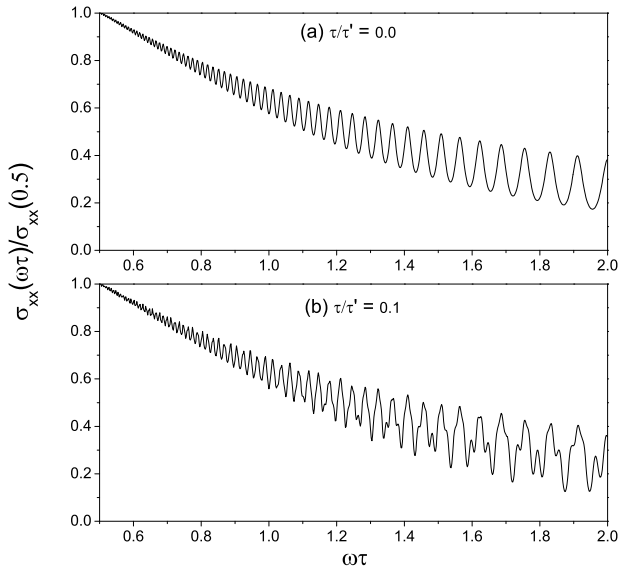


FIG. 4. The longitudinal conductivity σ_{xx} vs. magnetic field $\omega\tau$ for various intersubband scattering intensities: (a) $\tau/\tau' = 0.0$, (b) $\tau/\tau' = 0.1$ at a fixed Fermi energy $E_F\tau/\hbar = 0$. The exchange energy is $\Delta\tau/\hbar = 43$ and the SOI $\alpha = 0$.

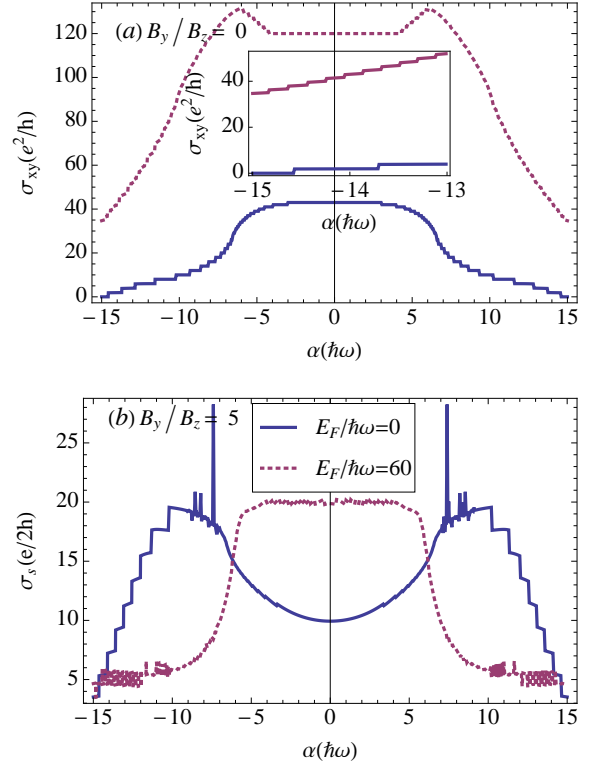


FIG. 5. (Top): The charge Hall conductivity, σ_{xy} vs. SOI parameter α with no in-plane component of the magnetic field ($B_y/B_z = 0$). Inset: a close-up of the quantization of σ_{xy} . (Bottom): The spin Hall conductivity, σ_s under $B_y/B_z = 5$. The exchange energy is $\Delta/\hbar\omega = 43$.

magneto-intersubband scattering (MIS) effect¹⁹), which behaves as $\cos(4\pi \frac{\langle \hat{\sigma}_z \rangle \Delta}{\hbar\omega})$ and is distinguishable from the background harmonic $\cos(2\pi \frac{\mu_{-}}{\hbar\omega} + \pi)$. Upon increasing the temperature, MIS dominates the oscillatory magnetoresistance due to its weak-temperature damping compared to SdH oscillations⁶.

V. HALL EFFECT

Following Rashba²³, the charge(spine) Hall conductivity is given by Kubo-Greenwood formula,

$$\sigma_{xy,s} = -\frac{e}{\pi l_B^2} \sum' \frac{\Re \langle \psi | \hat{y} | \psi' \rangle \langle \psi' | \hat{J}_{cx,s} | \psi \rangle}{E_{\psi'} - E_{\psi}}. \quad (12)$$

The prime over the sum indicates a summation only over the state $|\psi'\rangle$ below the Fermi level ($E_{\psi'} < E_F$) and $|\psi\rangle$ above the Fermi level ($E_{\psi} > E_F$). The numerical evaluation shows a quantization of the charge Hall conductance, $\sigma_{xy} = \nu e^2/2\pi\hbar$ (see in Fig.5a), for the charge current carried by each state is not changed by SOI⁷. A similar electric-field effect on the charge Hall conductivity has been reported in Graphene p-n junctions²⁴, in which the carrier type and density can be controlled by

external gates as well. Different from a semiconductor 2DEG, where a resonant spin Hall conductance is predicted for equal Rashba and Dresselhaus SOI²⁵, the spin Hall conductivity is zero in our system in the presence of only a perpendicular magnetic field due to the absence of a crossing of any adjacent energy-levels.

When the magnetic field is tilted in the $y - z$ plane with respect to the z axis, the in-plane y component B_y induces a nonzero spin polarization $\langle \sigma_y \rangle = 1/\nu \sum_{\psi} \langle \psi | \hat{\sigma}_y | \psi \rangle$, which renders the oxide magnetic order not exactly coplanar. A nonzero Berry's curvature is expected in the momentum space²⁶, and so is the spin Hall conductivity σ_s . The α -dependence of σ_s is different from that of σ_{xy} (see Fig.5b), which implies that the spin Hall conductance is not totally dominated by the spin-polarized charge current. A step-like structure is found in the spin-Hall conductivity for $E_F < \Delta$. It should be noted that the filling factor ν , the persistent spin current $\langle J_s^y \rangle$, and the charge Hall conductivity σ_{xy} are not sensitive to the in-plane magnetic field B_y . However, the competition between the Zeeman energy and the SOI results in sharp changes of the spin polarization $\langle \sigma_y \rangle$, which gives rise to resonant peaks around the degenerate point α_c (for $E_F = 0$) and to a quick oscillations (for $E_F = 60\hbar\omega$) of the spin Hall conductivity σ_s .

VI. SUMMARY

In summary, we studied the spin current, the magneto-oscillations, and the spin/charge Hall effect in 2DEG at oxide interfaces exposed to magnetic fields. The transport behavior is determined by an interplay between the exchange interaction, the magnetic field, and the effective SOI induced by the spiral geometry of the local magnetic order. The electrically tunable topological SOI allows a control of the system's magnetotransport by a small transverse electric field. Our predictions are accessible experimentally. In fact, very recently, the SdH effect has been experimentally observed for SrTiO₃/LaAlO₃²⁷, where an intense magnetic field up to 31.5T was applied. A collinear spin phase is thus achieved, and the spin helicity $q \rightarrow 0$, which corresponds to a special case described by the Hamiltonian Eq.3 with $\alpha = 0$ and $\Delta = g\mu_B B_z$.

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 - ¹⁶ The transformed exchange energy is $\tilde{h}_{ex} = U\tilde{\sigma}_z + g\mu_B B_z \mathbf{n}_r^q \cdot \tilde{\sigma}$ with $\mathbf{n}_r^q = (-\sin\theta_r, 0, \cos\theta_r)$. For multiferroics with spiral spins, the applying magnetic field \mathbf{B} should be as low as to preserve the local magnetic order, e.g. $B < 4T$ for experiments on Eu_{0.55}Y_{0.45}MnO₃¹⁵. The Zeeman splitting in \tilde{h}_{ex} is subsidiary as compared to the exchange field. For strong magnetic fields however, e.g. up to 31.5T in the experiment on SrTiO₃/LaAlO₃²⁷ a collinear spin phase is achieved. The relatively weak exchange interaction provides then a one-dimensional periodic modulation on the spins.
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