Observation of exchange Coulomb interactions in the quantum Hall state at $\nu = 3$

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Coulomb exchange interactions of electrons in the $\nu = 3$ quantum Hall state are determined from two inter-Landau level spin-flip excitations measured by resonant inelastic light scattering. The two coupled collective excitations are linked to inter-Landau level spin-flip transitions arising from the N=0 and N=1 Landau levels. The strong repulsion between the two spin-flip modes in the long-wave limit is clearly manifested in spectra displaying Coulomb exchange contributions that are comparable to the exchange energy for the quantum Hall state at $\nu = 1$. Theoretical calculations within the Hartree-Fock approximation are in a good agreement with measured energies of spin-flip collective excitations.

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The exchange Coulomb interaction energy of electrons on Landau levels (LL) plays key roles in quantum Hall systems, particularly at odd values of the filling factor, $\nu = nhc/eB$ (where n is the areal density), when the 2D electron system evolves into a quantum Hall ferromagnet. One way to probe the exchange interaction is by measurements of energies of collective spin-flip excitations. The simplest one is the spin-wave, in which Landau orbital quantization does not change. At odd filling factors the spin wave energy in the short wavelength limit is predicted to have a large exchange contribution, resulting in an enhanced spin gap[1,2]. However, the actual energy values measured in activated transport experiments turned out to be significantly below theoretical estimates. These discrepancies occur in both the integer and fractional quantum Hall regimes. Possible reasons for the discrepancies lie in impact of spin-textures (skyrmions)[4–6], and of weak residual disorder potential [7–9].

Experimental venues to access Coulomb exchange interactions also emanate from determinations of collective excitation modes in the spin degree of freedom. At odd integer values of filling factor the long-wavelength spin-wave is a minimum energy collective excitation. The long-wavelength spin-wave mode approaches the unshifted Zeeman energy[1,2,10] and carries marginal information about the electron-electron interaction.

In contrast, inelastic light scattering methods enable the direct determination of exchange Coulomb interactions from measurements of spin-flip collective excitations across cyclotron gaps [11–13]. In these spin-flip (SF) excitations there is simultaneous change in Landau quantization and in orientation of spin. The long wavelength SF excitations represent probes that are nearly insensitive to perturbations on length scales exceeding the characteristic size of the quasiparticle-quasihole pair magnetoexciton that is of the order of the magnetic length $l_o = (\hbar c/eB)^{1/2}$, where B is the perpendicular component of magnetic field.

At $\nu = 1$ the electron-electron interaction affects the energy of the long-wave cyclotron SF mode, which involves the change of the Landau quantization number by +1. This mode is shifted upwards from the cyclotron energy by about half the full exchange energy in the large momentum (small wave length) spin wave. Studies of the cyclotron SF mode at $\nu = 1$ have shown that the Coulomb exchange contributions to its energy scale as $\sqrt{B}$ and its value is softened by the spread of the electron wave-function in the direction normal to the 2D-plane. Theoretical predictions are in good agreement with measured mode energies determined as function of magnetic field and quantum well width [11,12].

We report inelastic light scattering measurements of collective inter-Landau level excitations in the quantum Hall state at $\nu = 3$. All collective excitations are identified in spectra of inelastic light scattering and their energies are compared with theoretical calculations. We identified two coupled cyclotron SF modes arising from the N=0 and N=1 Landau levels and interpreted the results in terms of Coulomb exchange interactions. We determined that these coupled cyclotron SF modes at $\nu = 3$ are subject to large Coulomb exchange interactions that are comparable to the exchange energy in the quantum Hall state at $\nu = 1$.

There is great current interest in the roles of the spin degree of freedom in the remarkable quantum Hall phases that emerge in the N=1 Landau level [14–16]. The finding reported here is that exchange Coulomb interactions in the N=1 Landau level are comparable to those in the N=0 level. This comparatively simple result suggest that the exotic collective states that emerge in the partially populated N=1 level are linked to the differences in cor-

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*Present address: Department of Electrical Engineering, Princeton University, Princeton, NJ 08544, USA relation effects between the two levels.

Figure 1a shows the schematic representation of five lowest energy collective excitations in case of filling factor ν=3. They are shown as magnetoexcitons consisting of an electron promoted from a filled Landau level and bound to an effective hole left in the "initial" LL. This representation is exact in the limit of strong magnetic field where the parameter $r_c = E_c/\hbar\omega_c$ is small enough[1–3]. Here E_c is the characteristic Coulomb energy scale and $\hbar\omega_c$ is the cyclotron energy. The set of dispersion curves of the collective modes can be described in the following way[2]:

$$E_{m,\delta S_z}(k) = m\hbar\omega_c + g\mu_B B\delta S_z + \Delta E_{m,\delta S_z}(k), \qquad (1)$$

where m is the change in the LL index, $g\mu_B B\delta S_z$ is the bare Zeeman energy associated with spin-flip. The last term $\Delta E_{m,\delta S_z}(k)$ is alone responsible for the dispersion and comprises all contributions from the many-body Coulomb interaction and exchange energies in the initial and the excited states. In the present discussion we focus on the excitation spectrum with $m = 0$ and $m = 1$.

At $\nu = 3$ the four inter-LL transitions with $m = 1$ shown in Fig.1a are not independent. They couple via the Coulomb interaction to yield two pairs of excitations. For the two inter-LL excitations with changes in the charge degree of freedom with $\delta S_z = 0$ we have the in-phase magnetoplasmon (MP) mode and the antiphase plasmon (AP) mode. For the the two excitations with changes in the spin degree of freedom with $\delta S_z = -1$ the two coupled modes are cyclotron spin-flip excitations SF1 and SF2.

In first-order perturbation theory the dispersion curves of the coupled modes are expressed as follows:

$$\begin{aligned} E_{1,2}(k) &= \frac{\mathcal{E}_1(k) + \mathcal{E}_2(k)}{2} \pm \\ &\pm \sqrt{\left(\frac{\mathcal{E}_1(k) - \mathcal{E}_2(k)}{2}\right)^2 + \Delta_{12}(k)^2}, \qquad (2) \end{aligned}$$

where $\mathcal{E}_{1,2}(k)$ are the energies of single transitions either with or without spin-flip, $\Delta_{12}(k)$ – is the term, responsible for coupling. For MP and AP excitations this theory yields a vanishing Coulomb term $\Delta E(k)$ in the long-wavelength limit. Unlike MP, for which the Kohn's theorem[17] is valid, the experimental values of the energy of AP mode are red-shifted relative to the cyclotron energy at integer filling factors $\nu \geq 2$. The experimental results were reported in Refs.[18,19,22] and the explanation was given in the framework of the second-order perturbation theory[20,22].

We calculated the wave vector dispersions of SF1 and SF2 at $\nu = 3$ in terms of matrix elements $\tilde{V}^{(1)}_{\alpha\beta\gamma\delta}(k)$ introduced in Ref.[2]:

$$\begin{aligned} \mathcal{E}_1(k) &= \hbar\omega_c + |g\mu_B B| + \Sigma_{0\uparrow,1\downarrow} - \tilde{V}^{(1)}_{1001}(k) \qquad (3) \\ \mathcal{E}_2(k) &= \hbar\omega_c + |g\mu_B B| + \Sigma_{1\uparrow,2\downarrow} - \tilde{V}^{(1)}_{2112}(k) \\ \Delta_{12}(k) &= \tilde{V}^{(1)}_{1102}(k) \end{aligned}$$

where $\Sigma_{0\uparrow,1\downarrow} = \tilde{V}^{(1)}_{0000}(0) + \tilde{V}^{(1)}_{0101}(0) - \tilde{V}^{(1)}_{1010}(0)$ and $\Sigma_{1\uparrow,2\downarrow} = \tilde{V}^{(1)}_{1010}(0) + \tilde{V}^{(1)}_{1111}(0) - \tilde{V}^{(1)}_{2020}(0)$ are the differences of exchange-self energies in the excited and ground states for two single spin-flip transitions between adjacent LLs depicted on Fig.1a. The calculated dispersion curves for all four inter-Landau level excitations at $B_\perp$ =5.3 T are plotted on Fig.1b by solid lines. For comparison with the experiment, performed on the 24nm quantum well, it was essential to take into account the finite thickness of the 2D-electron system. For this the Fourier component of the effective e-e interaction potential $\vartheta(q) = 2\pi e^2/\varepsilon q$ was multiplied by the geometric form-factor $F(q)$ calculated via the self-consistent solution of the Poisson's and Schrödinger's equations[21].

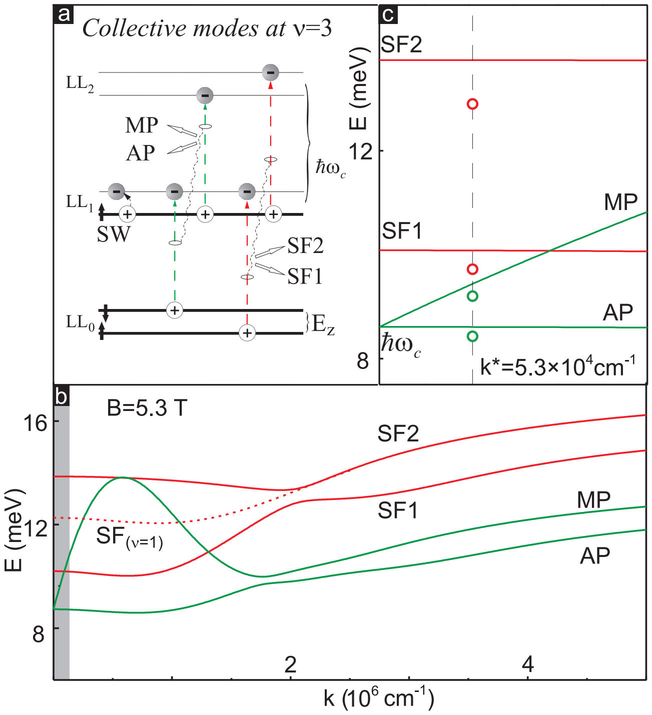

FIG. 1: (a): Schematic representation of the formation of collective modes at $\nu = 3$ from single-electron transitions. Spin-wave (SW) is described as a single spin-flip transition within half-filled LL$_1$. MP and AP are formed as inphase and antiphase combinations of two inter-LL transitions with $\delta S_z = 0$ (painted in green). Cyclotron spin-flip modes SF1 and SF2 arise from analogous combinations of inter-LL transitions with $\delta S_z = -1$ (painted in red). (b): Dispersion curves of inter-LL excitations calculated at $B_\perp = 5.3$ T within the first-order Hartree-Fock approximation are shown. Here the finite thickness of the 2D electron system is taken into account via the geometric form-factor. Dashed line represents the dispersion of cyclotron spin-flip mode at $\nu = 1$ and the same magnetic field. (c): The zoomed-in image of the long-wavelength region of Fig.1b, painted in light grey. The dashed vertical line indicates the experimental in-plane momentum $k^* = 5.3 \times 10^4$ cm^{-1}. Open circles represent the experimental data.

Both cyclotron spin-flip modes at $\nu = 3$ are significantly blue-shifted from the cyclotron energy and are nearly dispersionless in the long-wavelength limit (Fig.1b,c). Furthermore, they strongly repulse each other especially at small momenta. As a result, the Coulomb energy of SF2 in the long wavelength limit is even larger than that of analogous inter-LL spin-flip mode in a fully spin-polarized quantum Hall state $\nu = 1$[11,12]. The Coulomb energy of the long-wavelength mode SF2 is just 15% smaller than that of spin wave at $k \to \infty$ (shown on the inset to Fig.2) being the exchange energy of electrons on the LL_1. On the contrary, the energy of SF1 proves out to be pushed down. One of the intriguing results of this calculation is that the highest energy spin-flip excitation SF2 corresponds to the antiphased combination of two single electron transitions and SF1 corresponds to the inphase combination. In this aspect the situation is opposite to the case of $\delta S_z = 0$ modes MP (inphase) and AP (antiphase). As was shown in case of AP[19,20] the first-order perturbation theory gives somewhat overestimated energy in the long-wavelength limit. Although second-order corrections are exactly computed only for AP at $k = 0$, they are likely to be of the same order also for SF1 and SF2.

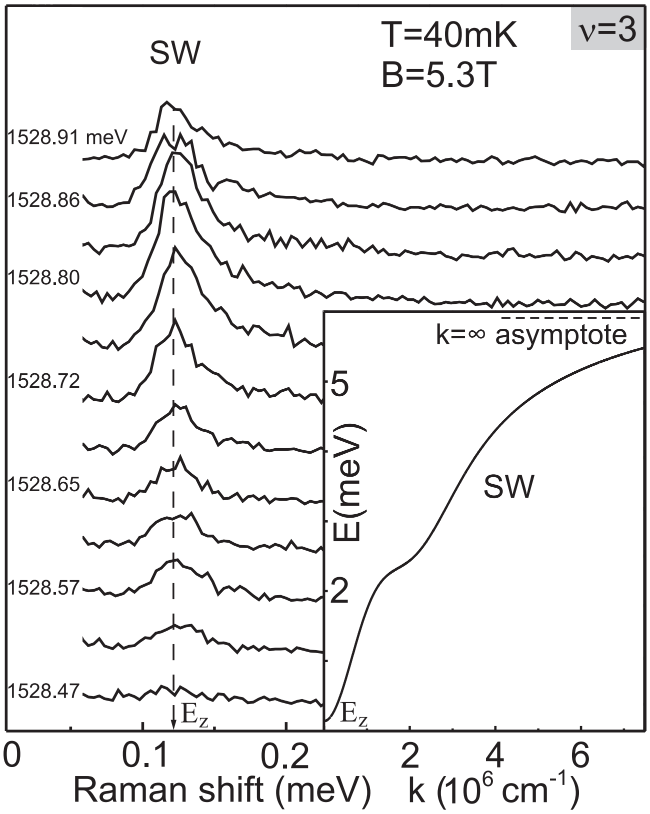

FIG. 2: Inelastic light scattering spectra of intra-LL SW mode at $\nu = 3$ and $B = 5.3$ T taken at different laser photon energies (shown on the left). The inset shows the SW dispersion curve calculated within the Hartree-Fock approximation[2], for a 24nm- wide quantum well. At the experimental in-plane momentum the energy of SW is indistinguishable from $\mathrm{E_Z}$, shown by a dashed arrow.

The inelastic light scattering measurements were performed on a high quality $\mathrm{GaAs/Al_{0.3}Ga_{0.7}As}$ single quantum well of width 24 nm. The electron density is $n = 3.85 \times 10^{11}\,\mathrm{cm}^{-2}$ and low temperature mobility above $17 \times 10^6\,\mathrm{cm^2/V\cdot sec}$. The sample was mounted on the cold finger of a $^3\mathrm{He}/^4\mathrm{He}$ dilution refrigerator that is inserted in the cold bore of a superconducting magnet. The refrigerator is equipped with windows for optical access. Cold finger temperature was held mostly at $T = 40\,\mathrm{mK}$ or 1.7 K for one part of the experiment. The backscattering geometry was used at an angle $\theta = 20^\circ$ with the normal of the sample surface. The perpendicular component of the magnetic field is $B = B_T \cos\theta$ and B_T is the total magnetic field. Resonant inelastic light scattering spectra were obtained by tuning the incident photon energy of a Ti:sapphire laser close to the fundamental optical gap of GaAs to enhance the light scattering cross section. The power density was kept below $10^{-4}\,\mathrm{W/cm^2}$ for the measurements at temperatures around 40 mK. The in-plane momentum, transferred to the excitations at the employed experimental geometry was about $5.3 \times 10^4\,\mathrm{cm}^{-1}$. The scattered signal was dispersed by a triple grating spectrometer T-64000 working in additive and subtractive modes and analyzed by a charge-coupled device camera. The combined resolution of the system was about 0.02 meV. In order to distinguish between inelastic light scattering and luminescence lines in spectra, the special test was employed – when varying the incident photon energy, inelastic light scattering lines traced the laser path, while luminescence lines did not change their spectral position.

The resonant enhancement of the intensities of light scattering spectra of the long wavelength spin-wave (SW) mode at $\nu = 3$ is displayed in Fig.2. The SW is at the bare Zeeman energy with $|g| = 0.37$ and corresponds to the leftmost part of the mode dispersion shown in the inset to Fig.2. Very small changes in the laser photon energy (by $\sim 0.5\,\mathrm{meV}$) dramatically affect the line intensity, indicating the importance of resonant excitation in these experiments. The strong SW seen in Fig.2 is consistent with the ferromagnetic character of the quantum Hall state at $\nu = 3$.

Similar resonant excitation conditions prevail in the observation of the inter-Landau level excitations reported below. To capture light scattering spectra of inter-LL excitations, the incident photon energies were chosen in such a way as to excite electrons from the valence band to the second or third Landau levels in the conduction band. Typical spectra are measured at two laser positions (1538.2 meV and 1550.0 meV) shown on Fig. 3.

The magnetoplasmon and antiphased plasmon are seen shifted from the cyclotron energy $\hbar\omega_c$=8.65 meV (depicted by an arrow on Fig.3). The blue shift of the MP results from the 2D-plasma energy at the non-zero in-plane momentum used in the experiment. In fact, the MP is the only dispersive mode in the range of experimentally accessible momenta (see Fig. 1c). The energy of AP is below the CR by 0.19 meV. This energy shift is somewhat smaller than that measured for 18 nm quantum well in Ref.[22], which is a consequence of the strong dependence of the effective Coulomb interaction strength on the quantum well width. Developed in Ref.[22] theory gives $\Delta E_{\mathrm{AP}}(0) \approx -0.25\,\mathrm{meV}$ for this magnetic field and quantum well width.

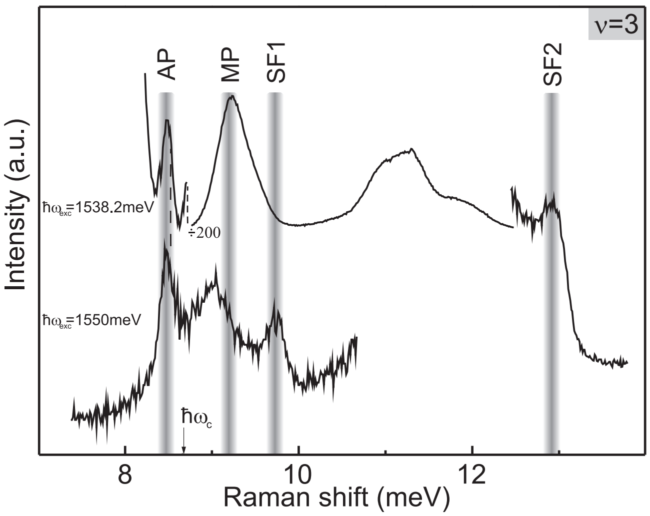

FIG. 3: Inelastic light scattering spectra of inter-LL excitations at $\nu = 3$ and $B_\perp = 5.3$ T. Upper spectrum is taken at the incident photon energy $\hbar\omega_{exc} = 1538.2$ meV, the lower one – at 1550.0 meV. The position of $\hbar\omega_c$=8.65 meV is shown by the arrow. Grey vertical columns mark inelastic light scattering lines. The rest of the spectrum is composed of the luminescence bands.

We focus here on the two cyclotron spin-flip modes SF1 and SF2 which are blue-shifted from $\hbar\omega_c$ by 1.13 meV and 4.3 meV respectively. We have compared these experimental values to those calculated theoretically within the first-order Hartree-Fock approximation taking into account the actual width of the quantum well (see Fig. 1c). The discrepancy is of the order of the negative second order corrections such as for AP plasmon. The Coulomb energy of SF2 is close to the estimated full exchange energy of electrons on LL$_1$. The latter is represented by the energy limit of shortwave SW at $\nu = 3$. This asymptotical value is about three fourth of analogous quantity at fully spin polarized state $\nu = 1$. We also find a marked dependence on magnetic field in which lines SF1 and SF2 are observed only in the narrow interval $\Delta B \simeq 0.3\,\mathrm{T}$ around $\nu = 3$. Outside this range of fields and filling factors the lines disappear from the spectrum. From this fact we conclude that these excitations are inherent to filling factor $\nu = 3$.

To summarize, by means of inelastic light scattering we have observed and identified four inter-Landau level collective excitations and intra-LL spin-wave at $\nu = 3$. Among these excitations there are two cyclotron spin-flip modes, which interact repulsively in the long-wave limit. As a result, the upper of them (SF2) acquires a huge exchange contribution to the energy, comparable with the theoretically estimated exchange energy of electrons on the first Landau level. The experimentally measured energies of all excitations are in a good agreement with the Hartree-Fock calculations taking into account the finite thickness of the 2D-electron system.

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