Testing gravity using the growth of large scale structure in the Universe

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ABSTRACT

Future galaxy surveys hope to distinguish between the dark energy and modified gravity scenarios for the accelerating expansion of the Universe using the distortion of clustering in redshift space. The aim is to model the form and size of the distortion in order to infer the rate at which large scale structure grows. We test this hypothesis and assess the performance of current theoretical models for the redshift space distortion using very large volume N-body simulations of the gravitational instability process. We simulate competing cosmological models which have identical expansion histories - one is a quintessence dark energy model with a scalar field and the other is a modified gravity model with a time varying gravitational constant - and demonstrate that they do indeed produce different redshift space distortions. This is the first time this approach has been verified using a technique that can follow the growth of structure at the required level of accuracy. Our comparisons show that theoretical models for the redshift space distortion based on linear perturbation theory, which are currently in widespread use, give a surprisingly poor description of the simulation results. Furthermore, the application of such models can give rise to catastrophic systematic errors leading to an incorrect interpretation of the observations. We show that an improved model is able to extract the correct growth rate. Further enhancements to theoretical models of redshift space distortions, calibrated against simulations, are needed if we are to fully exploit the forthcoming high precision clustering measurements.

Subject headings: Methods: numerical — Cosmology: theory — dark energy

1. Introduction

The accelerating expansion of the Universe can be explained by a dark energy component or a modification to gravity. In both of these alternatives, the cosmic expansion history can be described using an effective equation of state, $w(a)$, where a is the scale factor. If two models have the same $w(a)$, then, as a consequence, it is not possible to distinguish between them using a measurement of the expansion history alone. However, cosmic structures are expected to collapse under gravity at different rates in dark energy and modified gravity cosmologies. The hypothesis we test in this paper is that the growth rate of structure can be measured using distortions in the galaxy clustering pattern induced by the peculiar motions of galaxies, called redshift space distortions.

In the case of general relativity, the growth of density perturbations depends only on the expansion history through the Hubble parameter, $H(a)$, or equivalently, $w(a)$ (Linder 2005). This is not the case in modified gravity theories. By using the measured expansion history to predict the growth rate of structure and comparing this estimate to a direct measurement of the growth rate, it has been argued that it is possible to determine whether the physical origin of the accelerating cosmic expansion is dark energy or modified gravity (Lue et al. 2004; Linder 2005). If there is no discrepancy between the observed growth rate and the theoretical prediction assuming general relativity, this implies that a dark energy component alone can explain the accelerated expansion.

Here we test this hypothesis using large volume N-body simulations which are the only way to accurately follow the growth of cosmic structure and hence to probe the limits of linear perturbation theory. Previous simulations of gravitational instability in hierarchical cosmologies have shown that linear theory gives a surprisingly poor description of fluctuation growth and the distortion of clustering due to peculiar motions, even on large scales (e.g. Angulo et al. 2008; Smith et al. 2008; Jennings et al. 2010). We simulate the growth of structure in a modified gravity model and a dark energy model which, by construction, have the same expansion history. The growth rate is measured from the appearance of the power spectrum of clustering in redshift space. The goals of this paper are as follows: Firstly, we will determine if these competing cosmologies can be distinguished from the distortion of clustering as measured in redshift space, using the simulation results. Secondly, we will test several theoretical models of the power spectrum in redshift space against the simulation results to assess how well they can recover the correct value for the growth rate in each cosmology.

This letter is set out as follows. In Section 2 we review the growth of perturbations in linear theory and describe the modified gravity model. The clustering in redshift space is measured in Section 3, where different theoretical models are applied to describe the simulation results. In Section 4 we present our conclusions.

2. The cosmological models and simulations

Here we recap how the growth of perturbations depends on the expansion history and the strength of gravity (Section 2.1), before outlining the modified gravity model (Section 2.2) and our N-body simulations (Section 2.3).

2.1. The linear growth rate

In the framework of general relativity, the growth of a density fluctuation, described by $\delta \equiv (\rho(x,t) - \bar{\rho}_{\rm m})/\bar{\rho}_{\rm m}$, where $\bar{\rho}_{\rm m}$ is the average matter density, depends only on the expansion history, $H(a)$. Using the perturbed equations of motion, within general relativity, the growth of density perturbations is set by

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \rho_{\rm m}\delta = 0\,, \tag{1}$$

where G_N is the present value of the gravitational constant found in laboratory experiments and a dot denotes a derivative with respect to time. The growth rate, f, is defined as $f \equiv {\rm dln}\delta/{\rm dln}a$, where $\delta(a)$ is the growing mode solution to Eq. 1. If we change variables to $g \equiv \delta/a$ and allow the gravitational constant to vary in time, denoted by $\tilde{G}$, this equation becomes (Linder 2005)

$$\frac{{\rm d}^2 g}{{\rm d}a^2} + \left(5 + \frac{1}{2}\frac{{\rm dln}H^2}{{\rm dln}a}\right)\frac{1}{a}\frac{{\rm d}g}{{\rm d}a} + \left(3 + \frac{1}{2}\frac{{\rm dln}H^2}{{\rm dln}a} - \frac{3}{2}\frac{\tilde{G}(a)}{G_N}\Omega_{\rm m}(a)\right)g = 0\,, \tag{2}$$

where $\Omega_{\rm m}(a)$ is the ratio of the matter density to the critical density as a function of scale factor, a. It is clear from Eq. 2 that in the framework of general relativity, $\tilde{G}(a)/G_N = 1$ and the growth of perturbations depends only on the expansion history, $H(a)$. In theories of modified gravity, on the other hand, the growth of perturbations will depend on both $H(a)$ and $\tilde{G}(a)$.

2.2. Time variation of Newton's constant

Modifications to general relativity, referred to as modified gravity theories, provide an alternative explanation to dark energy for the observed accelerating expansion of the Universe. Several classes of modified gravity theories exist which generally can be divided into models which introduce a new scalar degree of freedom to Einstein's equations, e.g. scalar tensor or $f(R)$ theories, and those which modify gravity by changing dimensionality of space, e.g. braneworld gravity. In many modified gravity models, the time variation of fundamental constants, such as Newton's gravitational constant, G_N, is naturally present.

Self consistent scalar tensor theories are viable alternatives to Einstein's theory of gravity and give rise to a cosmic expansion that accelerates at late epochs, as suggested by observations. We refer to these as 'extended quintessence' models. Calculations with a mesh to allow spatial variations of the scalar field have shown that, in practice, a broad range of extended quintessence models can be effectively described as a theory which features a time varying Newton's constant (Pettorino & Baccigalupi 2008; Li et al. 2010).

The variation of Newton's constant is constrained by various astrophysical and cosmological observations. Increasing G_N shortens the life span of stars by causing them to burn faster (Teller 1948). The age of globular clusters sets a limit on the time variation of G_N (degl'Innocenti et al. 1996). The mass of a neutron star depends on G_N through the balance between gravity and the neutron Fermi pressure: $M \propto G_N^{3/2}$. Measurements of neutron star masses at different redshifts therefore limit the time dependence of G_N (Thorsett 1996). A change in G_N in the early Universe would alter the cosmic expansion rate and change the time interval over which light nuclei are synthesized (Umezu et al. 2005; Clifton et al. 2005). A time-varying G_N would also modify the temperature fluctuations measured in the cosmic microwave background, shifting the peaks to larger (smaller) scales on increasing (decreasing) G_N. This leads to a constraint on the variation of G, $\dot{G}/G = (-9.6 \sim 8.1) \times 10^{-12}$ yr^{-1} (Chan & Chu 2007).

In this paper we consider a simple model for $\tilde{G}$ (Zahn & Zaldarriaga 2003; Umezu et al. 2005; Chan & Chu 2007),

$$\tilde{G} = \mu^2 G_N \,, \tag{3}$$

where

$$\mu^2 = \begin{cases} \mu_0^2 & \text{if } a < a_* \\ 1 - \frac{a_s - a}{a_s - a_*}(1-\mu_0^2) & \text{if } a_* \le a \le a_s \\ 1 & \text{if } a > a_s \,. \end{cases} \tag{4}$$

This parametrization describes a smoothly varying $\tilde{G}$ which converges slowly to its present value, G_N, and which can be considered to be more physical than those based on step functions (e.g. Cui et al. 2010). The scale factor, a_*, denotes the time of photon decoupling and the parameters μ_0 and a_s quantify the deviation of $\tilde{G}$ from the present laboratory measured value, G_N, and the scale factor at which $\tilde{G}$ and G_N are equal, respectively. The background evolution is given by

$$H^2 = H_0^2 \frac{\tilde{G}}{G_N}\left(\frac{\Omega_{\rm m}}{a^2} + \Omega_{\rm DE} e^{3\int_a^1 {\rm d}\ln a'[1+w(a')]}\right) . \tag{5}$$

Note that here we assume an equation of state $w(a) = -1$ in the modified gravity model to match ΛCDM. In Eq. 5, $\Omega_{\rm DE}$ is the ratio of the dark energy density to the critical density today. In the left panel of Fig. 1, we plot the ratio of the Hubble rate for two different cosmological models with varying $\tilde{G}$, to the Hubble rate in a ΛCDM cosmology as a function of redshift. The green dot dashed line corresponds to $\tilde{G}$ with $\mu_0^2 = 1.13$ and $a_s = 1$ in Eq. 4, while the black solid line uses the parameters $\mu_0^2 = 1.075$ and $a_s = 0.5$. We chose to simulate the model with the maximum deviation of $\tilde{G}$ from G_N which is still compatible with CMB measurements and Solar System constraints ($\tilde{G} \to G$ as $a \to 1$), which occurs for a stabilization redshift corresponding to an expansion factor of $a_s = 1$ (i.e. as shown by the green dot dashed line in Fig. 1).

2.3. N-body simulations

We use large volume N-body simulations to carry out the first direct test of the idea that a dark energy cosmology and a modified gravity model which, by construction, have exactly the same expansion history, can be distinguished by a measurement of the rate at which cosmic structure grows. The modified gravity model we choose to simulate is the one with the maximum deviation from Newton's constant that is still compatible with current observational constraints, as discussed in the previous section. We can then construct a quintessence model by fitting the parameters w_0 and w_a (Linder 2003) in

$$H^2(a) = H_0^2(\frac{\Omega_{\rm m}}{a^3} + \Omega_{\rm DE} e^{-3w_a(1-a)} a^{-3(1+w_0+w_a)}) \tag{6}$$

to the expansion history of the varying $\tilde{G}$ model using MPFIT (Markwardt 2009). With a two parameter model for w, it is necessary to perform separate fits over limited redshift bins in order to obtain an accurate description of the expansion history over the redshift range considered in the simulations. We found that with four bins over

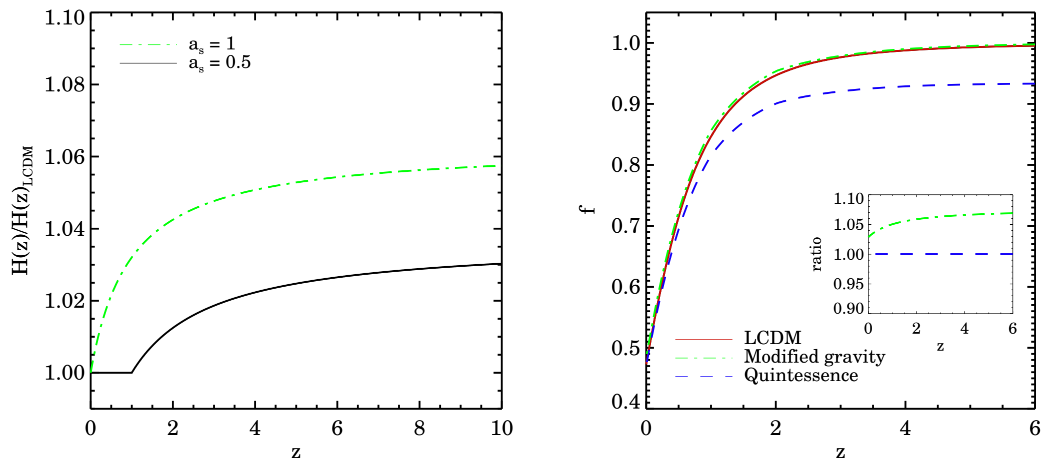

Fig. 1.— Left panel: The lines show the ratio of the expansion rate, $H(z)$, for two choices of parameters in the modified gravity model specified by Eq. 4 to the expansion rate in ΛCDM: dot dashed green line: $a_s = 1, \mu_0^2 = 1.13$ and solid black line : $a_s = 0.5, \mu_0^2 = 1.075$. Right panel: The linear growth rate, f, as a function of redshift for ΛCDM (red solid), a modified gravity cosmology (green dot dashed) and a quintessence model (blue dashed). The modified gravity model has parameters $a_s = 1$ and $\mu_0^2 = 1.13$ in Eq. 4. The quintessence model has the same expansion history as the modified gravity model. The inset panel shows the ratio of f for the modified gravity model to that in the quintessence model as a function of redshift (green dot dashed line).

the interval $0 \leq z \leq 200$, we are able to reproduce the expansion history of the varying $\tilde{G}$ model to better than 0.25%. This quintessence model is consistent with current constraints on dynamical dark energy (Komatsu et al. 2009; Sánchez et al. 2009).

The simulations were carried out at the Institute of Computational Cosmology using a memory efficient version of the TreePM code `Gadget-2`, called `L-Gadget-2` (Springel 2005). The simulation used $N = 1024^3 \sim 1 \times 10^9$ particles in a computational box of comoving length $1500h^{-1}$Mpc. The comoving softening length was $\epsilon = 50h^{-1}$kpc and the present day linear theory rms fluctuation in spheres of radius 8 h^{-1} Mpc is $\sigma_8 = 0.8$. Simulations of extended quintessence cosmologies need to account for both the gravitational correction due to a varying $\tilde{G}$ in the Poisson equation and a modified expansion history (see Pettorino & Baccigalupi 2008). In the modified gravity simulation, both the long and short-range TreePM algorithm force computations were modified to include a time-dependent gravitational constant. For both the modified gravity and the quintessence dark energy simulations the Hubble parameter computed by the code was also modified as in Jennings et al. (2010).

The linear theory power spectrum used to generate the initial conditions was obtained using CAMB (Lewis & Bridle 2002). Following previous authors (Laszlo & Bean 2008; Bertschinger & Zukin 2008), we assume that the Jeans length is smaller than the scales of interest at our starting redshift, $z = 200$, and that modified gravity has not yet become important. We therefore assume a ΛCDM cosmology to generate the linear theory power spectrum.

To obtain errors on our measurements we ran 10 lower resolution simulations with 512^3 particles, also in a computational box of comoving length $1500h^{-1}$Mpc, each with a different realisation of the density field. The power spectrum was computed by assigning the particles to a mesh using the cloud in cell (CIC) assignment scheme and performing a fast Fourier transform of the density field. For the initial conditions the linear growth rate for each model and ΛCDM was obtained by solving Eq. 2 numerically and is plotted in the

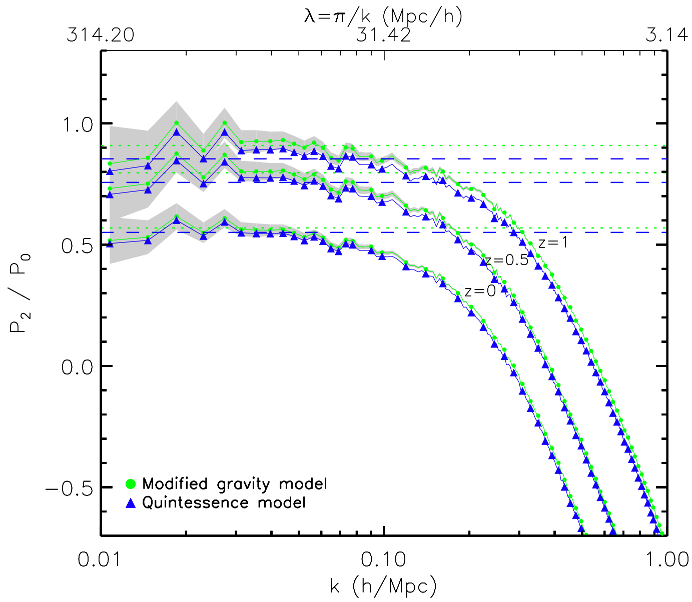

Fig. 2.— The ratio of the quadrupole and monopole moments of the redshift space power spectrum, P_2/P_0, as a function of wavenumber at redshifts $z = 0$, 0.5 and 1 (in order of ascending amplitude). The points show measurements from the N-body simulations, with green circles showing the results from the modified gravity model and blue triangles the quintessence model. The shading indicates the error on the ratio, estimated from the scatter over 10 lower resolution simulations. The horizontal lines show the predictions of the linear theory model, with the colours having the same meaning as those used for the points.

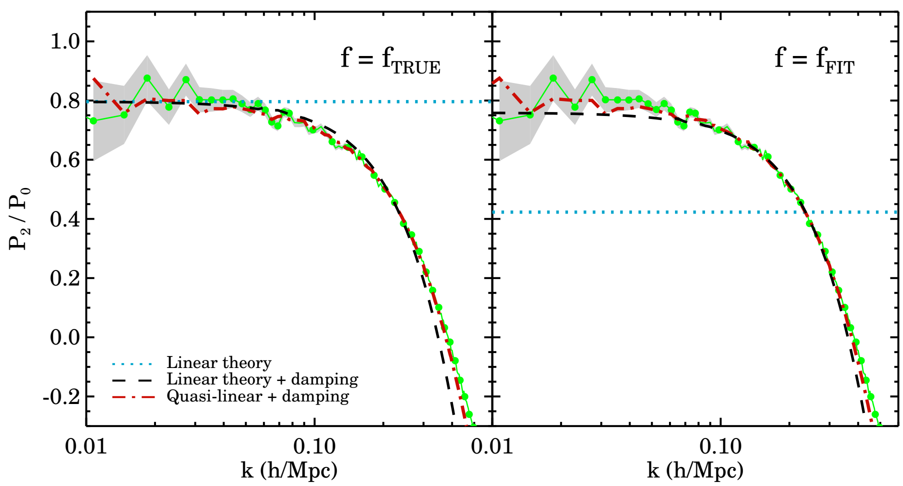

Fig. 3.— The ratio of the quadrupole and monopole moments of the redshift space power spectrum for the modified gravity cosmology measured from the high resolution simulation (green points) together with three models for P_2/P_0, using the correct linear growth rate, $f = f_{\rm TRUE}$ (left panel), and the value of f obtained in a χ^2 fit over the wavenumber range $0.01 \le k(h/{\rm Mpc}) \le 0.25$, $f = f_{\rm FIT}$ (right panel). The shaded region shows the propagated errors from the ten lower resolution simulations. The models plotted are indicated by the key: linear theory - blue dotted line, linear theory plus damping - black dashed line and quasi-linear plus damping - red dashed line. In the left panel the best fit value for σ_p (σ_v) obtained in the range $0.01 \le k(h/{\rm Mpc}) \le 0.25$,with fixed f, was used for the linear theory plus damping (quasi-linear plus damping) model.

right hand panel of Fig. 1 as a function of redshift. For all the models we used the following cosmological parameters: $\Omega_{\rm m} = 0.26$, $\Omega_{\rm DE} = 0.74$, $\Omega_{\rm b} = 0.044$, $h_0 = H_0/100{\rm km\ s^{-1}Mpc^{-1}} = 0.715$ and a spectral index of $n_{\rm S} = 0.96$ (Sánchez et al. 2009). We have verified that our modifications to Gadget-2 are accurate by checking that the growth of the fundamental mode in the simulations agrees with the linear theory predictions at several redshifts in both the modified gravity and quintessence model simulations.

3. Results

In this section we briefly recap the models used to describe the redshift space distortion of the matter power spectrum. In Section 3.2 we fit these models to the moments of the power spectrum measured in redshift space in our simulations.

3.1. Redshift space distortions

The matter power spectrum in redshift space can be decomposed into multipole moments using Legendre polynomials. The ratio of the quadrupole and monopole moment of the matter power spectrum is plotted in Fig. 2 at three output redshifts, $z = 0, 0.5$ and 1. The simulation results show that this ratio has a strong dependence on wavenumber. This can be contrasted with the prediction of linear perturbation theory, (Cole et al. 1994),

$$\frac{P_2(k)}{P_0(k)} = \frac{4\beta/3 + 4\beta^2/7}{1 + 2\beta/3 + \beta^2/5}, \quad (7)$$

where $\beta = f/b$ and b is the linear bias, which is unity for dark matter; Eq. 7 is independent of scale (horizontal lines in Fig. 2). The quadrupole to monopole ratio increases in amplitude with redshift, due to the associated evolution in the matter density parameter. At $z = 0$ there is a 2.5% difference between the linear theory growth rates in the two models. However, at this level, the measured ratios P_2/P_0 are indistinguishable on the very largest scales $k < 0.02h/$Mpc where our measurements match the linear perturbation theory predictions (green dotted and blue dashed horizontal lines). At $z = 0.5$ and $z = 1$ the linear theory predictions for the growth rates in the two models differ by 4% and 6% respectively. The error on this ratio measured from the ten lower resolution simulations is shown as a grey shaded region in Fig. 2.

In addition to the linear theory model described above, we consider two variants. The first is the Gaussian model (Peacock & Dodds 1994),

$$P^s(k, \mu) = P^r(k)(1 + \beta\mu^2)^2 e^{(-k^2\mu^2\sigma_p^2)}, \quad (8)$$

where σ_p is the pairwise velocity dispersion along the line of sight, which is treated as a parameter to be fitted. We refer to Eq. 8 as the "linear theory plus damping" model. The damping introduces a scale dependence into the ratio P_2/P_0. The second variant model takes into account departures from linear theory, as well as including small scale damping (Scoccimarro 2004):

$$P^s(k, \mu) = (P_{\delta\delta}(k) + 2f\mu^2 P_{\delta\theta}(k) + f^2\mu^4 P_{\theta\theta}(k)) \times e^{-(fk\mu\sigma_v)^2}, \quad (9)$$

where σ_v is the 1D linear velocity dispersion and $P_{\theta\theta}$ and $P_{\delta\theta}$ are the velocity divergence auto and cross power spectrum respectively (see Jennings et al. 2010). We refer to Eq. 9 as the "quasi-linear plus damping" model.

3.2. Measuring the growth rate

We now apply the models listed above to the measurements from the simulations. We model redshift space distortions in the distant observer approximation by perturbing the particle positions down one of the cartesian axes, using the suitably scaled peculiar velocity along the axis. In Fig. 3, we plot the measured ratio P_2/P_0, for the modified gravity cosmology at $z = 0.5$, together with the predictions for this ratio using the linear theory (cyan dotted line), the linear theory plus damping model (black dashed line) and the quasi-linear plus damping model (red dot dashed line). In the left panel, the correct value of f for this cosmology together with the best fit value for σ_p and σ_v in the range $0.01 \leq k(h/{\rm Mpc}) \leq 0.25$ was used in the linear theory plus damping and quasi-linear plus damping models respectively. In the right panel, the best fit value for f obtained by fitting over the same range of wavenumbers has been used for all models plotted. The value of f obtained for the linear theory model is set by the maximum value of k used in the fit. It is clear that both the linear theory and the linear theory plus damping models fail to predict the correct value for f,

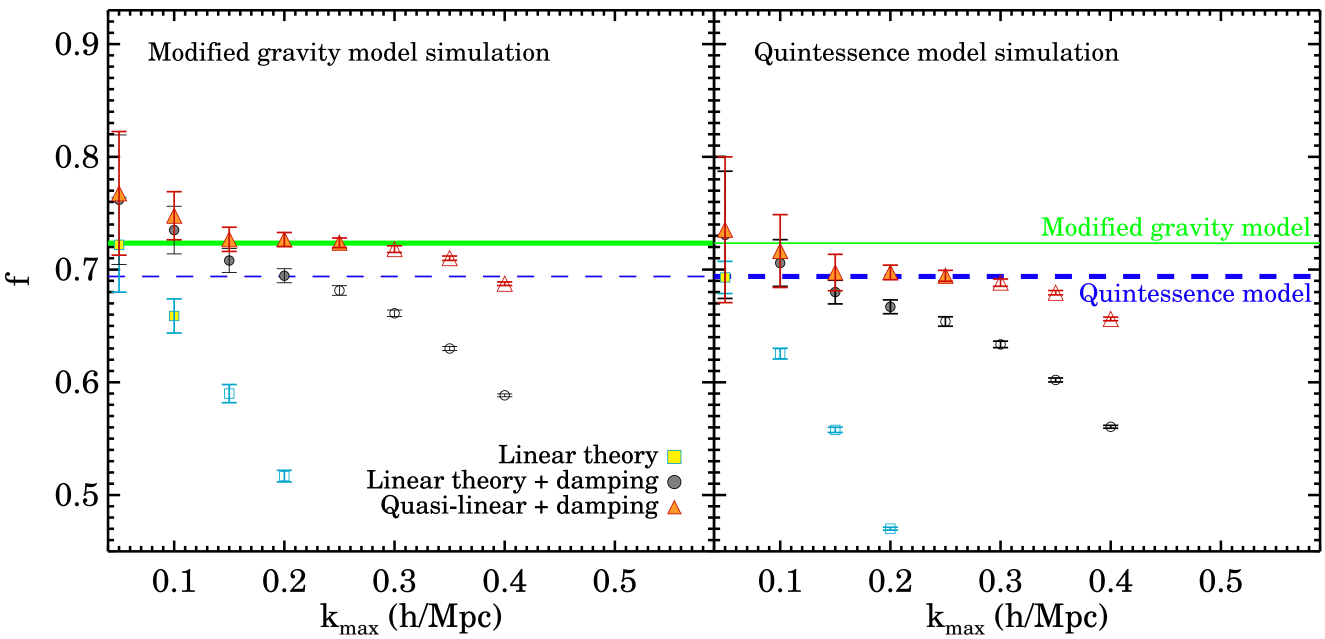

Fig. 4.— Measurements of the linear growth rate of cosmic structure, f. The results are plotted as a function of the maximum wavenumber, $k_{\rm max}(h/{\rm Mpc})$, used in the fit. The different symbols show the results of fitting to P_2/P_0 at $z = 0.5$ (see Fig. 2) using different models: linear theory - squares, linear theory plus damping - circles, quasi-linear plus damping - triangles. The symbols are filled in for scales over which the model is a good description of the measured ratio. The error bars represent the 1σ uncertainty in the fit. In the left hand panel we fit to the modified gravity model and aim to recover the true growth factor shown by the thick green horizontal line. In the right panel we fit to the ratio measured from the quintessence model, in which case the target growth factor is shown by the thick blue dashed line.

with the best fitting values differing by $\sim 40\%$ and $\sim 6\%$ respectively from the true value. All the models plotted in the right panel in Fig. 3 uses the value of f recovered when $k_{\rm max} = 0.25h/\rm Mpc$. The quasi-linear plus damping model recovers the correct value of f over this wavenumber range to a precision of $\sim 0.64\%$.

To test these models for the redshift space power spectrum further we vary the maximum wavenumber, $k_{\rm max}$, used in the fit and plot the recovered growth rate as a function of $k_{\rm max}$ in Fig. 4. With an accurate model for P_2/P_0, we would recover the correct value for the growth rate f and the answer would be independent of the value of $k_{\rm max}$ adopted, with the only sensitivity to $k_{\rm max}$ being displayed in the error on the growth rate. Fig. 4 shows that the quasi-linear plus damping model comes closest to meeting this ideal. Even this model breaks down beyond $k_{\rm max} \sim 0.3h/\rm Mpc$, which suggests that the modelling of the small scale velocity dispersion needs to be improved. Most importantly, this model recovers the correct value for f and can distinguish between the two cosmologies. The models based on linear theory perform less well. In fact, the answer depends strongly on the maximum wavenumber retained in the fit. In Fig. 4 filled symbols are plotted for scales over which the model is a good description of the measured ratio (i.e. $\chi^2/\nu \sim 1$, where ν is the number of degrees of freedom). We note that the expression for P_2/P_0 in the quasi-linear plus damping model is more sensitive to changes in f than the ratio of the monopole moment of the redshift space to real space power spectrum, and as a result the 1 sigma error bars for f in Fig. 4 are smaller when fitting to P_2/P_0 than they are when fitting to the monopole moment ratio.

4. Conclusions

The next generation of galaxy redshift surveys aim to resolve some of the fundamental questions in modern cosmology, such as whether general relativity needs to be modified or if a dark energy component is driving the accelerating expansion. We have measured the redshift space distortions from two simulations with different cosmologies and demonstrated that a modified gravity model, described by a time varying Newton's constant, and a dark energy model, which have identical expansion histories, have measurably different growth rates. We have tested several models for redshift space distortions of clustering including the commonly used linear theory and linear theory plus Gaussian damping models. We find that these two models fail to recover the correct value for the growth rate. A quasi-linear model which includes non-linear velocity divergence terms is far more accurate and would allow us to distinguish between these two competing cosmologies.

Even though the scales we consider are large it is clear that there are important departures from linear theory which can only be modelled accurately using an N-body simulation (Jennings et al. 2010). There is a real chance that without guidance from a simulation, the application of the linear theory or linear theory plus damping models could lead to systematic errors of the same order as the difference in f between the two competing cosmologies. In this event, these models would give the wrong conclusion about the physics driving the cosmic acceleration. We find that an improved model is able to recover the correct growth factor and hence to tell the two models apart. This model can be applied to the measured power spectrum over a wider range of scales than those based on linear theory, making better use of the available data. Our simulation tests show that a further improvement to this model is possible. Nevertheless our results suggest that with such improved models that have been validated against simulations, the prospects of distinguishing between modified gravity and dark energy models using measurements of galaxy clustering are encouraging.

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