

Chaotic neuron clock

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Abstract

A chaotic model of spontaneous (without external stimulus) neuron firing has been analyzed by mapping the irregular spiking time-series into telegraph signals. In this model the fundamental frequency of chaotic Rössler attractor provides (with a period doubling) the strong periodic component of the generated irregular signal. The exponentially decaying broad-band part of the spectrum of the Rössler attractor has been transformed by the threshold firing mechanism into a scaling tale. These results are compared with irregular spiking time-series obtained in vitro from a spontaneous activity of hippocampal (CA3) singular neurons (rat's brain slice culture). The comparison shows good agreement between the model and experimentally obtained spectra.

Keywords: Chaos, neuron, spikes, spontaneous activity

In order to work together the brain neurons have to make adjustment of their rhythms. The main problem for this adjustment is the very noisy environment of the brain neurons. If their work was based on pure periodic inner clocks this adjustment would be impossible due to the noise. The nature, however, has another option. This option is a chaotic clock. In chaotic attractors certain characteristic frequencies can be embedded by broad-band spectra, that makes them much more stable to the noise perturbations [1]. The question is: How can these inner chaotic clocks survive the threshold firing mechanism, and what spiking series will they produce as a result of their transition through this mechanism? In any way it is clear that the output signal will have an irregular shape. This will be the main problem we

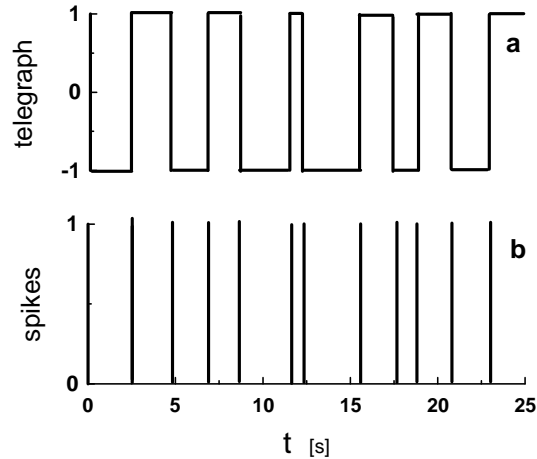


Figure 1: Mapping of a spike train (figure 1b) into a telegraph signal (figure 1a).

will address in present paper.

Another problem is spontaneous (without external stimulus) neuron firing. This process should be more simple and self-consistent than a reaction of a neuron to the external stimulus. This can allow to analyze the inner neuron clock in its free state. In the experimental *in vitro* situations the brain is known to produce a spontaneous activity [2],[3]. Therefore, the results of this analysis can be compared with the *in vitro* spontaneous activity of the brain neurons.

All types of information, which is received by sensory system, are encoded by nerve cells into sequences of pulses of similar shape (spikes) before they are transmitted to the brain. Brain neurons use such sequences as main instrument for intercells connection. The information is reflected in the time intervals between successive firings (interspike intervals of the action potential train, see Fig. 1b). There need be no loss of information in principle when converting from dynamical amplitude information to spike trains [4] and the irregular spike sequences are the foundation of neural information processing. Although understanding of the origin of interspike intervals irregularity has important implications for elucidating the temporal components of the neuronal code and for treatment of such mental disorders as depression and schizophrenia, the problem is still very far from its solution (see, for instance, Ref. [5] and references therein). The mighty Fourier transform method, for instance, is practically non-applicable to the spike time trains. The spikes are almost identical to each other and the neural information is coded in the length of the interspike intervals and the interspike intervals positions on the time axis, therefore it is the most direct way to map the spike train into a

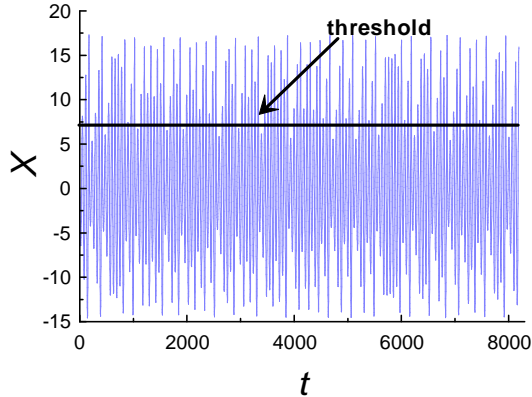


Figure 2: X-component fluctuations of a chaotic solution of the Rössler system Eq. (1) ($a = 0.15$, $b = 0.20$, $c = 10.0$).

telegraph time signal, which has values -1 from one side of a spike and values $+1$ from another side of the spike with a chosen time-scale resolution. An example of such mapping is given in figure 1. While the information coding is here the same as for the corresponding spike train, the Fourier transform methods are quite applicable to analysis of the telegraph time-series.

Using this mapping we will try to give certain answer on the above mentioned question. But first of all let us recall some basic electrochemical properties of neuron. Nerve cells are surrounded by a membrane that allows some ions to pass through while it blocks the passage of other ions. When a neuron is not sending a signal it is said to be "at rest". At rest neurons exhibit very small conductance of sodium ions and slightly larger potassium conductance against a high concentration of intracellular potassium ions. The resting *membrane* electrochemical potential (the voltage difference across the neural membrane) of a neuron is about -70mV . If some event (a stimulus) causes the resting potential to move toward 0mV and the depolarization reaches about -55mV (a "normal" threshold) a neuron will fire an *action* potential. The action potential is an explosive release of charge across plasma membrane and its surrounding that is created by a depolarizing current. If the neuron does not reach this critical threshold level, then no action potential will fire. Also, when the threshold level is reached, an action potential of a *fixed* size will always fire (for any given neuron the size of the action potential is always the same). Depending on different types of voltage-dependent ion channels, different types of action potentials are generated in different cells types and the qualitative estimates of the potentials and time periods can be varied.

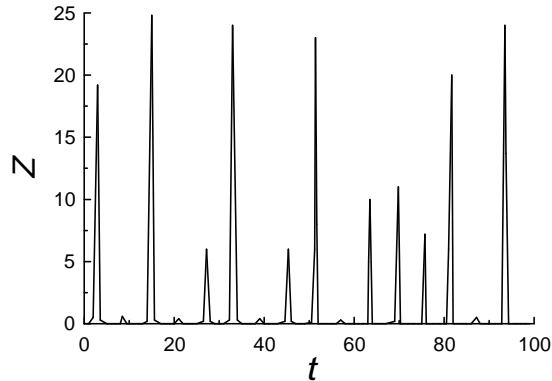


Figure 3: Z-component fluctuations of a chaotic solution of the Rössler system.

Recent reconstructions of the dynamics of *membrane* potential using the neuron spike trains indicate the Rössler oscillator as the most probable (and simple) candidate (see, for instance, Refs.[6]-[12]). Figure 2 shows as example the x-component fluctuations of a chaotic solution of the Rössler system [13]

$$\frac{dx}{dt} = -(y + z); \quad \frac{dy}{dt} = x + ay; \quad \frac{dz}{dt} = b + xz - cz \quad (1)$$

where a, b and c are parameters. At certain values of the parameters a,b and c the z-component of the Rössler system is a *spiky* time series Fig. 3 (see also Refs. [14],[15]).

It can be shown that the Rössler system and the well known Hindmarsh-Rose model [16] of neurons are subsystems of the same differential model with a spiky component [15]. Previously the 'spiky' component of such models was interpreted and studied as a simulation of a neuronal *output*. For the *spontaneous* neuron firing (without external stimulus), however, we suggest to reverse the approach and consider the spiky variable as the main component of the electrical *input* (which naturally should have a 'spiky' character, see above) to the neuron under consideration. For any given neuron the height of the spikes, which the neuron generates, is about the same. However, the heights of the spikes generated by different neurons are different. Also the signals coming from different neurons to the neuron under consideration have to go through the electrochemical passes with different properties. Therefore, the spiky z-time-series (Fig. 3) can naturally represent a multineuron signal, which can be considered as a spontaneous input for the neuron under consideration. If we still use the usual interpretation of the x-component as the membrane potential and the y-component as that taking into account the

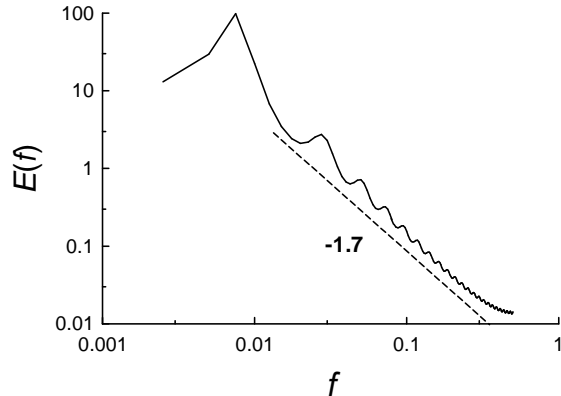


Figure 4: Spectrum of the telegraph signal corresponding to the spike train generated by the x-component fluctuations overcoming the threshold $x = 7$. The dashed straight line indicates a power law Eq. (3) in the log-log scales.

transport of ions across the membrane through the ion channels [16], then the position of the input (the component z) in the first equation of the system Eq. (1) has a good physical background (cf. Ref. [16]). Then, the quadratic nonlinearity in the third equation of the system Eq. (1) can be interpreted as a simple (in the Taylor expansion terms) feedback of the neuron to the main component of the neuronal input. This model with the strong nonlinear feedback can be relevant to the most active neurons of a spontaneously active brain (see below results of an *in vitro* experiment with a spontaneous brain activity). In this simple model the membrane potential x may overcome its threshold value (Fig. 2) due to the deterministic (chaotic) spontaneous stimulus. Let us consider a spike signal resulting from overcoming a threshold value $x = 7$, for instance. Fig. 4 shows spectrum of the telegraph signal corresponding to the spike train. In order to understand what is going on here we show in figure 5 spectrum of the x-component itself. The semi-log scales are used in these figures in order to indicate exponential decay in the spectra (in the semi-log scales this decay corresponds to a straight line):

$$E(f) \sim e^{-f/f_e} \quad (2)$$

While the low-frequency peak in the spectrum corresponds to the fundamental frequency, f_0 , of the Rössler chaotic attractor, the rate of the exponential decay (the slope of the straight line in Fig. 5) provides us with an additional characteristic frequency f_e Eq. (2).

It should be noted that for a wide class of deterministic systems a broadband spectrum with *exponential* decay is a generic feature of their *chaotic*

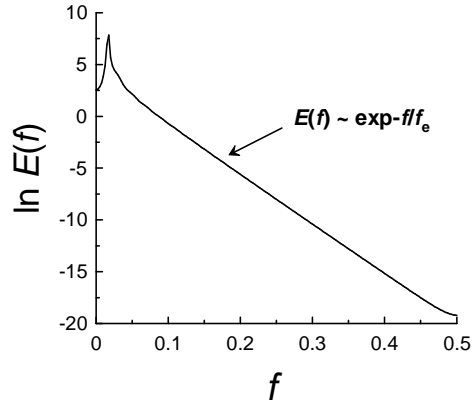


Figure 5: Spectrum of the x-component fluctuations shown in Fig. 2. We used the semi-log axes in order to indicate exponential decay of the spectrum.

solutions Refs. [17]-[20].

Thus Rössler chaotic attractor has two clocks: fundamental with frequency f_0 and decaying with frequency f_e . If one compares Fig. 4 and Fig. 5 one can see that the fundamental clock survived the threshold crossing (with a period doubling, of course, because the only odd threshold crossings in Fig. 2 result in the spike generation). The decaying clock, however, did not survive the threshold crossing: the exponential decay in Fig. 5 has been transformed into a scaling (power law) decay in Fig. 4

$$E(f) \sim f^{-\alpha} \quad (3)$$

(with $\alpha \simeq 1.7$), which has no characteristic frequency (scale invariance).

In order to compare this simple model consideration with the experimental data we have analyzed spike trains obtained *in vitro* from a spontaneous activity in CA3 hippocampal slice culture of a Wistar/ST rat (the raw data and the detail description of the experiment can be found online at <http://hippocampus.jp/data> and in Refs. [2],[3]), In the *in vitro* experiment a functional imaging technique with multicell loading of the calcium fluorophore was used in order to obtain the spike trains of spontaneously active singular neurons (hippocampal pyramidal cells) in the absence of external input. In this experiment different levels of activity were observed for different neurons [2],[3]. We take for our analysis the two most active neurons (<http://hippocampus.jp/data> - Data-006, cell-21, with 800 spikes in the time-series; and a cell-25, with 692 spikes). The spike trains were mapped to telegraph signals as it is described above. Figure 6 shows au-

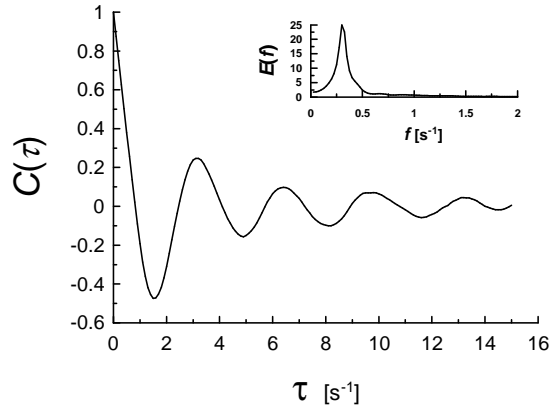


Figure 6: Autocorrelation function for the telegraph signal corresponding to the cell-21 (800 spikes). The insert shows corresponding spectrum.

Autocorrelation function for the telegraph signal corresponding to the cell-21 (800 spikes). Insert in the Fig. 6 shows corresponding spectrum. Both the correlation function and the spectrum provide clear indication of a strong periodic component in the irregular signal (the oscillations in the correlation function and the peak in the spectrum). The periodic component can be seen at frequency $f_0 \simeq 0.3\text{Hz}$. Figure 7 shows the spectrum in log-log scales. One can see that at high frequencies the spectrum exhibits a scaling behavior Eq. (3) (power law: $E(f) \sim f^{-1.7}$, as indicated by the dashed straight line). The real power law can be more pronounced but under the experimental conditions individual spikes emitted at firing rates higher than 5Hz were experimentally inseparable [2],[3]. Figure 8 shows spectrum of the telegraph signals corresponding to the spike train obtained for the cell-25 (D-006, 692 spikes). The spectrum is rather similar to the spectrum shown in Fig. 7 (for cell-21). The more broad peak in Fig. 8 can be related to the poorer statistics for the cell-25 (number of spikes 692) in comparison with cell-21 (number of spikes 800). One can compare Figs. 7 and 8 with Fig. 4 to see very good reproduction of the main spectral properties. It should be noted that *in vivo* neuron signals can be much more complex. The relatively simple firing of the *in vitro* spontaneously active hippocampal neurons can be used in order to reveal the underlying neuron dynamics.

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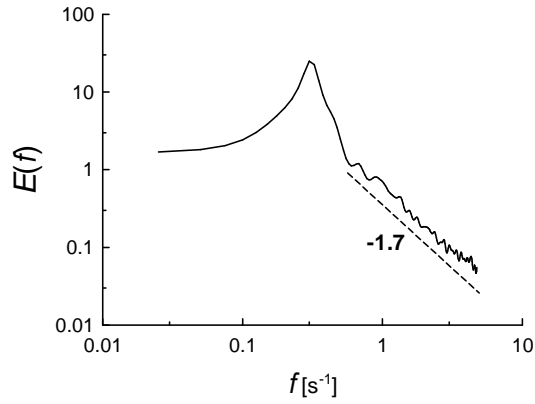


Figure 7: Spectrum of the telegraph signal corresponding to the cell-21 (800 spikes) in log-log scales. The dashed straight line indicates a power law Eq. 4: $E(f) \sim f^{-1.7}$.

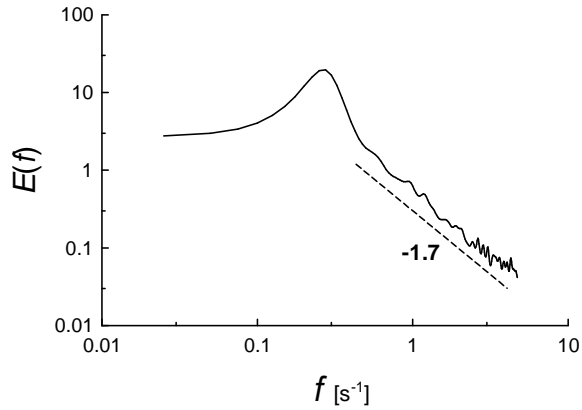


Figure 8: As in Fig. 7 but for cell-25 (D-006, 692 spikes).

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