Complexity Adjusted Soft-Output Sphere Decoding by Adaptive LLR Clipping

Konstantinos Nikitopoulos, *Member, IEEE,* and Gerd Ascheid,

Senior Member, IEEE

arXiv:1011.2113v1 [cs.IT] 9 Nov 2010

Abstract

A-posteriori probability (APP) receivers operating over multiple-input, multiple-output channels provide enhanced performance at the cost of increased complexity. However, employing full APP processing over favorable transmission environments, where less efficient approaches may already provide the required performance at a reduced complexity, results in waste of resources (e.g, processing energy/power). Therefore, for slowly varying channel statistics, substantial complexity savings can be achieved by simple adaptive schemes which perform performance tracking and adjust the complexity of the soft output sphere decoder by adaptively setting the related log-likelihood ratio (LLR) clipping value.

Index Terms

MIMO systems, soft-output detection, sphere decoding

I. INTRODUCTION

Multiple-input, multiple-output (MIMO) systems with spatial multiplexing offer increased spectral efficiency and therefore, they have been adopted by several upcoming wireless communication standards like IEEE 802.11n and IEEE 802.16e. *A-posteriori probability* (APP) receivers, which exploit soft information content, can efficiently decode such schemes but at the cost of increased processing requirements. Although accurate (full complexity) APP receiver processing

may be demanded for achieving the required performance when transmitting over unfavorable channels (i.e., ill conditioned and of low signal-to-noise ratio (SNR)), it may be unnecessary for favorable transmission conditions where the required performance can be achieved even by approximate, reduced complexity soft information processing.

In this context, a low overhead adaptive approach is proposed aiming at avoiding such unnecessary soft-output detection processing. It targets practical scenarios where the channel statistics lead to only slowly varying (over subsequent code blocks) *average* achievable bit-error-rate (BER) performance. The proposed output detector is realized by means of the depth-first sphere decoder (SD) of [1] which can ensure the (exact) *max-log* MAP performance, when required. The scheme adjusts the complexity/performance characteristics of the SD by adaptively setting the *log-likelihood ratio* (LLR) clipping value after tracking and comparing the provided BER performance to the target-error-rate (TER). LLR clipping bounds the dynamic range of the LLRs and decreases the detector's complexity at the cost of reduced performance. It has been originally proposed for aligning the detector's complexity to the unavoidable performance loss originating from the fixed-point implementation [1]. Therefore, typically the LLR clipping value is determined by the selected fixed-point accuracy (selected via extensive simulations which link the fixed-point accuracy to the achievable performance).

By adjusting the soft-output demapping complexity via adaptive LLR clipping the proposed method avoids the need for supporting several soft-output detection methods of different fixed complexity/performance characteristics. Additionally, any *performance prediction* burden related to the detector's complexity minimization for a given transmission scenario (by choosing the less complex detector or by selecting the optimal LLR value) is avoided. In this work, the efficiency of employing such adaptive schemes is demonstrated, without claiming any optimality for the specific tracking algorithm.

II. APP Receiver Processing for MIMO Systems

In MIMO transmission with M_T transmit and $M_R \geq M_T$ receive antennas, at the u-th MIMO channel utilization, the interleaved coded bits are grouped into blocks $B_{t,u}$ ($t = 1, ..., M_T$ and $u = 1, ..., U$ with U being the number of channel utilizations per code block) in order to be mapped onto symbols $s_{t,u}$ of a constellation set S of cardinality $|S|$. The bipolar k-th bit resides in block $B_{\lceil k/\log_2 |S| \rceil, u}$ and the blocks $B_{t,u}$ are mapped onto the symbols $s_{t,u}$ by a given mapping

function (e.g., Gray mapping). The corresponding received $M_R \times 1$ vector $\mathbf{y}_u$ is, then, given by

$$\mathbf{y}_u = \mathbf{H}_u \mathbf{s}_u + \mathbf{n}_u, \tag{1}$$

where $\mathbf{H}_u$ is the $M_R \times M_T$ complex channel matrix which can be QR decomposed into $\mathbf{H}_u = \mathbf{Q}_u \mathbf{R}_u$, with $\mathbf{Q}_u$ a unitary $M_R \times M_T$ matrix and $\mathbf{R}_u$ an $M_T \times M_T$ upper triangular matrix with elements $R_{i,j,u}$ and real-valued positive diagonal entries. By $\mathbf{s}_u = [s_{1,u}, s_{2,u}, ..., s_{M_T,u}]^T$ the transmitted symbol vector is denoted, while $\mathbf{n}_u$ is the noise vector consisting of i.i.d., zero-mean, complex, Gaussian samples with variance $2\sigma_n^2$.

The soft-output detector calculates the *a-posteriori* LLRs for all the symbols residing in the frame to be decoded. Assuming statistically independent bits (due to interleaving) and under the *max-log* approximation, the LLR calculation problem can be reformulated to the following constraint tree-search problem, known as SD [2]

$$L_D (c_{b,i,u}) \approx \frac{1}{2\sigma_n^2} \min_{\mathbf{s}_u \in S_{b,i,u}^{-1}} \|\mathbf{y}'_u - \mathbf{R}_u \mathbf{s}_u\|^2 -$$

$$\frac{1}{2\sigma_n^2} \min_{\mathbf{s}_u \in S_{b,i,u}^{+1}} \|\mathbf{y}'_u - \mathbf{R}_u \mathbf{s}_u\|^2 \tag{2}$$

where $c_{b,i,u}$ is the b-th bit of the i-th entry of $\mathbf{s}_u$, $S_{b,i,u}^{\pm 1}$ are the sub-sets of possible $\mathbf{s}_u$ symbol sequences having the b-th bit value of their i-th $\mathbf{s}_u$ entry equal to ± 1 and $\mathbf{y}'_u = \mathbf{Q}_u^H \mathbf{y}_u = \left[y'_{1,u}, y'_{2,u}, ..., y'_{M_T,u} \right]^T$. Then the corresponding tree search problem can be solved according to [1].

The resulted soft information is de-interleaved and fed to the soft-input, soft-output (SISO) channel decoder as *a-priori* information, with $\tilde{L}_A (\tilde{c}_k)$ denoting the *a-priori* information of the $\tilde{c}_k$ encoded bit. This information is employed for calculating the channel decoder's *a-posteriori* soft information $\tilde{L}_D (\tilde{c}_k)$ whose sign provides the corresponding decoded bit $\hat{c}_k$. Practically, this *a-posteriori* information can be efficiently calculated by the BCJR-MAP algorithm [3].

By employing the *max-log* approximation (for easier inspection and without loss of generality), and since

$$P[\tilde{c}_i] = \frac{\exp\left(-\left|\tilde{L}_A(\tilde{c}_i)\right|/2\right)}{1 + \exp\left(-\left|\tilde{L}_A(\tilde{c}_i)\right|\right)} \exp\left(\tilde{c}_i \tilde{L}_A(\tilde{c}_i)/2\right), \tag{3}$$

after extracting some mutually exclusive terms, it can be easily shown that

$$\tilde{L}_D (\tilde{c}_k) \approx \frac{1}{2} \max_{\tilde{\mathbf{c}}:\tilde{C}_k^{+1}} \left\{ \sum_{i=1}^{k} \left(\tilde{c}_i \tilde{L}_A(\tilde{c}_i) - \left|\tilde{L}_A(\tilde{c}_i)\right| \right) \right\} -$$

$$\frac{1}{2} \max_{\tilde{\mathbf{c}}:\tilde{C}_k^{-1}} \left\{ \sum_{i=1}^{k} \left(\tilde{c}_i \tilde{L}_A\left(\tilde{c}_i\right) - \left| \tilde{L}_A\left(\tilde{c}_i\right) \right| \right) \right\}, \tag{4}$$

with $\tilde{C}_k^{\pm 1}$ being the set of possible encoded sequences $\tilde{\mathbf{c}}$, of length K, with their k-th bit equal to ± 1. The candidate sequences for maximizing each of the terms in (4) are those with the corresponding non-positive sums being as close to zero as possible. Therefore, it is not expected that highly unlikely bits (i.e., with high $\left| \tilde{L}_A\left(\tilde{c}_i\right) \right|$ value and sign opposite to $\tilde{L}_A\left(\tilde{c}_i\right)$, see (3)) will belong to those sequences (except possibly the bit under constraint $\tilde{c}_k$) since they contribute with highly negative values. In addition, since the candidate bits (except for $\tilde{c}_k$) are expected to contribute with values close to zero, bounding the $\left| \tilde{L}_A\left(\tilde{c}_k\right) \right|$ value of the bit under constraint by a large value will not affect the final sign of (4) and, consequently, the decoded bit. Therefore, approximate calculation of the *a-priori* information of the the highly unlikely bits is not expected to significantly affect the resulting performance. In the same way, highly likely bits (i.e., with high $\left| \tilde{L}_A\left(\tilde{c}_i\right) \right|$ value and of the same sign with $\tilde{L}_A\left(\tilde{c}_i\right)$) will contribute with a zero value independently of their actual *a-priori* information (see (4)). The last observations lead to the conclusion that approximate (and thus of lower complexity) calculation of the strong soft information, (i.e., of high $\left| \tilde{L}_A\left(\tilde{c}_k\right) \right|$) is not expected to significantly affect the outcome of the SISO channel decoder and, therefore, the provided performance.

III. ADAPTIVE LLR CLIPPING

The problem to be addressed is to adaptively find the minimum clipping value, and thus to minimize the SD complexity, which does not jeopardize the TER performance. To this direction, a coarse LLR clipping value $L_{cl}^{(0)}$ is initially set, large enough to safely preserve the TER. Then, L_{cl} is adaptively reduced so that the provided performance reaches the TER. In detail, the following steps are performed:

1) *BER Evaluation*: According to [4] the error probability of the hard decision of the bit c with *a-posteriori* LLR L_c is

$$P_e(c) = \left(1 + \exp\left(\left| L\left(c\right) \right|\right)\right)^{-1}. \tag{5}$$

Since the average error rate is dominated by the bits with the smaller LLR magnitudes, the BER of the m-th code block can be approximated as

$$\hat{P}_b^{(m)} \approx \frac{1}{N} \sum_{r=1}^{n} P_e\left(\breve{c}_{r,m}^{(I)}\right), \quad n_I \leq N_I \tag{6}$$

with $\breve{c}_{r,m}^{(I)}$ being the information bit of the m-th code block with the r-th smallest $\left| \tilde{L}_D \left(\tilde{c}_k \right) \right|$ value, N being the number of information bits and with the accuracy of the estimate increasing with n.

2) *Clipping Value Initialization*: For the bits which meet the TER constraint already before channel decoding (i.e., $\left| \tilde{L}_A \left(\tilde{c}_k \right) \right| > \tilde{L}_{TER} = \ln(TER^{-1} - 1) \approx -\ln(TER)$, see (5)) their average BER performance after channel decoding is expected to be even better. Therefore, since no significant effect on the SISO decoding output is expected when clipping the high magnitude LLRs, setting $L_{cl}^{(0)} = \tilde{L}_{TER}$ is not expected to compromise the TER performance. This is verified in by simulations in Section IV.

3) *Clipping Value Adaptation*: The following simple adaptive algorithm can be used to adjust the LLR clipping value to the TER performance

$$L_{cl,c}^{(m)} = L_{cl}^{(m-1)} - \mu \left[\ln \left(TER \right) - \ln \left(\hat{P}_b^{(m-1)} \right) \right] \tag{7}$$

$$L_{cl}^{(m)} = \max \left\{ \min \left\{ L_{TER}, L_{cl,c}^{(m)} \right\}, |L|_{\min} \right\} \tag{8}$$

with μ being the step size parameter and $|L|_{\min}$ being the minimum possible LLR magnitude determined by the fixed point accuracy. Then, by (8) non-positive L_{cl} values are avoided ($\max$ operation). Additionally, unnecessary L_{cl} (and thus complexity) increase is avoided for those cases where the best achievable performance is worse than the TER ($\min$ operation).

The main complexity overhead of the proposed approach originates from the BER evaluation part and in detail in the calculations of (5)-(6) and in the compare-select process required to identify the dominant n bits, which are performed once per code block. However, as shown in Section IV, a small n (≤ 50) provides adequate accuracy. Therefore, the overhead is very small compared to the (per code block) SD complexity gain.

IV. SIMULATIONS

A 4×4 MIMO system is assumed, operating over a spatially and temporally uncorrelated Rayleigh flat-fading channel. The encoded bits are mapped onto 16-QAM via Gray coding. A systematic $(5/7)_8$ recursive convolutional code of rate 1/2 is employed with a code block of $N = 1152$ information bits. The log-MAP BCJR algorithm has been employed for SISO

channel decoding. Tracking takes place over 100 consequent frames and $\mu = 0.1$ unless otherwise specified.

In Fig. 1 the BER performance of the proposed approach is shown for various TER values (10^{-4}, 10^{-3}, 10^{-2}). The examined n values equal 100 and N with a significant performance difference for low TERs since BER's underestimation leads faster to the small $L_{cl}^{(m)}$ values (but still with performance below the TER). To validate the efficiency of the L_{cl} initialization, $\mu = 0$ is also considered. Then, as expected, a performance divergence is observed only when the TER performance is worse than the best achievable. However, the resulted degraded performance still significantly outperforms the TER which denotes that a great amount of unnecessary processing takes place. The last is reduced for $\mu = 0.1$ and $n = N$, but with a provided performance which can be slightly worse than the TER in the high SNR regime and for high TER values (see TER=10^{-4} and SNR > 12 dB) due to the oscillations of the clipping value around the optimal. In practice, this performance loss can be reduced by setting a smaller μ value, at the cost of reduced convergence speed, or by setting a TER value slightly lower than the actual.

In Fig. 2 the complexity is depicted in terms of average visited nodes per MIMO channel utilization (i.e., 144 MIMO channel utilizations form a code block). Significant complexity gains can be observed over the whole SNR range even for $\mu = 0$ when compared to the non-clipped case. For example, for SNR=14 dB and TER=10^{-4} there is gain of 92% compared to the non-clipped SD (requiring $1.49 \cdot 10^3$ visited nodes, by simulations not depicted here). Substantial complexity gains can be further achieved by the proposed scheme when the best achievable performance is significantly better than the TER. For example, as shown in Fig. 2, an additional gain of about 53% is provided for SNR=14 dB, TER=10^{-4} and $\mu = 0.1$, while the additional gain is about 40% for TER=10^{-2}.

As discussed in Section I, the proposed approach avoids any performance prediction burden. However, it would be interesting to examine the additional complexity gains under optimal L_{cl} knowledge, even without considering the computational overhead to calculate it. Fig. 2 shows that a clipping value of $\ln\left(1/10^{-2} - 1\right)$ without tracking achieves a TER of 10^{-3} at 12 dB. Then, additional complexity gains of only 5% could be achieved compared to the proposed scheme targeting the same TER (see Fig. 2). The same clipping value achieves a TER of 10^{-4} at 13.75 dB. In this case, the proposed approach would be even slightly less complex (due the small BER degradation) and with a gain of about 50% compared to the $\mu = 0$, TER=10^{-4} case.

V. Conclusion

It is demonstrated that the complexity of the soft-output sphere decoding can be efficiently adjusted to the required BER performance by simple schemes which track the provided performance and adaptively adjust the LLR clipping value, when the channel statistics vary slowly. Then, substantial complexity savings can be achieved under favorable transmission conditions without requiring computational intensive *performance prediction* methods which would reduce the practicality and applicability of the approach.

References

[1] C. Studer, A. Burg, and H. Bölcskei, "Soft-output sphere decoding: algorithms and VLSI implementation," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 290–300, 2008.

[2] B. M. Hochwald, and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, no. 3, pp. 389–399, Mar. 2003.

[3] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, Mar. 1974.

[4] I. Land, and P. A. Hoeher, "Log-likelihood values and Monte Carlo simulation - some fundamental results," in *Proc. of Int. Symposium on Turbo Codes and Related Topics*, 2000.
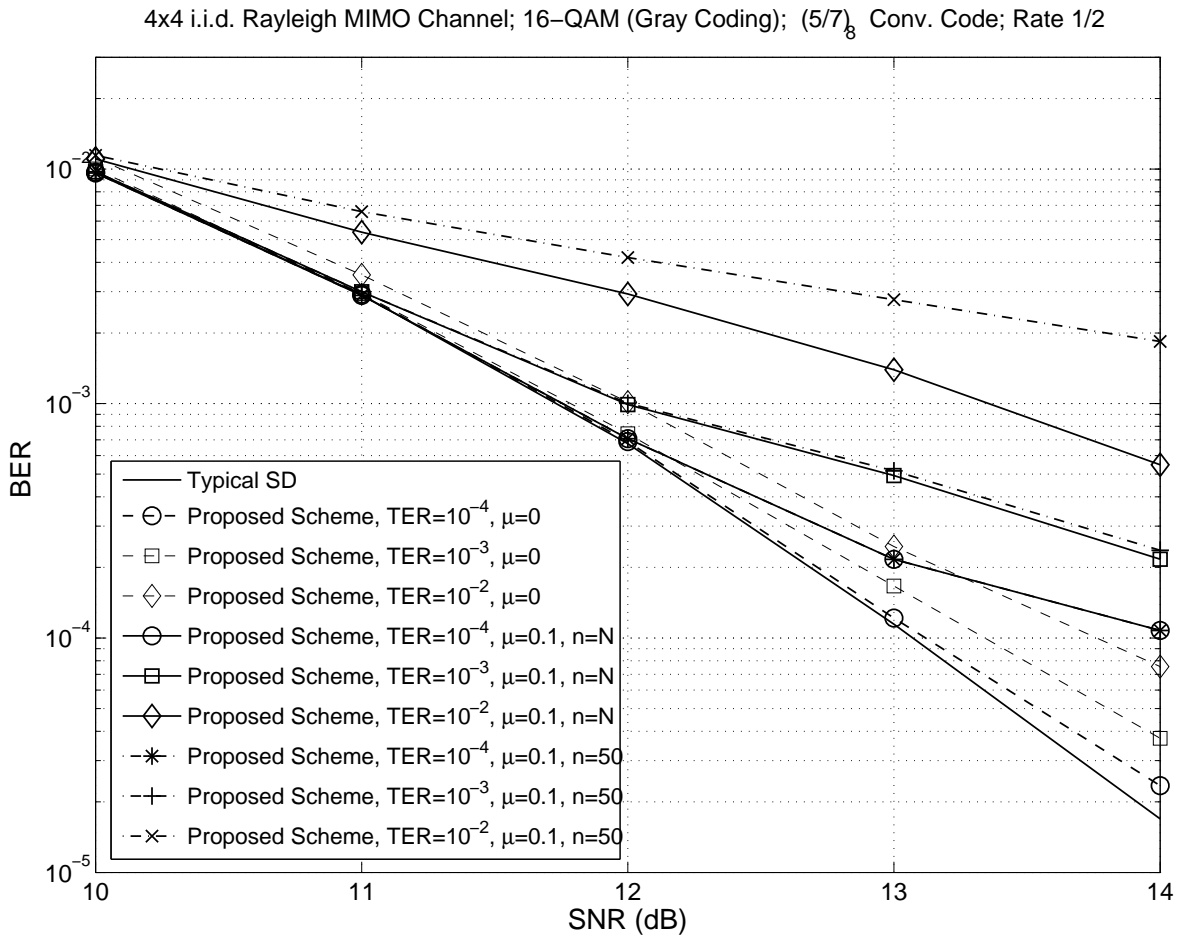
4x4 i.i.d. Rayleigh MIMO Channel; 16−QAM (Gray Coding); $(5/7)_8$ Conv. Code; Rate 1/2

Legend:
- Typical SD
- Proposed Scheme, TER=10^{-4}, $\mu=0$
- Proposed Scheme, TER=10^{-3}, $\mu=0$
- Proposed Scheme, TER=10^{-2}, $\mu=0$
- Proposed Scheme, TER=10^{-4}, $\mu=0.1$, n=N
- Proposed Scheme, TER=10^{-3}, $\mu=0.1$, n=N
- Proposed Scheme, TER=10^{-2}, $\mu=0.1$, n=N
- Proposed Scheme, TER=10^{-4}, $\mu=0.1$, n=50
- Proposed Scheme, TER=10^{-3}, $\mu=0.1$, n=50
- Proposed Scheme, TER=10^{-2}, $\mu=0.1$, n=50

Fig. 1.   BER performance for several TER values.

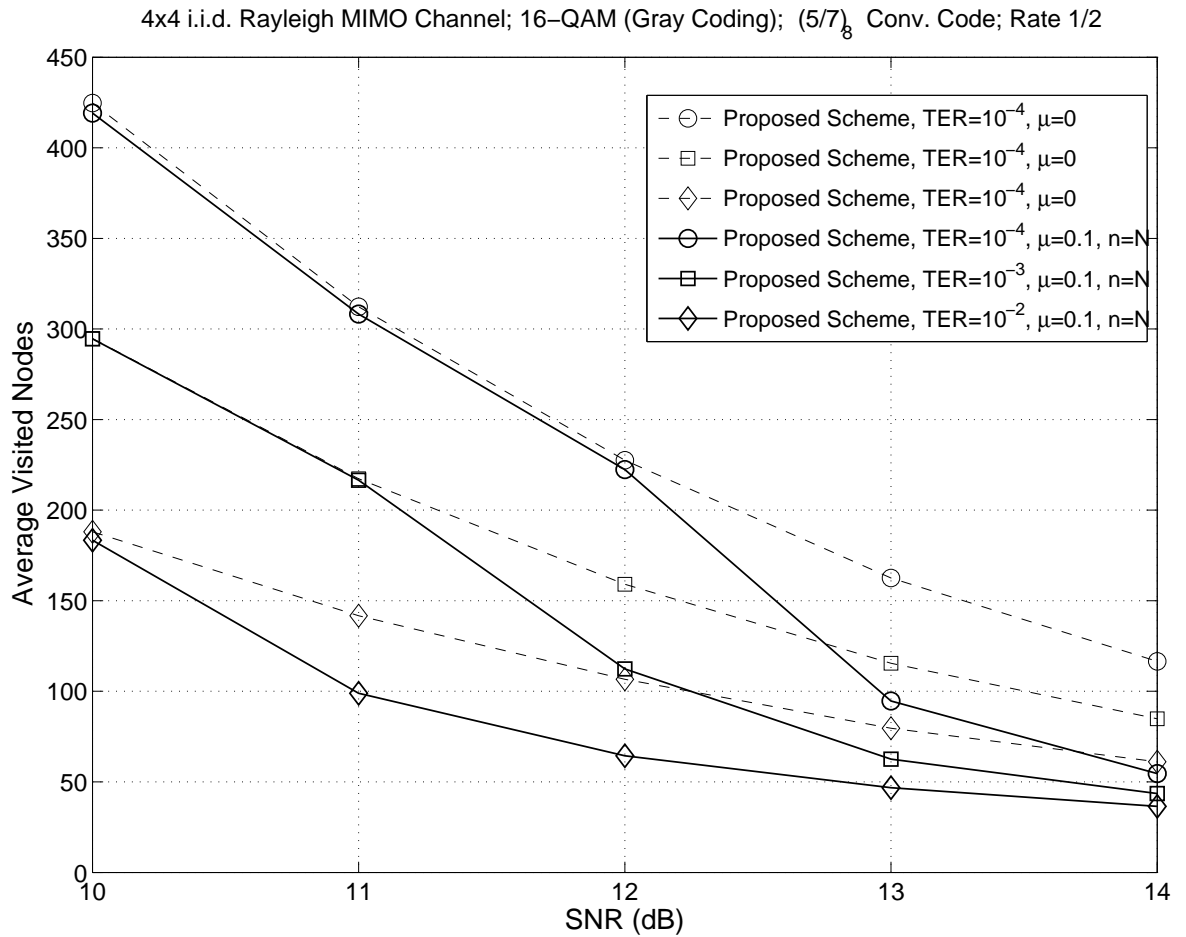4x4 i.i.d. Rayleigh MIMO Channel; 16–QAM (Gray Coding); $(5/7)_8$ Conv. Code; Rate 1/2

Fig. 2.   Complexity, in terms of average visited nodes, for several TER values.