Left lane road electrification

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(Dated: November 2, 2010)

We propose the partial electrification of multilane motorways in suburban areas as a practical alternative in order to reduce the weight and price of electrically-powered automobiles. These could then be supplied energy en-route, thus significantly reducing the weight and cost of vehicle-mounted batteries. Our theoretical study is based on the circumstances of Madrid (Spain), a six million inhabitant region, but should be easily adaptable to other metropolitan areas.

PACS numbers:

I. MOTIVATION

As is well known, transportation consumes a large fraction of the crude oil resources in the world, as reprinted in table I.

TABLE I: Percentage of oil consumption in transport, per year and geographic region, and projected growth. Source: [1]

Annum	1980	2000	2015	2030
USA	52	66	71	72
Western Europe	31	50	57	58
Pacific	27	38	44	44
Japan	23	36	39	38
Eastern Europe/Eurasia	24	37	45	45
Asia (not OCDE)	25	36	42	51
Middle East	32	36	39	43
Africa	46	51	51	54
Latin America	39	47	52	54
World	**38**	**51**	**54**	**57**

The conclusion from the table is obvious. Half of the world's crude oil consumption is already due to transportation, and the trend (should no action be taken) is to increase further. Proven oil reserves can satisfy human demand at current levels for a good 40 years, but long term transportation solutions will have to be based on alternative energy sources.

A. Madrid's dependence on private commuting and import oil

Our modern society is fully dependent on private automobiles for road transportation. Although the city of Madrid is endowed with an excellent city rail system, the regional railways are clearly insufficient and most of the commuting is, as in many other urban agglomerations, done on the roads. Table II presents the number of utility vehicles, amounting to about half a percent of the world's total estimated at 650 million operating automobiles.

TABLE II: Spain's and Madrid's automobile number [2].

Category	Madrid's region	Total in Spain
Family cars	3 376 000	22 145 000
Trucks and Pick-ups	669 000	5 192 000
Buses	11 300	62 000
Motorcycles	259 000	2 500 000
Industry and agriculture	17 000	213 000
Other motorized units	78 000	855 000

80% of homes in the Madrid region own an automobile, and 50% of those not able to walk to work employ it daily, averaging to over two million vehicles daily on the road.

Madrid's energy dependence is absolute. The region produces barely 4% of the energy it consumes. In the bigger national picture, Spain reaches 20.9% of energy self-sufficiency, while a negligible 0.2% of all oil consumed is domestically produced. Between 1985 and 2007 the consumption of Diesel fuels soared by a factor 4, while gasoline consumption remained stable. We estimate that the spanish economy pays foreign suppliers 92500 million euro for oil imports [13].

Madrid hosts an eighth of the total automobile fleet of Spain, yielding up to 5000 million euro per year in the transportation sector. The total budget of the state's administration, for comparison, is a factor of two higher. All this money is handed out to oil producers (Nigeria, Lybia, Saudi Arabia, Iran, Irak, Mexico, Venezuela and Russia are the major ones).

B. Pollution by automobile fleet

A diesel-powered vehicle typically emits to the atmosphere 150-200 *grams* of CO_2 per kilometer. The 22 million vehicles in Spain, assuming some 10000 km of yearly usage, are responsible for 33 million metric tons of carbon dioxide per year. This is about a fourth of the total emission (a figure also similar to the European total percentage). Best modern vehicles can lower the carbon

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emission to the level of 30-40 *grams* (from well to wheel) per kilometer, much improvement is possible, but there seems to exist a lowest floor for optimization.

If the vehicle fleet could be made electric, a large part of the energy consumption could be turned to alternative sources. In the case of Spain, 35% of the electricity production mix is not based on carbon sources, and this gain would be immediate.

In addition, the rest of the energy would be produced at industrial installations outside the city ("pollute elsewhere" principle) which would drastically reduce the levels of Carbon monoxide (87% of Madrid totals due to vehicle emissions) or Nitrogen oxides (66%).

Sound pollution could also be reduced by adopting an electric vehicle fleet. For speeds above 50 km/h (city speed limit inside the inner belt of Madrid) there is little possible gain since most of the friction is produced in the tire-pavement contact [4] leading to 70-80 dB sound levels, but below that speed the explosion motor is the main source of noise. Electrical motors are remarkably silent, rated at about 10 dB.

C. High cost of the electric vehicle

By the end of 2010, the spanish government hopes to reach 2000 electric vehicles and 540 static battery charging points. Looking ahead, China plans to have 50 000 electric cars in its territory by the end of 2012. A similar goal for Spain would seem achievable. Israel's "Better Place" programme, a world referent, scaled to Madrid's size, would entail 140 000 light electric vehicles and a cost of about 400 million euro of public expenditure (not very likely to be carried out). However, it is clear that the stumbling block to widespread adoption of the electric vehicle is its high cost at the dealer.

TABLE III: Estimated cost of several electrical vehicles upon reaching a consumer in Spain (not necessary official manufacturer prices) and several significant data about their battery's capacity and autonomy. Notes and abbreviations: Mitsubishi Innovative Electric Vehicle*. Renault Zero Emission Fluence†. Available only for monthly rental $^{\&}$.

Model	MIEV*	Tesla S	Nissan Leaf	RZE†	Mercedes E-Smart	BYD e6
Price (euro)	$>$ 34000	$\simeq$ 40 000	33 400	28000	700/month$^{\&}$	30 000
Battery storage	16 kwh	42 kwh	24 kwh	22 kwh	16.5 kwh	60 kwh
Range (km)	120	260	160	160	135	300
Power (kw)	47	-	80	70	30	75
Weight (kg)	1100	1815	1270	1540	900	2020

Table III presents estimates of current prices for electrical vehicles in the final consumer market.

These prices are well in excess of the typical 12000 euro that a consumer pays for a small family utility vehicle conventionally powered by a diesel engine. Fiscal incentives to purchase electric cars, of order 2000 euros, can hardly overcome the price step. While some reduction of the price is achievable with mass-production and sales in the future, one should expect electric cars to still remain a luxury product for a while.

A two-battery set with 70 kwh energy storage and 180 km range can cost up to 14 000 euro; to achieve a price reduction of 10 000 euro per vehicle to curb it into marketability, it is necessary to significantly reduce the cost of its electric batteries. This is not foreseeable by any price reduction of Lithium, other metals, or manufacturing costs [5] New ideas are clearly a necessity if the benefits of electric automotion are to be realized.

The advantages of the electric vehicle, once purchased, are significant in terms of cheaper energy consumption, lower maintenance and operating costs, lower pollution, etc. But the public, generally not holding large amounts of capital to spend upfront, prefers a cheaper initial deal at the expense of the higher maintenance and operating costs of diesel fueled vehicles.

Can one find a scheme where the upfront purchase is significantly cheapened, even if later the expenses are higher? We think providing the electricity on the road, in analogy with the railways, is a possible answer. Batteries would then store much less energy and make the car cheaper.

In this article we discuss this possibility at a theoretical level and present preliminary investigations that suggest that the option should be further considered.

II. THE FLEET'S ENERGY SUPPLY

A. Reasonable consumption

Simple estimates of well-to-wheel efficiencies [6] for very efficient fossile fuel vehicles (models similar to the Honda Civic) yield about 2.2 kilometers per *kwh* of gasoline energy. Electric vehicles with proven efficiencies (such as the Tesla Roadster) can reach 4.8 *km/kwh*, a good factor of 2 above their fossile fueled counterparts. Low-weight hybrid vehicles fare somewhere between the two estimates. However a lot of the energy expense associated to an electric vehicle is at the power plant and distribution. The actual range per *kwh* in the battery is 9.1 *km*, which is a key figure for us.
If these nine kilometers are run at a moderate highway speed of 90 *km/h*, the power consumption is 10 *kw*. This figure is for optimal vehicle circulation.

In table IV we give some further estimates of the power consumed by actual vehicles reaching the consumer market.

TABLE IV: Power in *kw* of several electric vehicles that are available in the market in Spain, or will shortly be so.

MIEV	47
Nissan Leaf	80
Renault ZE Fluence	70
Mercedes E-Smart	30
BYD e6	75

Let us now check these maker's estimate with simple physical arguments and see how much more power should we wish delivered to a vehicle.

Aerodynamic resistance to the vehicle's advance can be estimated as

$$F = \frac{C_d}{2}\rho v^2 S \tag{1}$$

where the air density is about $\rho \simeq 1.22\ kg/m^3$, and the effective area perpendicular to the advance (area times the aerodynamic coefficient) is, for a small family car such as the Opel Astra, $C_d S \simeq 0.68\ m^2$. Altogether, at the reference speed of 90 *km/h*, this implies a power consumption of $P = 6.5\ kw$.

Ground friction can be estimated as

$$F_r = C_r \times (mg) \tag{2}$$

which, for a rolling motion friction coefficient $C_r = 0.03$ makes a vehicle loaded to 1500 *kg* of mass spend an additional 11 250 *watt*. Assuming a 70 % efficiency in the electric motor and the transmission, we obtain an estimate for the power consumption in optimal rolling of $P = 25\ kw$, for a car which is analogous to convenience vehicles in current use.

Finally one needs to plan for the additional stress caused on the battery upon acceleration, necessary upon joining the highway, but also upon conditions of dense traffic when braking and reacceleration become necessary.

A 1500 *kg* automobile accelerating in 10 *s* from rest to 90 *km/h* will need some 46.9 *kw*.
Let us therefore adopt the figure of 50 *kw* as the desirable maximum power to be provided to an average small automobile by an energy delivery system.

Can one exceed this demand by sustained periodic traffic circumstances? We can negatively answer with a simple model with a cosinusoidal profile

$$v(t) = v_0 \cos^2\left(\frac{2\pi t}{T}\right) \ . \tag{3}$$

The root-mean squared average acceleration is (after a simple integration over a half-period)

$$\bar{a} = \frac{\sqrt{2}\pi v_0}{T} \ .$$

To exceed 50 *kw* one needs, for the 1500 *kg* vehicle, with 40 meter races between two full stops, and peak velocity of 47 *km/h*, acceleration and breaking times of about 6 seconds. We do not see this intensity of traffic as realistically sustainable in a motorway and therefore conclude that 50 *kw* can be taken as a standard in most circumstances. If a vehicle instantaneously needs more power delivered (air condition and on-board amenities are further expenses), it can detract it from the batteries; if in smaller need, it can reload them. All in all, we think that 50 *kw* is a reasonable demand on the external road energy supplier.

B. Electricity supply

With a horizontal area seldom exceeding 5-6 m^2, it is clear that vehicle mounted photovoltaic panels are not an option for family vehicles in motion, since only 1-2 *kw* of energy can be obtained. Hydrogen fuel cells are a more practical option for producing energy on board the vehicle by chemical means, but they are pricy.

The current bet in many countries, including Spain, is for the deployment of electric vehicles that do not produce their own power but instead store it in batteries. Supply will be provided by both charging points at home or parking lots, and battery pack replacement stations, "electrolineras" in spanish language, in analogy with currently used "gasolineras", following the battery renting model. We find this model unpractical if scaled to a large fraction of the current automobile fleet, given the enormous inmobilized capital invested in batteries that the owner of the electric supply station needs to have at customer's disposal.

What we propose is to have an electric line running parallel to the highway, such that the driver can hook to the same and obtain the standard 50 *kw* of power from the line with wich to operate the vehicle and recharge its

batteries without stopping. This makes middle and long range travel possible with very small battery packs.
This concept seems a reasonable option and is the model employed by developed railway lines. Many cities also operated electric buses in the past, that were supplied by aerial electric lines. Its adoption would imply the possibility of much reducing battery packs in electric vehicles.

In the particular case of Madrid, the municipaities most distant from a highway susceptible of electrification are the (sparsely populated) towns of Rascafría (26 *km* to A1 near Buitrago), Valdemaqueda (21 *km* to M-501 near Navas) and Valle San Juan (20 *km* to A3 near Villarejo). Commuters from these towns would need batteries with a range of 50 *km* at least. For most of the six million inhabitants, it would be sufficient to have batteries with ranges of the order 20-30 *km* for all regional driving purposes.

III. THE ELECTRIC LINE

Railways are often supplied by a suspended catenary line placed directly on top of the rails. This is possible by the large standardization of rail wagons, since the operators are one or few companies per rail line and acommodate their vehicles to the height of the infrastructure. We think this is a less favorable option for a road electrification given the many different vehicle heights. If large trucks were to circulate by the same electrified lane ever, small family cars would be forced to carry extremely long poles to contact the electricity carrier.

To have the electricity line on the right side of the road, where it would naturally fit in countries with wheel on the vehicle's left by the smaller vehicle speeds on the right lane, has the inconvenience of numerous line interruptions due to the highway exits on that side.

Many underground railways and airport cars take electricity from the surface railing. This we do not deem safe enough in the open highway, given the possibility of people, animals, or machines shortcircuiting a system on the ground.

Therefore we opt for temptatively proposing an electrification of the left lane of the road, with two suspended cables on the side.

A. Electric power necessary

At a speed of 90 km/h the maximum occupation of the lane that should be allowed is 25 vehicles (implying a very short breaking distance of 40 meters). At 50 *kw* of standard supply per vehicle, this gives us an estimate of 1.25 *Mw* supplied per kilometer of line. If one lane for each of the two traffic directions is electrified, the total power needed would be 2.5 Mw/km.

A standard 1 *Gw* Westinghouse nuclear reactor can therefore supply 400 *km* of thus partially electrified highway. Tha main network of highways in the region of Madrid has 632 *km* as of mid-2010 (out of a 2610 *km* network that includes secondary roads). Therefore we would conceive this road electrification to be supplied by one additional power plant with two reactors or equivalent generation capacity by other means.
This power generation (and more) will be necessary in any case if the electric vehicle is adopted as a standard. The only difference with our proposal is that the energy be transfered on the road instead of static charging points.

B. Voltage choice

The choice of a specific voltage is very much region-dependent and worldwide carmakers will need to be flexible in adapting transformers to their vehicles to feed typical electrical motors with 100 V of DC.

For a road electrification it seems natural to adopt the local railway standards, to easily find civil contractors with the necessary expertise. Turning to Madrid, the underground metro system (5 *km* characteristic distance) is electrified at 600 V of DC. Most regional railways (50 *km* ranges), to reduce Ohmic losses, are fed at the higher 3000 V DC. Finally, high speed railways (with 500 *km* lines) built in the last two decades operate at 25 kV AC.

Let us briefly examine Ohmic losses. Combining Joule's and Ohm's laws, the handed-out power and the lost power are related by means of the voltage and the electrical resistance of the line

$$P_{\rm Joule} = \frac{RP^2}{V^2} \ . \tag{4}$$

For a wire of uniform cross section, the resistance is elementarily given by $R = \rho \frac{L}{S}$.

Steel at a temperature of 18 Celsius has a resistivity coefficient 0.098×10^{-6} $Ohm\ m$, and Aluminum at equal temperature 0.028×10^{-6} $Ohm\ m$ [7]. A cable of section 500 mm^2, yields a resistance per kilometer of 0.2 and 0.05 *Ohm* respectively.
Often used is the so called "Condor" cable, a composite of both materials, with a resistance of about 0.07 Ohm/km, somewhat intermediate.

If losses are to be kept below 1%, there is a maximum distance along which one wants to transport a given amount of power, obtainable from Eq. 4 as

$$L \le \sqrt{1\% \frac{V^2}{(R/km)(P/km)}} \ . \tag{5}$$

Considering the three railway voltages mentioned above, we find that, if we want to guarantee the supply of 1.25 Mw to each kilometer of cable with resistance 0.07 Ohm/km, the approximate maximum lengths L are

- For 600 V DC, 200 m.
- For 3000 V DC, 1000 m.

- For 25 kV AC, 8.4 km.

The final choice would have to be made by car designers in cooperation with the utility company. However, in view of these numbers, an interesting possibility is to combine an underground high-voltage cable (at 25 kV or up) connected at kilometer intervals to a transformer and rectifier that steps down the voltage to 3000 V and feeds the aerial voltage-carrying line. This guarantees acceptable losses and makes automobile-mounted transformers rather simple.

The system would share elements with a common road illumination setup, with buried cables and a vertical steel bar every 25-50 meters. Except, more like in railway electrifications, two catenaries (one grounded and one carrying the 3000 V voltage) would hang from them. Every kilometer or so the underground cable would feed the signal-carrying aerial. The situation is sketched in Fig. 1.

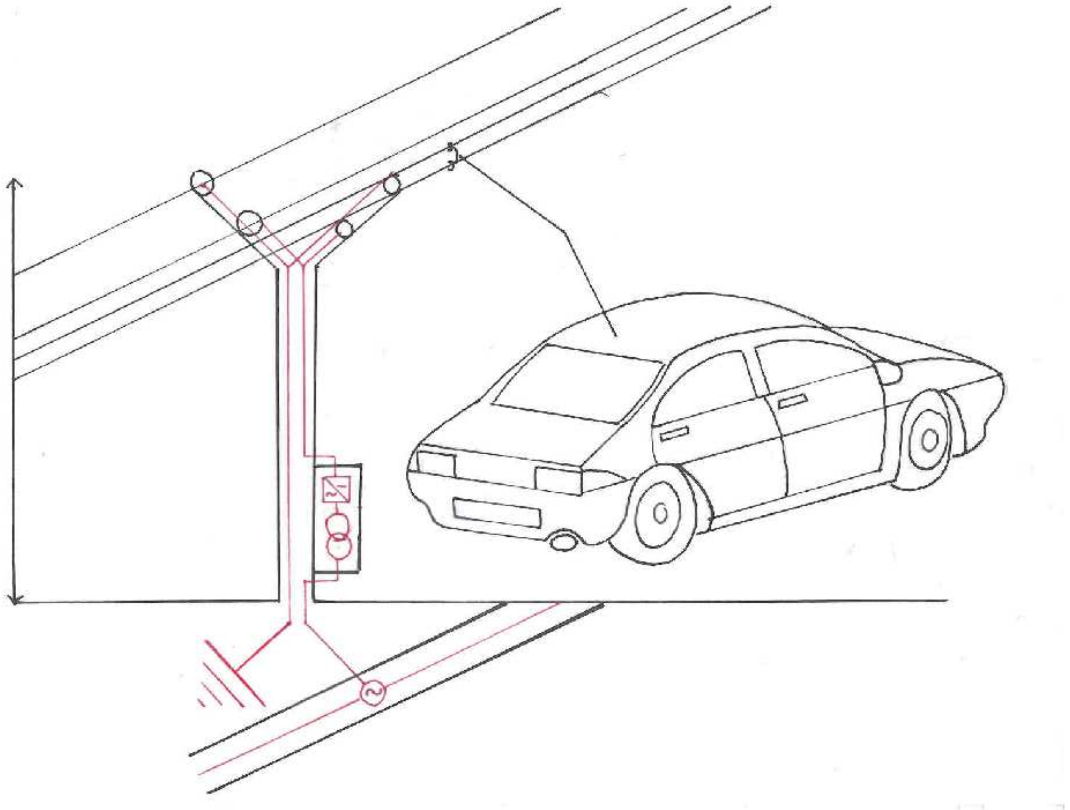

FIG. 1: Sketch of the electric line feeding a rolling automobile. An underground AC cable feeds an aerial 3000 volt DC line, a third cable provides the ground voltage. A Y-disposition allows simultaneous feeding two lanes in opposite directions of the highway where this be possible. Otherwise the system supplies only the left lane in one direction. Automobiles should connect to the cable by a side pantograph-antenna.

C. Electricity grid

Urban areas typically already count on good electricity networks. The Madrid region is supplied with 220 and 400 kV aerial lines that can be found near every major highway. Of course, the generalization of electric transport would require a reinforcement of this electric grids, which requires much specialized research.

Highway side electric service stations for electric cars can serve a dual role. Currently they are conceived as containing battery-changing points and fast-charge outlets at about 800 V.
One could easily conceive them also as hosting transformers to receive very high voltage from the power grid and feeding it, once stepped down, to the road electrification at intervals of order 10 km (again sensible in densely populated areas).

D. Automobile connector

Railways are connected to their top electricity supply by a pantograph. Sideways connection for automobiles could be effected by a pole, not much unlike a side-mounted radio antenna of somewhat larger length and thickness than usually found in cars. This automobile pantograph should be articulated with two knees because automobiles are not much guided by a rail (see later in section IV). To reduce the risk of electricity shock, we would recommend a coaxial line with the inside cable carrying the voltage and the outside grounded. Many dielectrics can maintain the necessary electric field differences as seen in table V. The choice of a dielectric would in the end be made by mecanical considerations, durability and price.

TABLE V: Breakup voltage for several dielectric materials [8] easily allowing a 1 cm thick coaxial insulation.

Material	Breakup field ($Volt/cm$)
Polyester	340 000
Polyamid	360 000
Aramid	50 000
Epoxy	24 000

A sketch of the antenna's section is given in figure 2. We propose that the articulation should allow for a tolerance of half a meter in the sideways distance to the automobile. Larger separations should mean that the driver wanted to separate from the connection.

IV. DRIVING HABITS

We would recommend a speed limitation to 90 km/h or similar in the electrified left lane, to reduce mechanical stresses and simplify the design of the electrical line and the pantograph hooking to it from the vehicle. This is not overconstraining in many highways of metropolitan areas, with three or more lanes, where faster traffic can be carried by the middle lane.

In fact, in many states, for example California, the left lane is already reserved for vehicles with high passenger number, not for the fastest vehicles.

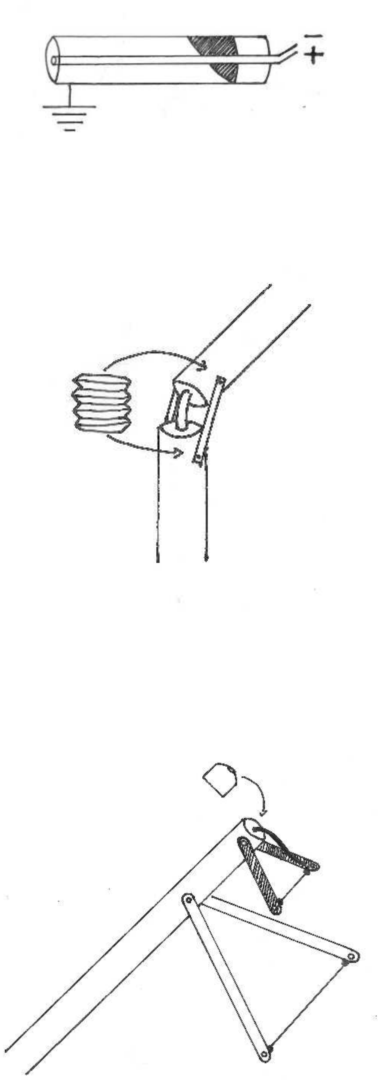

FIG. 2: Sketch of the coaxial antenna, which is somewhat thicker than usual radio antennae, supporting the DC current feeding the vehicle. The outside steel cylinder is grounded by the second aerial thereby providing ground voltage for the vehicle's mass for safety reasons as well. It can also be coated to avoid corrosion and other damage. The inner wire carries the 3000 V potential and is protected by an insulating polymer (colored black). The antenna needs to be articulated to allow for slight distance changes between automobile and aerial. At the contact end dampers should be provided to reduce noise, and the contact wires should be engineered to reduce arcing upon contact.

In the case of Madrid, the average speed in its beltlines has varied between 50 and 65 km/h in the period 2004-2008, so that 90 km/h would not be a real restriction except at night hours.

A greater difficulty lies in the driver maintaining the vehicle aligned with the electric line to avoid loosing contact. We recommend for this purpose a small depression in the left end of the road of about 2 *cm* depth and 1 *m* width (to allow for a 50 *cm* standard variation of mean distance between vehicle and electric line). This substitute of a railway's track offers no real resistance to a driver wishing to abandon it, but with proper visual and pavement sound aid, it is effective to help him stay connected to the aerial. The situation is depicted in Fig. 3.

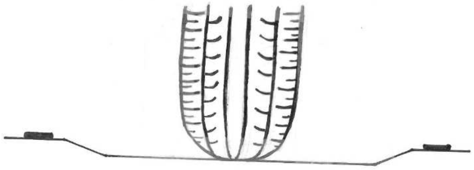

FIG. 3: A small depression in the pavement with side visual and sound bands helps the driver keep the vehicle at constant distance from the electric line.

V. FINANCIAL FEASIBILITY

A. General considerations

We would like to examine the question of whether the electric car economic model can closely mimic the current oil-based model in terms of financing, with an equal upfront expense upon purchase and similar energy costs later. This can work assuming that the manufacture price for both is quite the same, which can be achieved once freed from expensive battery packs.

Since there is no direct experience with such electrifications, we will take as orientation the cost of electrifying a railway line or illuminating a road. We have consulted the budgets of several such civil undertakings in Spain and conclude that a budget of 1 Million euro per kilometer amply allows for the electrification we propose. We will call this variable capital cost per unit length, C/L. The cost of electrical off-road grid infrastructure is accounted for in the electricity price and need not concern us any further.

By current oil prices, a commuter in Spain is paying about 9 euro cents per kilometer. If her vehicle would be moved by electricity, her energy cost would instead be about 4.2 cents per kilometer. Thus, the bare energy price is cheaper for electricity by more than four cents per kilometer. This price difference will be denoted by $\frac{\delta p}{L}$.

What we want to argue next is that these excess four cents per kilometer can finance the needed on-road electric infrastructure. The state acts only as a regulator and the private exploitation of the electrification is possible. This will be feasible with certain minimum traffic levels which we will establish later.

We will, for an order of magnitude estimate, neglect the very much fluctuating weekend and vacation traffic, and concentrate on the $N_d = 235$ yearly working days where commuter traffic is quite constant, leaving the rest as surplus payments.

It is clear that near the city, denser traffic makes the electrification more profitable. Let the variable x run from 0 at the city center through positive values of the distance to it along the highway. The number of cars at any one point of the highway will be the variable $n(x)$, with $\frac{dn(x)}{dx}$ being a continuous approximation to the actual number of cars entering and leaving the way at the exits. The number of vehicles that have adopted electricity as an energy means can be represented by multiplying with a ratio smaller than 1, $r(x)$ or $r(t)$ (depending on whether one wishes to stress or not its time dependence). As a function of time one expects $r(t)$ to be a population-logistic curve, with initial exponential growth and later saturation.

B. Loan repayment

Let the infrastructure capital $C(t)$ be financed by a state-guaranteed loan, and assume for the sake of the discussion a yearly interest rate of $TAE = 5\%$. To avoid increasing the debt, the yearly repayment must be $P(t) \geq TAE \times C(t)$. The revised owed capital will therefore be, at year $t+1$,

$$C(t+1) = C(t) - (P(t) - TAE \times C(t)) \,. \qquad (6)$$

If the time for repayment is chosen to be τ, so that $C(\tau) = 0$, and solving Eq. 6 for a constant payment $P(t) = P$, we find

$$P = \frac{C(0) \times (1+TAE)^{\tau}}{\sum_{i=0}^{\tau-1} (1+TAE)^{i}} \,. \qquad (7)$$

Taking $\tau = 30\ yr$ (a generation's span), the payment will be about $P/L = 65\ 050$ euro/annum. This is the typical revenue that each kilometer of highway needs to yield for the investment to break even.

This cost needs to be shared among the users. To keep the total user payment at or below the current cost of operating with fuel-based cars, we obtain the condition

$$\frac{P}{N_d n(x)} < \delta p \,, \qquad (8)$$

from which the minimum number of cars needed daily on the road falls out as

$$n(x) \geq \frac{P}{N_D \delta p} \,. \qquad (9)$$

For the reader's ease we have collected the resulting number of cars as a function of the energy source price difference in table VI. All of these numbers are very far from

TABLE VI: Number of circulating electric cars necessary on each work day at kilometer x of a motorway's electrified lane, assuming a yearly interest rate on capital investment of $TAE = 5\%$ and an electrification cost of $1M$ euro per kilometer, as function of the price difference between fossile fuel and electricity needed to drive that kilometer.

$n(x)$ (car number)	δ_p (oil-less electricity in cents/km)
13 840	2
9 226	3
6 920	4
3 460	8

saturation; note that with a speed of 90 km/h and car separation of 50 m, the maximum number of cars that a highway lane operating twelve hours can take in a given day is 21 600, well above those figures.

C. Case study: A6 in Madrid

A6 is one of eight major radial roads and gives commuter access to the city from the northwest. We here consider a stretch of about 50 km inside the region of Madrid.

The Mean Daily Traffic Intensity was measured in 2002 to be 71 438 incoming vehicles and 78 872 outgoing vehicles at kilometer $x = 19$ from the center, where the road had four useful lanes in each direction of circulation.

Rounding off to 70 000 daily commutes, and with $\delta P \simeq 4$ $cents/km$ (that is the current situation), the electrification of the left lane would become profitable for $r \geq 10\%$, that is, with one in ten vehicles being electric.

The regional planner would face the question of when and how far to provide the electrification. One can sensibly think that vehicles within 10 km of the city would opt for commuting on their batteries and not paying the electrification fee, recharging at night instead. Therefore, if the number of vehicles that make the electrification profitable is calculated to be N, it must be that

$$\begin{cases} \text{if } x > 10\ km & \text{then } N < r(t + 2yr) \times n(x) \\ \text{if } x < 10\ km & \text{then } N < r(t + 2yr) \times n(10) \end{cases} . \qquad (10)$$

Recall that r is the fraction of electric vehicles, which we evaluate at retarded time by an estimated 2 years (typical time span between decision and operativity of a civil work of these characteristics).

We do not have at hand traffic data for all points of A6, so we have made an estimate with the known data (traffic at city accesses, at the furthest point of A6 where two tunnels go under the mountain chain in the limit of the Madrid region, and others) and filled the missing traffic data by making it proportional to the population that potentially accesses that highway (the census is known) at each kilometric point x. The outcome is shown in Fig. 4.

From these data for $n(x)$ and our prior financial discussion, it follows that it is profitable to electrify the entire 50 km length of the highway when the percentage of electric vehicles reaches 20%.

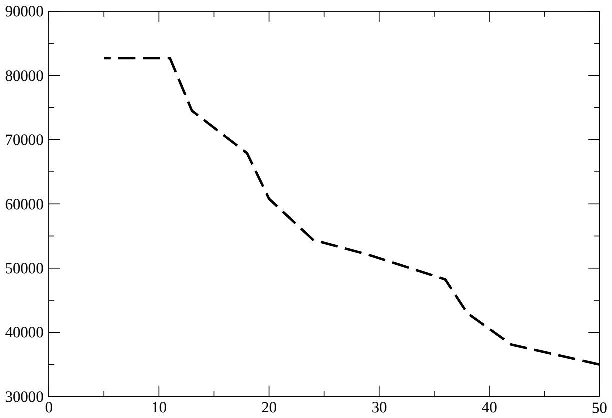

FIG. 4: Estimate of Mean Daily Value of car number on A6 as function of the distance to the center, saturating at kilometer $x = 10$ from the center.

To decide on the electrification of a given kilometer all that is needed is to know the rate $r(t)$ at which the electric vehicle will be adopted (if it finally is). The initial growth phase of a technology revolution is exponential, but we need to know the characteristic time.

It might be of help to consider the adoption of cellular telephones, the latest wide-reaching technology revolution. The characteristic time of its logistic curve happens to be about $\tau = 2$ yr.

Coming to the electric car, we consider $\tau = 8$ $months$ through $\tau = 3$ yr in table VII.

It is seen that the electrification will not be profitable in the four-year horizon of an administration.

D. Net Present Value

Looking farther ahead, one may still ask when is the electrification expected to be profitable. A useful analysis tool is the Net Present Value, defined as

$$VNP = \sum_0^N \frac{C_n}{(1+d)^n} . \qquad (11)$$

TABLE VII: Percentage and total number of electric vehicles estimated for the next years according to the unknown value of τ (time in which the number is multiplied by $e \simeq 2.718$). The adoption of cellular phones was characterized by $\tau = 2\ yr$. The initial condition is Spain's goal for 2010 of 2000 electric cars.

Year	$\tau =$	8 *month*	$\tau =$	1 *yr*	$\tau =$	2 *yr*	$\tau =$	3 *yr*
2010	0.0091	2 000	0.0091	2 000	0.0091	2 000	0.0091	2 000
2011	0.041	8 960	0.025	5 440	0.015	3 300	0.013	2 790
2012	0.183	40 180	0.067	14 780	0.025	5 440	0.018	3 900
2013	0.82	180 120	0.18	40 170	0.041	8 960	0.025	5 440
2014	3.7	807 340	0.50	109 200	0.067	14 780	0.034	7 590
2015	16	3 618 800	1.3	296 830	0.11	24 360	0.048	10 590

Here C_n are future cash inflows in year n due to the infrastructure, and d is the discount rate (whose meaning is that future income could be obtained by alternative means, for example buying bonds, and therefore the gain C_n very far in the future is less competitive). If the Net Present Value turns positive, the infrastructure should be undertaken, as it will be more profitable than alternative investments. However the value of d is usually uncertain and depends on market conditions.

We plot in figure 5 the resulting Net Present Value for the electrification of A6 following from the analysis of subsection V C. From the figure one concludes that, although in 2015 the electrification would incur losses, it would break even in the 30 yr span of the capital repayment. If the electric vehicle is indeed adopted, construction undertaken after that date is profitable.

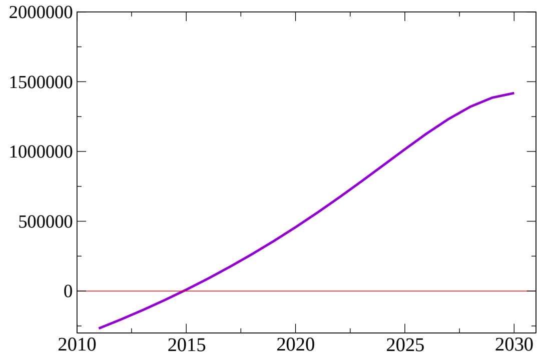

FIG. 5: Net present value of the electrification of one kilometer of A6 in Madrid, in Euro, as a function of the year of construction, employing $\tau = 2\ yr$ for the characteristic time of adoption of the electric vehicle and a discount rate of 6%.

E. User payment

If the electrification is to be self-sustainable, as we have argued it can be in metropolitan areas, and exemplified with the A6 highway in the Madrid region, charge of the service has to be made to the user.

This would be a great application of Power Line Communication [10], that is already being deployed for electricity fees incurred by vehicles at a stationary charging point. The on-board electronics can easily pick up a modulated high frequency signal through the electric network (again, the electric service stations spaced every ten kilometers could host the corporation's computer billing the charge).

Other ways of charging drivers for their use of the electric line more in line with current toll collection (such as a radiowave device, or credit card charge) are also possible but less attractive.

VI. CONCLUSIONS AND OUTLOOK

To summarize the article, we have proposed to supply electricity to electric cars on highways by means of an aerial line suspended at about 3 m height on the left side of the left-most lane (in right-circulating countries such as continental Europe or the USA) .

Reasonable parameters to planify the infrastructure include a provision of 50 kw per vehicle, taking into account that consumption of a small family car in ordinary circumstances will be in the range of 25-35 kw at a reference speed of 90 km/h.

A possible voltage to be employed is 3000 volts in direct current, with an auxiliary high voltage line underground, feeding the first at kilometer intervals.

There is no obvious technological roadblock to the deployment of such systems, although much engineering work and standardization would have to be undertaken.

We have shown the profitability of the electrification under reasonable assumptions on financial markets, if the electric car is indeed adopted. As a gross reference, one should like to have 7 000 electric cars employing the electrified lane in each working day.

As the user is concerned, his transportation costs would closely parallel those of the current gas-based model, with a smaller upfront investment in the vehicle purchase than current electric cars require, but a higher operating cost.

Many questions remain and grant further research, but many are of a social nature and common to other methods of supplying electric cars. One for example is whether the commuters would be limited to regional travel with their low storage-capacity vehicle if the road electrification was not extended. One should like to carry out a similar analysis to interstate or national-level road systems taking into account the data of larger areas with population spread in several nodes. In the case of Spain, a length of order 8 000 *km* of road should be electrified to have a national impact (we estimate the cost of this deployment as equivalent to a fifth of the current profit of the entire electricity sector over a decade).

It is also interesting to compare the analysis with the railway electrification. The last plan undertaken in Spain for conventional railways dates back to 1979 (high-speed railways are automatically electric-powered) [11]. As of Dec. 31st, 2008, 60% of the conventional rail network was completely electrified, with 5350 *km* missing. The total electrification of the system was encouraged and claimed to be profitable already in 1945 [12].
Based on train circulation numbers and power consumed per unit we estimate the total power in the high-speed line Madrid-Seville to be of order 140 *MW* for a 500 *km* stretch of railway. The consumed power is a fourth of the requirement for our proposed road electrification. This shows that, while technologically challenging, what we suggest is not far from currently existing infrastructure.

Since transport is responsible for the consumption of 50 % of crude oil, any electrification achieved is a way to reduce the world's dependence on this resource. We hope that, given the importance of things that are at stake, our contribution at a theoretical level warrants further specialized investigation.

Acknowledgments

A spanish-language monograph from which this work is abstracted can be found at our University's webpage under
`http://teorica.fis.ucm.es/~ft11/LEFTLANE.DIR/coche.pdf`
All information that the scholar needs from that work has been collected in this english version, but a student may find the additional (if lengthy) discussion of use.

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