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Cosmography by GRBs : applications

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Abstract. In this work we present some applications about the use of the so-called Cosmography with GRBs. In particular, we try to calibrate the Amati relation by using the luminosity distance obtained from the cosmographic analysis. Thus, we analyze the possibility of use GRBs as possible estimators for the cosmological parameters, obtaining as preliminary results a good estimate of the cosmological density parameters, just by using a GRB data sample.

Key words. Gamma rays : bursts - Cosmology : cosmological parameters - Cosmology : distance scale

Cosmography by GRBs : applications

1. Introduction

It is a matter of fact that Gamma Ray Bursts (GRBs) are the most powerful explosions in the Universe; this feature makes them as one of the most studied objects in high energy astrophysics. The flux observed from their emission and the measurement of the redshift z from the observation of the afterglow (Costa et al. 1997), point out a very high value for the isotropic energy emitted in the burst, so that there are some GRBs observed at very high redshift. Up to date, the farthest GRB has a spectroscopic redshift of $\sim$ 8.2 (Tanvir et al. 2009; Salvaterra et al. 2009). These interesting features point out a possible use of GRBs as distance indicators; unfortunately our knowledge on the mechanisms underlying the GRB emission is not completely understood, so that their use as standard candles seems to be hard to be implemented. However, there exist some correlations among the observed spectroscopic and photometric properties of the GRBs, allowing us to put severe constraints on the GRBs distances.

What we need is an independent estimate of the isotropic energy E_{iso} emitted from a GRB. Indeed, by using the GRB's fluence S measured by a detector in a certain energy range, it becomes possible to determine the luminosity distance d_l as follows

$$d_l = \left(\frac{E_{iso}(1+z)}{S_{bolo}}\right)^{\frac{1}{2}}, \qquad (1)$$

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where S_{bolo} is the bolometric fluence emitted, obtained from the Schaefer formula (Schaefer 2007)

$$S_{bol} = S_{obs} \frac{\int_{1/(1+z)}^{10^4/(1+z)} E\phi dE}{\int_{E_{min}}^{E^{max}} E\phi dE}. \quad (2)$$

In literature it is possible to find several correlations between observed feature of the GRBs[1], each of them taking in account a different observed quantity.

In this work we assume the validity of the so-called Amati relation (Amati et al. 2002), for different reasons:

1 it relates just the isotropic energy with the peak energy in the νF(ν) spectrum without considering other quantity as other existing correlations,
2 it does not involves time quantity, that suffer from very large instrumental and cosmological biases,
3 all of the long GRBs satisfy this correlation, while the same is not true for the other correlations.

Although the Amati relation suffers of some biases, as the detector dependence of the observed quantity considered (Shahmoradi & Nemiroff 2009; Butler et al. 2009), it seems to be well verified from observations (Amati et al. 2009); hence, these issues may be not considered hereafter in our discussion. However, one of the most relevant challenge is represented by the calibration of the Amati relation, because a low redshift sample of GRBs is, up to now, lacking; a similar sample should be necessary in order to allow us to calibrate the relation too as well as the Supernovae Ia (SNeIa) procedure. Anyway, a first computation of the relation parameters has been performed by considering the *concordance* model, namely ΛCDM, obtaining a model-dependent luminosity distance. But this procedure leads naturally to the so-called circularity problem when we take in account a cosmological use of the GRBs with the Amati relation. A possible solution has been provided by the use of SNeIa, considered to be good *standard* candles (Perlmutter et al. 1999). In principle one can wonder if it is possible to calibrate GRBs by adopting at low redshift the SNeIa sample. This way has been already developed in literature; frequently we can find works trying to calibrate GRB correlations, (see e.g. Liang et al. (2008)). On the other hand, recently it has been proposed an alternative to solve this controversy, by adopting the model-*independent* procedure described by Cosmography, which shall be clarified later in the next section.

[1] For a review see Meszaros (2006)

2. The cosmographic Amati relation

As stressed in the introduction, the necessity to account a procedure which is based on a model-independent way for characterizing the Universe dynamics is essential; Indeed, different cosmological tests may be taken into account, unfortunately for any case, one of the major difficulty is related to choosing which may be considered the less model independent one. One of these, first discussed by Weinberg (Weinberg 1972) and recently by Visser (Visser 2004), proposes to consider the waste amount of kinematical quantities as constraints to discriminate if a model works well or not.

Cosmography is exactly what we mean for that; we refer to it as the part of cosmology trying to infer the kinematical quantities as the expansion velocity, the deceleration parameter and so on, just making the minimal assumption of a (here flat) Friedman-Robertson-Walker (FRW) metrics, being $ds^2 = c^2dt^2 - a(t)^2\left(dr^2 + r^2 \sin^2\theta d\theta^2 + d\phi^2\right)$, Weinberg (1972); in particular, it is based only on keeping the geometry by assuming the Taylor expansion of the scale factor $a(t)$.

In this way we do not predictions about the standard Hubble law, but only to its kinematical constraints; it is worth noting that once expanded as a Taylor series the Hubble law it is consequent to expand the luminosity distance d_l too and then the distance modulus $\mu(z)$ (Capozziello & Izzo 2010); unfortunately it is clear that a similar expansion diverges for $z > 1$, thus to circumvent this mathematical issue it should be necessary to change the variable, defining

$$y = \frac{z}{1+z}, \tag{3}$$

which limits the redshift range, i.e. $y \in (0,1)$.

With this model-independent formulation for the luminosity distance we can immediately determine the cosmographic parameters, in order to reconstruct the trend of the function d_l(y) also at high redshift. Indeed, our aim consists in assuming the luminosity distance obtained with a good distance indicators, (SNeIa), extending it also for high redshifts.

So far, as a first step we estimated the cosmographic parameters from a very large sample of SNe Ia, by adopting the Union 2 compilation (Amanullah et al. 2010); to perform this, we used a likelihood function $L \propto e^{-\chi^2/2}$ given by

$$\chi^2 = \sum_i \frac{(\mu(y) - \mu_i)^2}{\sigma^2_{\mu_i}}, \tag{4}$$

where μ_i are the distance modulus for each Union SNeIa and $\sigma\mu_i$ its correspondent error. The results are summarized in Table 1.

Once having an expression for d_l, in principle, it would be possible to calibrate the Amati relation too, by using the observed redshift and the bolometric fluence S_{bolo} of a GRB, computing the isotropic energy, by inverting eq. 1. Then, having as the Amati relation the formula $E_{iso} = A\, E^{\gamma}_{p,i}$, we evaluated the parameters A and γ, through the use of a sample of 108 GRBs (Capozziello & Izzo 2010), considering as estimator a log-likelihood function and taking into account the possible existence of an extra variability σ_{ext} of the y data, due to some hidden variables that we cannot observe directly (D'Agostini 2005). The cosmographic calibration gives as results the following values

$$A = 49.17 \pm 0.40, \qquad \gamma = 1.46 \pm 0.29, \tag{5}$$

and in Fig. 1 is showed the best fit curve in the $E_p - E_{iso}$ plane.

3. Cosmological applications

Although of its elegance, our calibration of the Amati relation has been obtained using a formulation of d_l which suffers from some theoretical misleading problems. First of all, since it is defined for low values of the redshift, the consequent extension to higher redshift may bring to deviations from the real cosmological picture. In order to check this discrepancy, we plotted in fig. 2 both the distance modulus obtained from the cosmography by SNeIa and from a standard ΛCDM paradigm, with $\Omega_\rho = 0.27$, being the matter density.

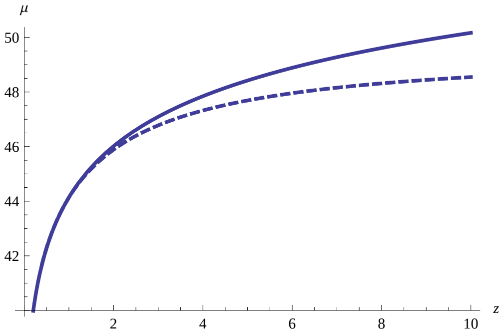

Fig. 2. Plot of the μ(y) computed for a fiducial ΛCDM cosmological model, the continuous line, and for the reconstructed μ(y) obtained from the cosmographic fit of the SneIa, the dashed line, in function of the z redshift.

We immediately note a difference of one magnitude at redshift z ≈ 4, increasing with z, due to different possible reasons

1 the propagation of the systematics in the analysis of the SNeIa used for calibration,
2 the large scatter in the data sample of the Amati relation,
3 the standard ΛCDM model fails at high redshift.

The latter assumption seems to be the less probable one, since the ΛCDM model is able to explain the growing of structure formations. In addition, in the following, we are going to present a cosmological application of the GRB sample to estimate the density parameters.

Let us first compute the isotropic energy E_{iso} for each GRB from the cosmographic Amati relation, obtaining the distance modulus for each of them, by using the bolometric fluence S_{bolo} of eq. 2. Thus, the GRB sample be-

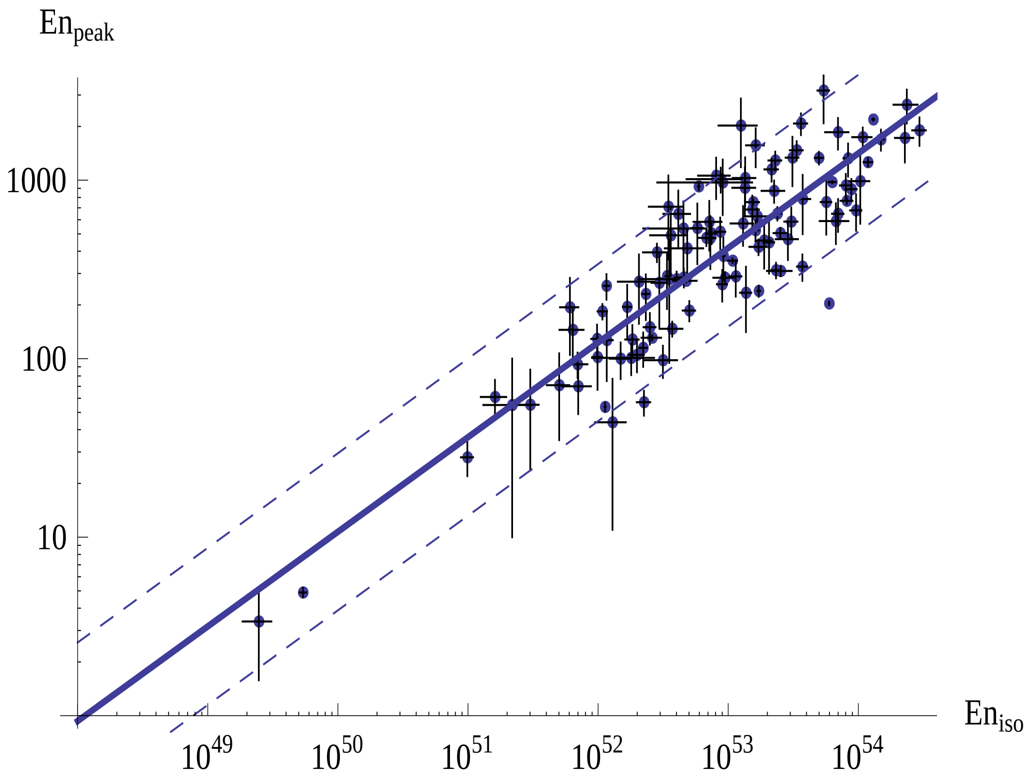

Fig. 1. Plot of the cosmographic Amati relation in the $E_p - E_{iso}$ one. The line of prediction bounds represents a deviation of $2\sigma_{ext}$ from the best fit line, the thick one.

Table 1. Cosmographic parameters obtained using the SneIa sample UNION2. Note that we have considered for the determination of the jerk j_0 and of the snap s_0 the flatness condition $\Omega_k = 0$. The error on s_0 does not include the contribute from covariance terms.

Parameter	value	error
H_0	69.90	0.027
q_0	−0.58	0.03
j_0	1.50	0.22
s_0	−2.96	1.58

comes related to the following theoretical distance modulus

$$D_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{|\Omega_k|}}\mathrm{sinn}\left(\sqrt{|\Omega_k|}\int_0^z \frac{d\xi}{E(\xi)}\right), \quad (6)$$

with $E(z)$ the reduced Hubble parameter, i.e. $E \equiv \frac{H(z)}{H_0}$, while Ω_k represents the fractional curvature density at $z = 0$, and

$$\mathrm{sinn}(x) = \begin{cases} \sin(x), & \text{if } \Omega_k < 0, \\ x, & \text{if } \Omega_k = 0, \\ \sinh(x), & \text{if } \Omega_k > 0. \end{cases}$$

We consider as likelihood $L \propto e^{-\chi^2_{GRB}/2}$ the function given by

$$\chi^2_{GRB} = \sum_{i=1}^{108} \frac{(\mu_{th} - \mu_{obs})^2}{\sigma^2_{\mu,i}}\,, \tag{7}$$

where μ_{obs} is the observed distance modulus for each GRB, with its error $\sigma_{\mu,i}$, derived from the Amati relation, while μ_{th} is the value of the distance modulus theorized by the considered cosmological model.

The constraints have been evaluated by a combined cosmological test, provided by the SNeIa, baryon acoustic oscillations (BAO) and cosmic microwave background (CMB). Hence the total χ^2 is given by (Wang & Mukherjee 2006)

$$\chi^2 = \chi^2_{GRB} + \chi^2_{SN} + \chi^2_{BAO} + \chi^2_{CMB}. \tag{8}$$

In order to perform it, we adopt the Union 2 compilation (Amanullah et al. 2010), deriving the constraints and confidence limits by using the same statistic employed for GRBs. In particular we adopt the CMB shift parameter R

$$R = \sqrt{\Omega_m H_0^2}\, r(z_{CMB}) \tag{9}$$

with $r(z_{CMB}) = \frac{c}{H_0}|\Omega_k|^{-1/2}\mathrm{sinn}(|\Omega_k|^{1/2}\int_0^{z_{CMB}}\frac{d\xi}{E(\xi)})$, while the χ^2 term is given by

$$\chi^2_{CMB} = \frac{(R - R_{obs})^2}{\sigma^2_R}, \tag{10}$$

where for R_{obs} and its error we consider the recent WMAP 7-years observations (**?**). For the SDSS baryon acoustic oscillations (BAO) scale measurement and in particular the distance parameter A

$$A = \left(r(z_{BAO})^2 \frac{c z_{BAO}}{H(z_{BAO})} \right)^{1/3} \frac{(\Omega_m H_0^2)^{1/2}}{c z_{BAO}} \tag{11}$$

with $r(z_{BAO}) = \frac{c}{H_0}|\Omega_k|^{-1/2}\mathrm{sinn}(|\Omega_k|^{1/2}\int_0^{z_{BAO}}\frac{d\xi}{E(\xi)})$, $A_{BAO} = 0.469\ (n_s/0.98)^{-0.35}$ and $\sigma_A = 0.017$ (Eisenstein et al. 2005). The redshift $z_{BAO} = 0.35$ while the spectral index is given in Table **??** as measured by WMAP7 (**?**). The minimization of the total χ^2 was done applying a grid-search method in the parameter space of the model considered. As a first analysis we considered again the case of the ΛCDM model, obtaining not good results, (see fig. 3). We conclude that this happened due to the lacking low-redshift GRB sample, so that we are not able to give a good accuracy for the best fit values obtained using only the GRB data sample.

A natural extension of ΛCDM is represented by the *w*CDM model, the so-called quintessence model (Sneden et al. 1991); here again the results are not in good agreement with respect what we expected, (see fig. 4). In order to show a good agreement with observations we expect that, since GRBs are generally at high redshift a varying Quintessence model, can provide the trend of the *w*-term, giving rise to a well-fitting procedure. Among all the possibilities we report below the so-called Chevallier-Polarski-Linder (CPL) (Chevallier & Polarski 2001) as

$$w(z) = w_0 + w_a \frac{z}{1+z}, \tag{12}$$

where $w \equiv \frac{p}{\rho}$. In this way the distance modulus curve is sensitive to variations at high redshift of the w quantity, and GRBs are the only source that can shed light on this topic. The performed analysis developed by using both the SNeIa and GRB data sample gives results quite in agreement with what we expected, see fig. 5 and tab. **??**; together with the other analysis, and seems to point out GRBs as fundamental tracers of an evolving Dark Energy EoS.

4. Conclusions

In this effort we wondered if the possibility of using GRBs as distance indicators can be a real resource of the modern *Precision Cosmology*; obviously this deals with the issue that up to now, we cannot admit that GRBs are standard candles. We developed a statistical (combined) analysis in which the calibration of the luminosity distance has been performed by a SNeIa sample, testing different models (ΛCDM, *w*CDM and CPL parametrization) with a more complete sample, including GRB data. We obtain satisfactory results especially

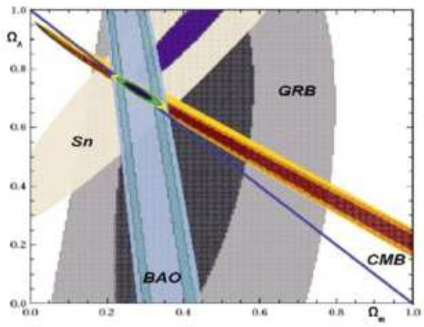

Fig. 3. 68%, 95%, and 98% constraints on Ω_ρ and Ω_Λ in the ΛCDM model obtained from CMB (red), BAO (blue), the Union 2 Compilation(gray and blue) and the GRB sample considered in this paper(gray and black). The superimposed contour plot represents the combined final results.

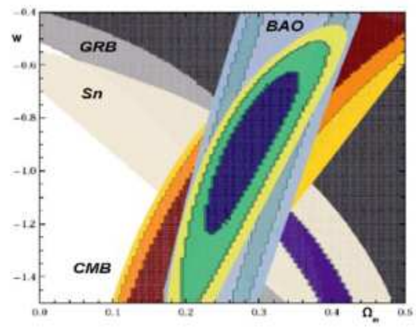

Fig. 4. 68%, 95%, and 98% constraints on Ω_ρ and w obtained from CMB (orange), BAO (green), the Union 2 Compilation(gray and blue) and the GRB sample considered in this paper(gray and black). The superimposed contour plot represents the combined final results.

in the CPL case. We conclude that the present data cannot suggest to us something new about the standard model, but the procedure must be seen as a first application of the use of GRBs in cosmology, for future developments. In a next paper we shall present intriguing results, studying with more accuracy the quoted models and other alternatives.

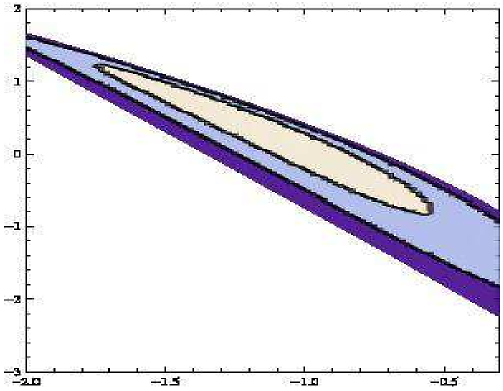

Fig. 5. 68%, 95%, and 99.7% constraints on the CPL parameters w_0 and w_a obtained from the Union 2 Compilation and the GRB sample.

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