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Radiative activity of magnetic white dwarf undergoing Lorentz-force-driven torsional vibrations

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We study radiative activity of magnetic white dwarf undergoing torsional vibrations about axis of its own dipole magnetic moment under the action of Lorentz restoring force. It is shown that pulsating white dwarf can convert its vibration energy into the energy of magneto-dipole emission, oscillating with the frequency equal to the frequency of Alfvén torsional vibrations, provided that internal magnetic field is decayed. The most conspicuous feature of the vibration energy powered radiation in question is the lengthening of periods of oscillating emission; the rate of period elongation is determined by the rate magnetic field decay.

Keywords: magnetic white dwarf, non-radial pulsations, magneto-dipole emission

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1. Introduction

The perfect conductivity of metal-like matter of magnetic white dwarfs (composed of ions suspended in the Fermi-gas of relativistic electrons pervaded by magnetic field of a fairly high intensity, $10^5 < B < 10^8$ Gauss) suggests that these solid stars are capable of sustaining Alfvén oscillations. It true, this property should manifest itself by quasi-periodic oscillations of optical and X-ray emission detected form pulsating magnetic white dwarf stars[1]. In recent work[2], this proposition has been investigated in the model of solid star with poloidal and toroidal static magnetic field with focus on frequency spectra of node-free regime of Alfvén vibrations. The main purpose of this last work was to elucidate the difference between frequency spectra of a-modes in white dwarfs having one and the same mass M and radius R, but different shapes of constant-in-time poloidal and toroidal magnetic fields.

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And it was found that each specific form of spatial configuration of static magnetic field is uniquely reflected in dependence of frequency upon overtone ℓ of nodeless vibrations. Continuing this line of investigation, in this paper we explore the impact of magnetic field decay in pulsating magnetic white dwarf on its radiative activity. In section 2, a brief outline of magneto-solid-mechanical energy method of computing frequency of node-free Lorentz-force-driven vibrations of white dwarf matter in time-varying magnetic field is given. In section 3, the theory of vibration-energy powered magneto-dipole emission produced by magnetic white dwarf vibrating in Alfvén mode is considered in some details. The obtained results are summarized in section 4.

2. Magnetic-field-decay effect on the energy of Alfvén vibrations

In what follows we confine our consideration to the solid-star model of magnetic (DA and DB) white dwarf with poloidal (uniform and/or non-uniform internal and dipolar external) magnetic field which is regarded as a slowly decreasing function time. The magnetic field decay is, most likely, determined by peculiarities of κ-mechanism of ionization[3] (of hydrogen in DA and helium in DB stars). The time-dependent field can be conveniently represented in the form

$$\mathbf{B}(\mathbf{r},t) = B(t)\,\mathbf{b}(\mathbf{r}), \quad \mathbf{b}(\mathbf{r}) = [b_r(\mathbf{r}) \neq 0, b_\theta(\mathbf{r}) \neq 0, b_\phi(\mathbf{r}) = 0], \tag{1}$$

where $B(t)$ is time dependent intensity and $b(\mathbf{r})$ is the time-independent vector-function of spatial distribution of the field in the which a perfectly conducting matter of volume. This leads to the following modification, compared to the case of constant in time field[2,4,5], of governing equations of magneto-solid-mechanics

$$\rho(\mathbf{r})\ddot{\mathbf{u}}(\mathbf{r},t) = \frac{B(t)}{c}[\delta\mathbf{j}(\mathbf{r},t) \times \mathbf{b}(\mathbf{r})], \quad \nabla\cdot\mathbf{u}(\mathbf{r},t) = 0, \quad \nabla\cdot\mathbf{b}(\mathbf{r}) = 0, \tag{2}$$

$$\delta\mathbf{j}(\mathbf{r},t) = \frac{c}{4\pi}[\nabla\times\delta\mathbf{B}(\mathbf{r},t)], \quad \delta\mathbf{B}(\mathbf{r},t) = B(t)\nabla\times[\mathbf{u}(\mathbf{r},t)\times\mathbf{b}(\mathbf{r})]. \tag{3}$$

These equations describe the Lorentz-force-driven shear (non-compressional) differentially rotational vibrations of a perfectly conducting solid-state plasma, regarded as non-flowing elastically deformable continuous medium, about axis of the time-varying magnetic field $\mathbf{B}(\mathbf{r},t)$. In what follows we focus on the regime of node-free shear Alfvén vibrations, which has recently been extensively studied in the context of a solid-star model of homogeneous density ρ and arbitrary spatial configuration of axisymmetric internal magnetic field, $\mathbf{b}(\mathbf{r})$. The frequency of such vibrations can be computed by using the following separable representation of toroidal field of differentially rotational material displacements[6,7]

$$\mathbf{u}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\,\alpha(t), \quad \mathbf{a}(\mathbf{r}) = \nabla\times[\mathbf{r}\,\chi(\mathbf{r})], \quad \nabla^2\chi(\mathbf{r}) = 0. \tag{4}$$

With account of all this, the basis dynamical equation (2) can be represented in the form

$$\rho\mathbf{a}(\mathbf{r})\ddot{\alpha}(t) = \frac{B^2(t)}{4\pi}[[\nabla\times[\nabla\times[\mathbf{a}(\mathbf{r})\times\mathbf{b}(\mathbf{r})]]]\times\mathbf{b}(\mathbf{r})]\,\alpha(t). \tag{5}$$

Scalar product of (5) with $\mathbf{a}(\mathbf{r})$ and integration over the star leads to equation for $\alpha(t)$

$$\mathcal{M}\ddot{\alpha}(t) + \mathcal{K}(t)\alpha(t) = 0, \quad \omega^2(t) = \frac{\mathcal{K}(t)}{\mathcal{M}}, \tag{6}$$

$$\mathcal{M} = \rho m, \quad m = \int \mathbf{a}^2 \, d\mathcal{V}, \tag{7}$$

$$\mathcal{K}(t) = \frac{B^2(t)}{4\pi} k, \quad k = \int \mathbf{a}(\mathbf{r}) \cdot [\mathbf{b}(\mathbf{r}) \times [\nabla \times [\nabla \times [\mathbf{a}(\mathbf{r}) \times \mathbf{b}(\mathbf{r})]]]] \, d\mathcal{V}. \tag{8}$$

Thus, the account of magnetic field decay leads to equation of non-isochronal vibrations with time-dependent spring constant $\mathcal{K}(t)$, contrary to harmonic in time vibrations in constant field $\mathbf{B}$ with constant frequency. It is easy to see that the magnetic field decay results in decreasing (slow-down) of circular frequency of vibrations

$$\omega(t) = \omega_A(t)\,\eta, \quad \omega_A(t) = B(t)\sqrt{\frac{R}{3M}} \quad \eta = \sqrt{\frac{k}{m}}\,R \tag{9}$$

and, hence, in the lengthening of the basic period of Alfvén vibrations at a rate which is determined by the rate of time-evolving suppression of magnetic field intensity $B(t)$

$$P_A(t) = \frac{2\pi}{B(t)}\sqrt{\frac{3M}{R}}, \tag{10}$$

$$\frac{dP_A(t)}{dt} = -\sqrt{\frac{3M}{R}}\frac{2\pi}{B^2(t)}\frac{dB(t)}{dt}. \tag{11}$$

The most conspicuous and important feature of the vibration process under consideration is that magnetic field decay is accompanied by the loss of energy of Alfvén vibrations

$$E_A(t) = \frac{\mathcal{M}\dot{\alpha}^2(t)}{2} + \frac{\mathcal{K}(t)\alpha^2(t)}{2}. \tag{12}$$

The rate of vibration energy loss

$$\frac{dE_A(t)}{dt} = \frac{d\mathcal{K}(t)}{dt}\frac{\alpha^2(t)}{2} = \frac{\mathcal{M}\alpha^2(t)}{2}\frac{d\omega^2(t)}{dt} \tag{13}$$

$$\frac{d\omega^2(t)}{dt} = \frac{2\eta^2 R^2}{3M}B(t)\frac{dB(t)}{dt}. \tag{14}$$

owes its origin to the time-evolving decay of intensity of magnetic field in the star.

3. Oscillating magneto-dipole radiation of white dwarf powered by energy of Alfvén vibrations

We consider a case when the white dwarf converts the energy of magneto-mechanical Alfvén vibrations into the energy of magneto-dipole emission. This means that rate of the vibration energy loss equals to the Larmorś luminosity of magneto-dipole emission produced by oscillating dipole magnetic moment $\delta\boldsymbol{\mu}(t)$ of magnetic white dwarf

$$\frac{dE_A(t)}{dt} = \frac{2}{3c^3}\delta\ddot{\boldsymbol{\mu}}^2(t). \tag{15}$$

The interrelation between variations in total magnetic moment $\delta\boldsymbol{\mu}(t)$ and the amplitude $\alpha(t)$ of magneto-mechanical oscillations can be consistently interpreted provided that both above characteristics oscillates with one and the same frequency $\omega(t)$. Such a possibility is realized when the node-free torsional magneto-mechanical oscillations are accompanied by fluctuations of total magnetic moment preserving its direction: $\boldsymbol{\mu} = \mu\,\mathbf{n} = \text{constant}$. If so, $\delta\boldsymbol{\mu}(t)$ and $\alpha(t)$ must obey equations of similar form, namely

$$\delta\ddot{\boldsymbol{\mu}}(t) + \omega^2(t)\delta\boldsymbol{\mu}(t) = 0, \tag{16}$$

$$\ddot{\alpha}(t) + \omega^2(t)\alpha(t) = 0, \quad \omega^2(t) = B^2(t)\kappa^2. \tag{17}$$

It follows

$$\delta\boldsymbol{\mu}(t) = i\,\boldsymbol{\mu}\,\alpha(t), \quad i^2 = -1. \tag{18}$$

On substituting (13), (14), (18) in (15) we arrive at

$$\frac{dB(t)}{dt} = -\gamma\,B^3(t), \quad \gamma = \frac{2\mu^2\kappa^2}{3\mathcal{M}c^3} = \text{constant}. \tag{19}$$

As a result, we arrive at the following law of magnetic field decay

$$B(t) = \frac{B(0)}{\sqrt{1+t/\tau}}, \quad \tau^{-1} = 2\gamma B^2(0). \tag{20}$$

The lifetime of magnetic field τ is regarded as a parameter whose value must be established from relations between the period $P(t)$ and its time derivative $\dot{P}(t)$ which are taken from observations. From above it follows that the time evolution of these latter quantities is determined by the time evolution of Alvén period $P_A(t)$. In view of this we confine our analysis to characteristic peculiarities of $P_A(t)$ and $\dot{P}_A$.

The immediate consequence of above line of argument is the magnetic-field-decay induced lengthening of vibration period

$$P_A(t) = \frac{2\pi}{\omega_A(t)} = \frac{C_A}{B(t)}, \quad C_A = 2\pi\sqrt{\frac{3M}{R}}. \tag{21}$$

The rate of the Alfvén period elongation is given by

$$\dot{P}_A(t) = -\frac{C_A}{B^2(t)}\frac{dB(t)}{dt}, \qquad \frac{dB}{dt} < 0. \tag{22}$$

Combining these equations we obtain

$$P_A(t)B(t) = \text{constant}, \tag{23}$$

$$\frac{\dot{P}_A(t)}{P_A(t)} = -\frac{\dot{B}(t)}{B(t)} \tag{24}$$

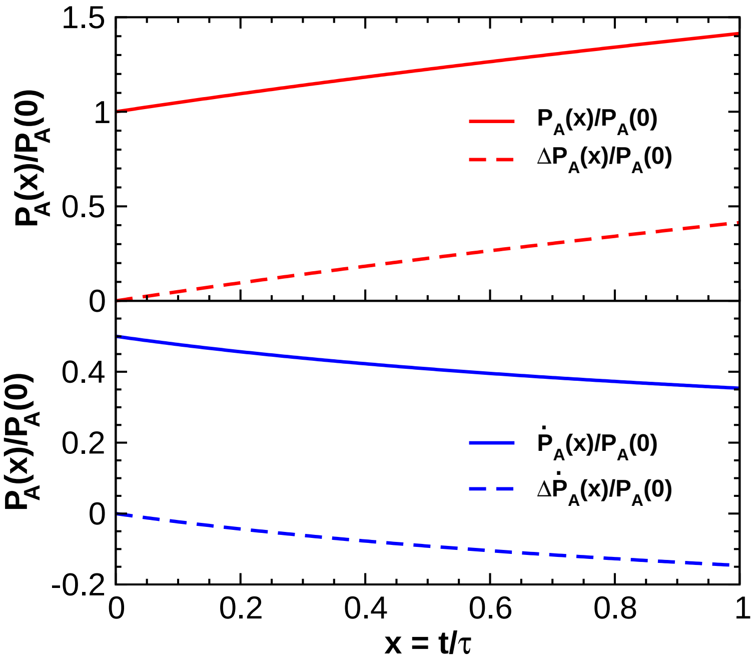

Fig. 1. The upper panel: the magnetic-field-decay induced elongation of the Alfvén period P_A and difference between periods ΔP_A of toroidal a- mode, defined in the text, plotted as functions of time t normalized to lifetime τ, $x = t/\tau$. The lower panel: the time derivatives of Alfvén period $\dot{P}_A$ and difference between periods $\Delta\dot{P}_A$ as functions of x.

These relation are independent of specific form of the magnetic field decay law and, therefore, can be used as general argument motivating interpretation of quasi-periodic oscillations of optical and X-ray emission from pulsating white dwarf as being produced by its global Alfvén torsional vibrations. The difference between periods evaluated at successive moments of time $t_1 = 0$ and $t_2 = t$ is given by

$$\Delta P_A(t) = P_A(t) - P_A(0) = -P(0)\left[1 - \frac{B(0)}{B(t)}\right] > 0, \quad B(t) < B(0). \tag{25}$$

Taking into account the obtained law of magnetic field decay we obtain

$$P_A(t) = P_A(0)\,[1 + (t/\tau)]^{1/2}, \tag{26}$$

$$\dot{P}_A(t) = \frac{1}{2\tau}\frac{P_A(0)}{[1 + (t/\tau)]^{1/2}} \tag{27}$$

$$\Delta P_A(t) = P_A(0)\left[1 - \sqrt{1 + t/\tau}\right], \tag{28}$$

$$\Delta\dot{P}_A(t) = \dot{P}_A(t) - \dot{P}_A(0) = -\frac{P_A(0)}{2\tau}\left[1 - \frac{1}{\sqrt{1 + t/\tau}}\right]. \tag{29}$$

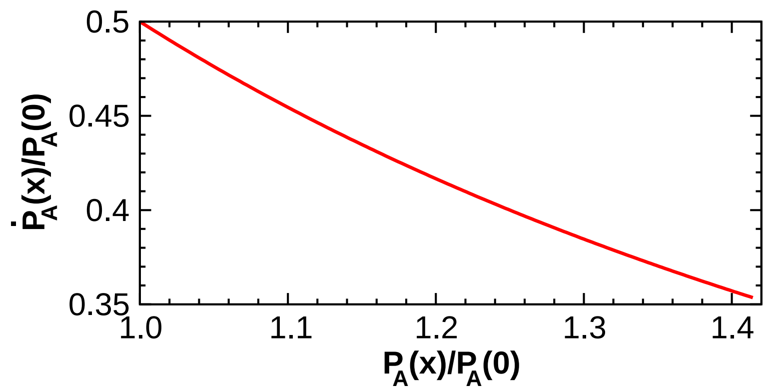

Fig. 2. The time derivative of period $\dot{P}_A(x)$ versus period $P_A(x)$, both normalized to period at initial moment of time $P_A(0)$, computed as functions of $x = t/\tau$.

It follows that parameter of lifetime τ is given by

$$\dot{P}_A(t)\, P_A(t) = \frac{P^2(0)}{2\tau} = \text{constant} \tag{30}$$

and for the ratio $\dot{P}_A$ to P_A we obtain

$$\frac{\dot{P}_A(t)}{P_A(t)} = \frac{1}{2\tau}\,[1 + (t/\tau)]^{-1}. \tag{31}$$

In Fig.1 and Fig.2 the above quantities are plotted as functions of $x = t/\tau$ which is ranged in the interval $0 < x < 1$.

4. Concluding remarks

The most important insight to be gained from above theory is that the magnetic field decay in magnetic white dwarf undergoing Lorentz-force-driven pulsations is inevitably accompanied by the loss of energy of Alfvén oscillations of the star that causes its vibration period to lengthen at a rate proportional to the rate of magnetic field decay. This theory rests on equations of magneto-solid mechanics lying at the base of asteroseismology of non-convective compact objects of final stage of evolutionary track. In view of this the considered solid-star model of vibration-energy powered emission is appropriate not only for white dwarfs but also for neutron stars[4,5] and for strange quark stars whose material is expected to be in solid aggregate state[8]. It must be noted that the star capability of converting the energy of Alfvén vibrations into energy of magneto-dipole electromagnetic emission has been considered long ago by Hoyle, Narlikar and Wheeler[9], before the discovery of pulsars, as one of plausible mechanisms of radiative activity of neutron stars. The extensive consideration of such possibility in the context of pulsating radiation from quaking magnetars is given in our recent work[10]. The general conclusion of this last and the present papers is that such possibility can be realized provided that

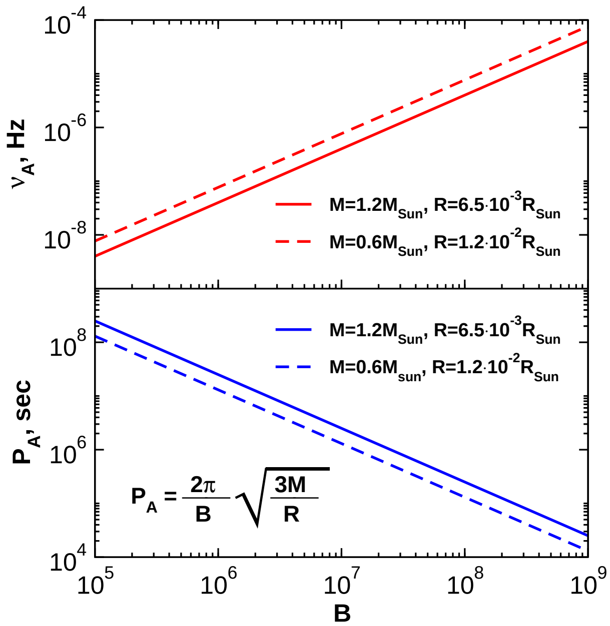

Fig. 3. Alfvén frequency and period of torsional magneto-mechanical oscillations of white dwarfs with indicated masses and radii as functions of intensity of internal magnetic field.

Lorentz-force-driven Alfvén vibrations of non-convective solid star are accompanied by decay of internal magnetic field. The characteristic frequencies and periods of such oscillations are ploted in Fig.3 for the solid-star model with typical for white dwarf masses and radii. The predicted elongation of vibration period should manifest itself in lengthening of period of oscillating optical and X-ray emission from white dwarf pulsating in a-mode and, hence, can be tested by observational means.

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