Quasi-biennial oscillations in the solar tachocline caused by magnetic Rossby wave instabilities

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ABSTRACT

Quasi-biennial oscillations (QBO) are frequently observed in the solar activity indices. However, no clear physical mechanism for the observed variations has been suggested so far. Here we study the stability of magnetic Rossby waves in the solar tachocline using the shallow water magnetohydrodynamic approximation. Our analysis shows that the combination of typical differential rotation and a toroidal magnetic field with a strength $\geq 10^5$ G triggers the instability of the $m = 1$ magnetic Rossby wave harmonic with a period of ~ 2 years. This harmonic is antisymmetric with respect to the equator and its period (and growth rate) depends on the differential rotation parameters and the magnetic field strength. The oscillations may cause a periodic magnetic flux emergence at the solar surface and consequently may lead to the observed QBO in the solar activity features. The period of QBO may change throughout the cycle, and from cycle to cycle, due to variations of the mean magnetic field and differential rotation in the tachocline.

Subject headings: Sun: oscillations —Physical Data and Processes: magnetic fields—MHD—waves

1. Introduction

Apart from the well known 11-year cycle, solar activity shows quasi periodic variations on shorter time scales. Two different time scales have been frequently observed in many solar

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activity indicators: several months and a few years. The oscillations with period of several months (mostly with 150-160 days) are known as Rieger-type periodicities (Rieger et al. 1984; Lean & Brueckner 1989; Carbonell & Ballester 1990; Oliver et al. 1998; Krivova & Solanki 2002; Kane 2005; Knaack et al. 2005). The oscillations with period $\sim$ 2 years are known as Quasi-Biennial Oscillations (QBO) and they modulate almost all indices of solar activity (Sakurai 1981; Gigolashvili et al. 1995; Knaack et al. 2005; Kane 2005; Danilovic et al. 2005; Forgács-Dajka & Borkovits 2007; Badalyan et al. 2008; Javaraiah et al. 2009; Laurenza et al. 2009; Vecchio & Carbone 2009; Vecchio et al. 2010; Sýkora & Rybak 2010).

The source(s) of these periodicities is still unclear. Several mechanisms have been suggested to drive the Rieger-type periodicities: interaction between $l = 2$ and $l = 3$ g-modes (Wolff 1983), the timescale for storage and/or escape of magnetic fields in the solar convection zone (Ichimoto et al. 1985), "clock" modeled by an oblique rotator (Bai & Sturrock 1991) and equatorially trapped Rossby-type waves in the photosphere (Lou 2000). Recently, Zaqarashvili et al. (2010) (hereinafter Paper I) suggested that the Rieger-type periodicities can be caused by unstable $m = 1$, two-dimensional ($\theta - \phi$ surface in spherical coordinates) magnetic Rossby waves in the solar tachocline. They show that a combination of the typical differential rotation parameters and the magnetic field strength $\leq 10^4$ G in the tachocline favor the strong growth of one particular harmonic with period of 150-160 days. The periodic modulation of the tachocline magnetic field due to the unstable harmonic triggers the periodic emergence of magnetic flux towards the surface, which leads to the observed periodicities in the solar activity. On the other hand, there is no clear mechanism for QBO reported in literature so far. Pataraya & Zaqarashvili (1995) supposed that the quasi 2-year impulse of shear waves can cause the 2-year periodicity of the differential rotation in the photosphere. However, this mechanism may work only near solar minima and cannot explain the long standing modulation of solar activity. Therefore, the question of the source for QBO is widely open.

In this letter, we show that the instability of magnetic Rossby waves in the tachocline could be the reason for QBO in solar activity. We consider a nonzero thickness of the tachocline and hence we use the shallow water magnetohydrodynamic (SWMHD) equations (Gilman 2000). We show that a stronger magnetic field ($\geq 10^5$ G) favors the growth of magnetic Rossby wave harmonic with period $\sim$ 2 years.

2. Shallow water MHD equations and unstable magnetic Rossby wave harmonics

In the following we use a spherical coordinate system (r, θ, ϕ) rotating with the solar equator, where r is the radial coordinate, θ is the co-latitude and ϕ is the longitude.

The solar differential rotation law in general is

$$\Omega = \Omega_0(1 - s_2\cos^2\theta - s_4\cos^4\theta) = \Omega_0 + \Omega_1(\theta), \tag{1}$$

where Ω_0 is the equatorial angular velocity, and s_2, s_4 are constant parameters determined by observations.

In the solar tachocline the magnetic field is predominantly toroidal, $\vec{B} = \Xi\hat{e}_\phi$, and we take $\Xi = B_\phi(\theta)\sin\theta$, where B_ϕ is in general a function of co-latitude. Then, the linear SWMHD equations (Gilman 2000) can be rewritten in the rotating frame (with Ω_0) as follows:

$$\frac{\partial u_\theta}{\partial t} + \Omega_1\frac{\partial u_\theta}{\partial \phi} - 2\Omega\cos\theta u_\phi = -\frac{g}{R_0}\frac{\partial h}{\partial \theta} + \frac{B_\phi}{4\pi\rho R_0}\frac{\partial b_\theta}{\partial \phi} - 2\frac{B_\phi\cos\theta}{4\pi\rho R_0}b_\phi, \tag{2}$$

$$\frac{\partial u_\phi}{\partial t} + \Omega_1\frac{\partial u_\phi}{\partial \phi} + \frac{u_\theta}{\sin\theta}\frac{\partial[\Omega\sin^2\theta]}{\partial\theta} = -\frac{g}{R_0\sin\theta}\frac{\partial h}{\partial\phi} + \frac{B_\phi}{4\pi\rho R_0}\frac{\partial b_\phi}{\partial\phi} + \frac{b_\theta}{4\pi\rho R_0\sin\theta}\frac{\partial[B_\phi\sin^2\theta]}{\partial\theta}, \tag{3}$$

$$\frac{\partial b_\theta}{\partial t} + \Omega_1\frac{\partial b_\theta}{\partial\phi} = \frac{B_\phi}{R_0}\frac{\partial u_\theta}{\partial\phi}, \tag{4}$$

$$\frac{\partial}{\partial\theta}(\sin\theta b_\theta) + \frac{\partial b_\phi}{\partial\phi} + \frac{B_\phi\sin\theta}{H_0}\frac{\partial h}{\partial\phi} = 0, \tag{5}$$

$$\frac{\partial h}{\partial t} + \Omega_1\frac{\partial h}{\partial\phi} + \frac{H_0}{R_0\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta) + \frac{H_0}{R_0\sin\theta}\frac{\partial u_\phi}{\partial\phi} = 0, \tag{6}$$

where u_θ, u_ϕ, b_θ and b_ϕ are the velocity and magnetic field perturbations, H_0 is the tachocline thickness and h is its perturbation, ρ is the density, g is the reduced gravity and R_0 is the distance from the solar center to the tachocline. Eqs. (5)-(6) are the solenoidal conditions of shallow water magnetic field and velocity respectively (Gilman 2000).

Fourier analysis with $\exp[im(\phi - ct)]$ and the transformation of variables $\mu = \cos\theta$ in Eqs. (2)-(6) lead to the equations

$$[(c-\Omega_1)^2 - \Omega_A^2](1-\mu^2)\frac{\partial H}{\partial\mu} - 2\mu[\Omega(c-\Omega_1) + \Omega_A^2]H = -im\Omega_g^2 h + im(1-\mu^2)[(c-\Omega_1)^2 - \Omega_A^2]h, \tag{7}$$

$$2\mu[\Omega(c-\Omega_1)+\Omega_A^2](1-\mu^2)\frac{\partial H}{\partial\mu} - \left[m^2(c-\Omega_1)^2 - m^2\Omega_A^2 + \mu(1-\mu^2)\frac{\partial\Omega^2}{\partial\mu} - \mu(1-\mu^2)\frac{\partial\Omega_A^2}{\partial\mu}\right]H =$$

$$= im\Omega_g^2(1-\mu^2)\frac{\partial h}{\partial \mu} + 2im\mu(1-\mu^2)[\Omega(c-\Omega_1)+\Omega_A^2]h, \tag{8}$$

where

$$\Omega_A = \frac{B_\phi}{\sqrt{4\pi\rho}R_0}, \;\; \Omega_g = \frac{\sqrt{gH_0}}{R_0},$$

are the Alfvén and surface gravity frequency respectively, h is normalized by H_0 and

$$H(\mu) = \frac{b_\theta(\mu)\sqrt{1-\mu^2}}{B_\phi} = \frac{u_\theta(\mu)\sqrt{1-\mu^2}}{R_0(c-\Omega_1)}. \tag{9}$$

In the remaining we use a magnetic field

$$B_\phi = B_0\mu, \tag{10}$$

which changes sign at the equator (Gilman & Fox 1997).

We expand H and h in infinite series of associated Legendre polynomials (Longuet-Higgins 1968)

$$H = \sum_{n=m}^{\infty} a_n P_n^m(\mu), \;\; h = \sum_{n=m}^{\infty} b_n P_n^m(\mu), \tag{11}$$

which satisfy the boundary conditions $H = h = 0$ at $\mu = \pm 1$ (i.e. at the solar poles). Then, we substitute Eq. (11) into Eqs. (7)-(8) and, using a recurrence relation of Legendre polynomials, we obtain algebraic equations as infinite series. The dispersion relation for the infinite number of harmonics can be obtained when the infinite determinant of the system is set to zero. In order to solve the determinant, we truncate the series at $n = 70$, and the resulting polynomial in ω is solved numerically. This gives the frequencies of the first 70 harmonics. The harmonics with real frequency are stable, while those with complex frequency are unstable (see the general technique of Legendre polynomial expansion in Longuet-Higgins (1968); Watson (1981); Gilman & Fox (1997); Zaqarashvili et al. (2010) and references therein).

The typical values of equatorial angular velocity, radius and density in the tachocline are $\Omega_0 = 2.7 \cdot 10^{-6}$ s^{-1}, $R_0 = 5 \cdot 10^{10}$ cm and $\rho = 0.2$ g $\cdot$ cm^{-3} respectively. We use a tachocline thickness $H_0 = 0.02R_0 = 10^9$ cm. The ratio between angular and surface gravity frequencies $\epsilon = \Omega_0^2/\Omega_g^2 = \Omega_0^2 R_0^2/(gH_0)$ is an important parameter in the shallow water theory. $\epsilon \ll 1$ means strongly stable stratification (main part of tachocline), while $\epsilon \gg 1$ considers weakly stable stratification (upper overshoot region). Here we consider the mean part of the tachocline and thus we take the limit $\epsilon \ll 1$.

The observed differential rotation parameters near the solar surface satisfy $s_2+s_4 \approx 0.28$, which may tend to $s_2 + s_4 \approx 0.26$ near the upper part of the tachocline (Schou et al. 1998).

The solar radiative interior rotates uniformly, therefore the latitudinal differential rotation parameters drop to zero from the upper part of tachocline to its base. The radial dependence of latitudinal differential rotation through the tachocline is not clear, and s_2, s_4 may also vary throughout the solar cycle (Howe et al. 2000). Therefore, s_2+s_4 may take any value between 0.26 and 0.

Figure 1 shows the real, mc_r, and imaginary, mc_i, frequencies of all $m = 1$ unstable harmonics for different combinations of ϵ (i.e. reduced gravity) and magnetic field strength. The differential rotation parameters are fixed to $s_2 = s_4 = 0.11$. The plot shows that each combination of the magnetic field strength, differential rotation parameters and reduced gravity leads to the occurrence of a particular harmonic whose growth rate is much stronger than that of other harmonics. This is similar to what happens in the two-dimensional case (Zaqarashvili et al. 2010). An increase of magnetic field strength leads to the reduction of the frequency of the most unstable harmonic. The unstable harmonics are mostly symmetric (defined by asterisks) with respect to the equator for a magnetic field strength $< 10^4$ G, while they become mostly antisymmetric (defined by circles) for a strength $> 10^5$ G. A magnetic field strength between $10^4 - 10^5$ G yields unstable harmonics for both symmetries. This can be explained in terms of magnetic and differential rotation energies. Equipartition between the magnetic energy and the kinetic energy of differential rotation occurs at $\sim 5 \cdot 10^4$ G for $s_2 = s_4 = 0.11$. When the magnetic field strength is smaller, then the differential rotation is the main energy source for instability and this obviously yields the symmetric harmonics as the differential rotation is symmetric around the equator. However, when the magnetic field is stronger, then the magnetic energy is the main source for the instability and the unstable harmonics are antisymmetric as the magnetic field is antisymmetric with respect to the equator.

The importance of the equipartition value of the magnetic field strength is clearly seen on Figure 2. This figure displays the periods and growth rates (defined as mc_i/Ω_0) vs magnetic field strength. The growth rates are higher for weaker ($< 10^4$ G) and stronger ($> 10^5$ G) magnetic fields. However, the growth rates are much lower when the magnetic field strength is inside the interval $10^4 - 10^5$ G. The weaker field ($< 10^4$ G) favors Rieger-type periodicities (150-160 days), while the stronger field ($> 10^5$ G) supports QBO. Increasing the magnetic field suppresses symmetric harmonics as it has been shown in Paper I.

Figure 3 displays the period of the most unstable symmetric and antisymmetric harmonic vs the value of reduced gravity (i.e. on ϵ) for a magnetic field strength of $8{\cdot}10^4$ G and the differential rotation parameters $s_2 = s_4 = 0.11$. It is seen that the oscillation period does not depend significantly on the reduced gravity.

Figure 4 displays the dependence of the periods of the most unstable symmetric and antisymmetric harmonics on the differential rotation parameters for a magnetic field strength of $8{\cdot}10^4$ G and for $\epsilon = 0.12$ (corresponding to a reduced gravity of $1.5 \cdot 10^2$ cm s^{-2}). The upper panel (circles) displays the antisymmetric harmonics and the lower panel (asterisks) displays the symmetric ones. The periods of unstable harmonics vs s_4 are plotted for different values of s_2. The values of s_2 vary from 0.13 (blue color) to 0.09 (red color). We can observe that the period of this harmonic depends on the differential rotation parameters significantly and takes values between 400-700 days. The period becomes shorter for stronger differential rotation.

3. Discussion

Our results show that the differential rotation and the magnetic field with a strength of $> 10^5$ G may lead to large-scale oscillations of tachocline with periods of ~ 2 years. The oscillation is due to the $m = 1$ unstable mode of magnetic Rossby waves. The magnetic Rossby waves are magnetohydrodynamic counterparts to usual hydrodynamic Rossby waves (Zaqarashvili et al. 2007, 2009). The period and growth rate of the unstable harmonics depend on the magnetic field strength and the differential rotation parameters (Figures 1, 2 and 4). The unstable harmonics with periods of ~ 2 years are antisymmetric with respect to the solar equator.

The unstable magnetic Rossby waves in the tachocline can be the reason for QBO observed in almost all indices of the solar activity. Recent papers suggest that QBO are not persistent but may vary from cycle to cycle (Vecchio & Carbone 2009) and throughout a cycle (Sýkora & Rybak 2010). Our analyses also suggest this behaviour as the period of unstable harmonics depends on magnetic field strength and differential rotation parameters, which may vary in time depending on phase and strength of a particular cycle.

The antisymmetric behaviour of unstable harmonics with respect to the equator may explain the recent observational results of Badalyan et al. (2008), which show that QBO are well recognizable in the N-S asymmetry of solar activity indices.

Our analysis suggests the reduction of growth rates of unstable harmonics when the magnetic field strength is inside the interval $10^4 - 10^5$ G (see Figure 2). It is clearly seen that the relatively weak magnetic field $< 10^4$ G leads to the occurrence of Rieger-type periodicities (see the same results in the paper I), while the field of $> 10^5$ G favors QBO. The upper overshoot region of the tachocline probably contains relatively weaker magnetic field comparing to the lower stable layers. Therefore, we may speculate that the Rieger-type periodicities are formed in the overshoot layer (this was also suggested in the paper

I), while QBO are formed in lower layers with strongly stable stratification. Therefore, the both periodicities may occur simultaneously.

The magnetic field of 10^5 G is unstable due to the buoyancy instability, which makes difficult to keep it in the tachocline. On the other hand, the emergence of magnetic flux at observed latitudes requires sufficiently strong magnetic field ($\sim 10^5$) below the convection zone (Fan 2004). Therefore, the storage of strong fields below the convection zone is still open question.

Significant simplifications in our approach is the linear stability analysis, which only describes the initial phase of instability. Intense numerical simulations are needed in the future to study the developed stage of magnetic Rossby wave instabilities.

4. Conclusions

We have shown that the unstable magnetic Rossby waves in the solar tachocline could be responsible for the observed intermediate periodicities in solar activity. The periods and growth rates of unstable harmonics depend on the differential rotation parameters and the magnetic field strength. The unstable harmonics are either symmetric or antisymmetric with respect to the equator. The latitudinal differential rotation is mainly responsible for the growth of symmetric harmonics, while, the antisymmetric toroidal magnetic field favors the growth of antisymmetric harmonics. A magnetic field with a strength $\leq 10^4$ G leads to oscillations with shorter period (150-170 days), while a stronger magnetic field $\geq 10^5$ G favors oscillations with longer periods (1-2.5 yrs). Hence, $\sim$ 2-year oscillations can be formed in the main part of the tachocline with stronger toroidal magnetic field and strongly stable stratification. The oscillations may trigger the periodic magnetic flux emergence at the solar surface and consequently QBO in solar activity.

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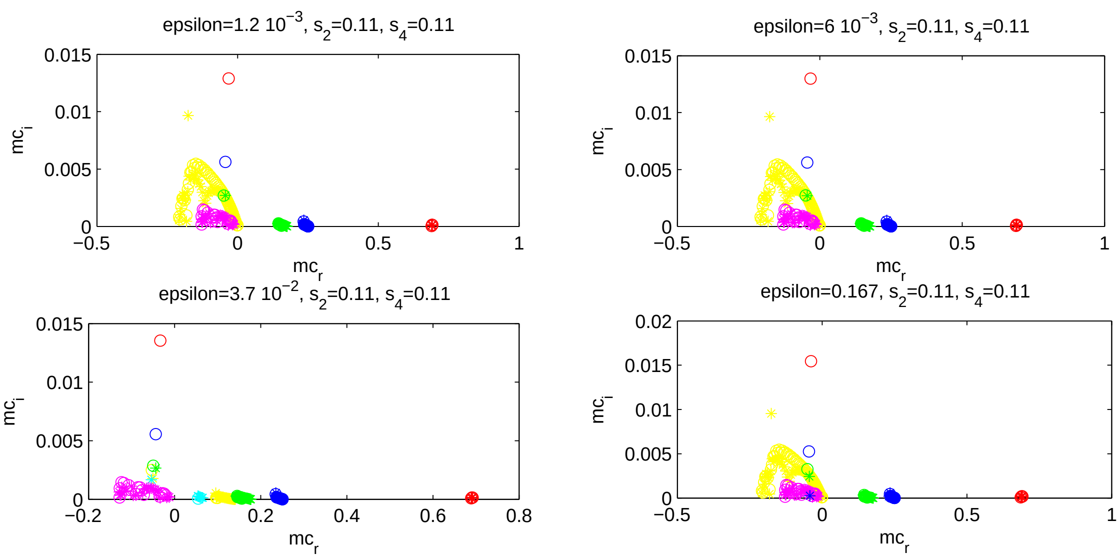

Fig. 1.— Real, mc_r, and imaginary, mc_i, frequencies of all $m = 1$ unstable harmonics for different combinations of ϵ (i.e. reduced gravity) and magnetic field strength. The differential rotation parameters are fixed to $s_2 = s_4 = 0.11$ for all four panels. The reduced gravity varies from $1.5\cdot 10^4$ cm s^{-2} ($\epsilon = 1.2\cdot 10^{-3}$, upper left panel) to $1.1\cdot 10^2$ cm s^{-2} ($\epsilon = 0.167$, lower right panel). Yellow, magenta, blue, green, dark blue and red colors correspond to magnetic field strengths of $2\cdot 10^3$ G, $2\cdot 10^4$ G, $6\cdot 10^4$ G, $8\cdot 10^4$ G, 10^5 G, and $2\cdot 10^5$ G, respectively. Asterisks (circles) denote the symmetric (antisymmetric) harmonics with respect to the equator.

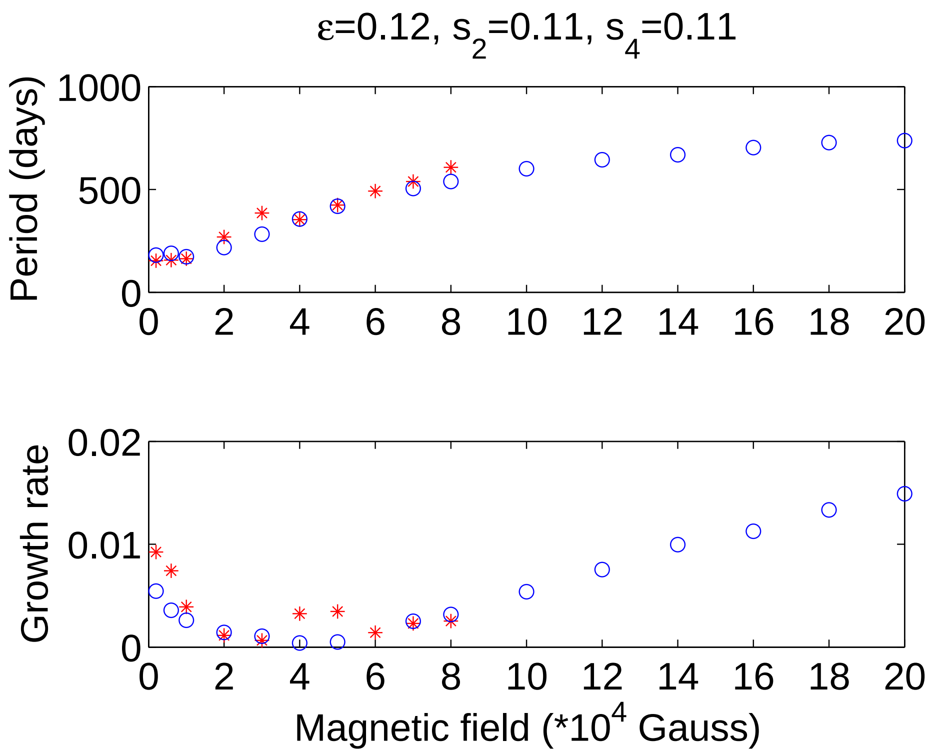

Fig. 2.— Period (upper panel) and growth rate mc_i/Ω_0 (lower panel) of the most unstable harmonics vs magnetic field strength. Asterisks (circles) define symmetric (antisymmetric) harmonics. Here ϵ=0.12 and the differential rotation parameters are $s_2 = s_4 = 0.11$.

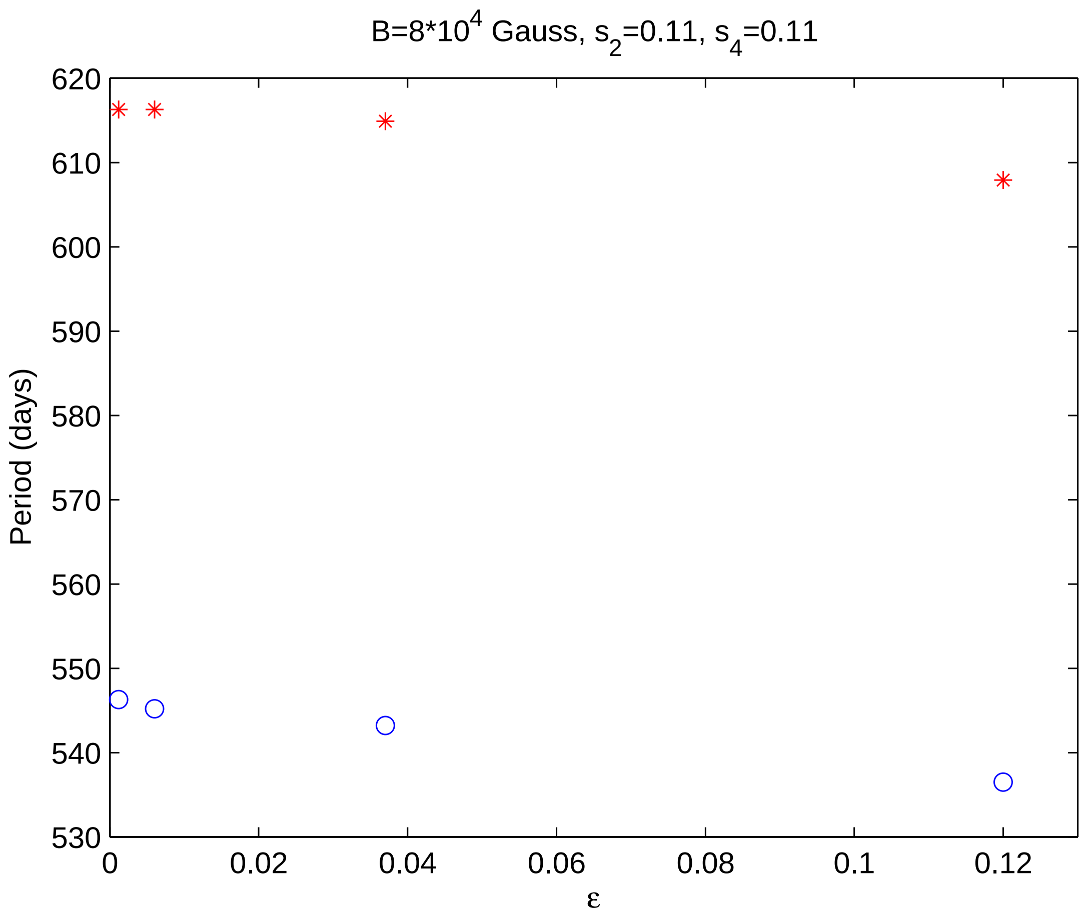

Fig. 3.— Periods of the most unstable symmetric (asterisks) and antisymmetric (circles) harmonics vs ϵ for a magnetic field strength of $8{\cdot}10^4$ G and the differential rotation parameters $s_2 = s_4 = 0.11$.

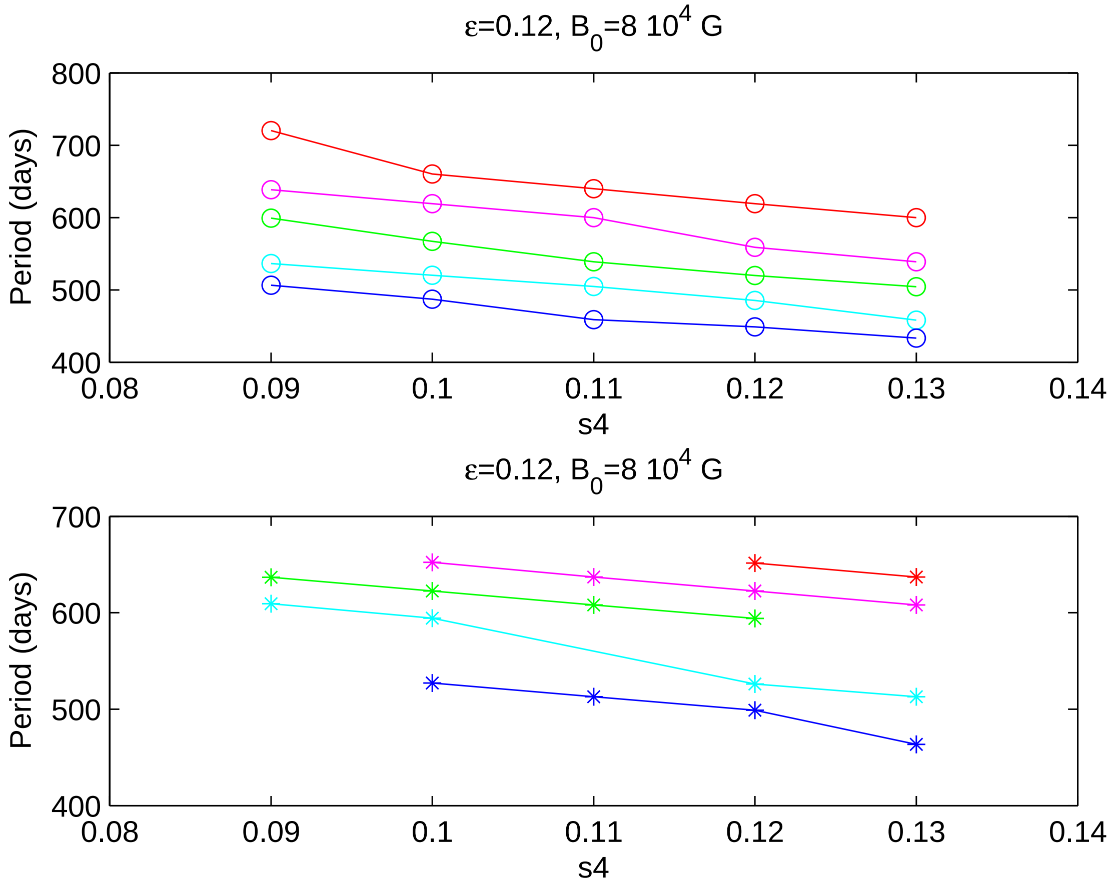

Fig. 4.— Periods of the most unstable symmetric (asterisks) and antisymmetric (circles) harmonics vs s_4 for different values of s_2. Dark blue, blue, green, magenta and red colors correspond to s_2=0.13, 0.12, 0.11, 0.10 and 0.09 respectively. The magnetic field strength equals $8{\cdot}10^4$ G and $\epsilon = 0.12$.